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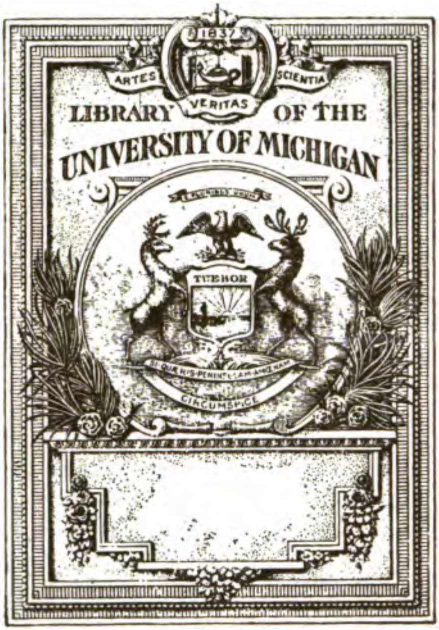

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Bath 19th Nov 1748

Philoſophia Britannica:

OR

A New and Comprehensive

SYSTEM

OF THE

Newtonian PHILOSOPHY, ASTRONOMY and GEOGRAPHY.

IN A

Course of Twelve LECTURES,
With NOTES;
CONTAINING
The Physical, Mechanical, Geometrical, and
Experimental Proofs and Illustrations
of all the Principal Propoſitions in every Branch of
NATURAL SCIENCE.
ALSO
A particular Account of the Invention, Structure,
Improvement and Uses of all the conſiderable
Instruments, Engines, and Machines,
With *new Calculations* relating to their
Nature, Power, and Operation.
The Whole collected and methodized from all the principal
Authors, and public Memoirs to the preſent Year;
And embelliſh'd with Seventy-five COPPER-PLATES.

By B. MARTIN.

Quæ toties Animos veterum torſêre Sophorum
Quæque Scholas fruſtra rauco Certamine vexant,
Obvia conſpicimus, Nubem pellente Matheſi. Hal. in Newt. Prin.

In Two Volumes.

VOL. I.

READING,
Printed by C. Micklewright and Co. for the AUTHOR;
and for M. Cooper in *Pater-noſter-row, London*; R. Raikes
at *Glouceſter*; B. Collins at *Saliſbury*; and J. Leake, and
W. Frederick at *Bath.* MDCCXLVII.

TO THE
RIGHT HONOURABLE
Sir William Lee, *Knt.*
Lord Chief Juſtice of His Majeſty's Court of King's Bench, *and one of His Majeſty's moſt Honourable Privy Council.*

My LORD,

AS *Britain* is the *Birth-Place* of PHILOSOPHY, ſo *Engliſh* is its *vernacular Tongue,* and

Rec'd 1-16-37 HRW

the NEWTONIAN *Matheſis* its peculiar and diſtinguiſhing Dreſs. In this, with all its native Elegance and Majeſty, it attends on YOUR LORDSHIP for Protection; perſuaded that He, whom the Wiſdom of the Legiſlature has conſtituted JUDGE of the *Laws of Nations* and *Rights of Mankind*, (which are founded in the *Primary Law of* REASON) muſt be particularly qualified and diſpoſed to patronize a SYSTEM of the *Secondary Laws*, and a *faint* DELINEATION of the DIVINE GEOMETRY of *Nature*.

IT is not, MY LORD, that *Philoſophy* has any thing to apprehend from the Attacks of *Ignorance* and *Pedantry*, that it thus recurs to

Your Lordship: All true *Science*, and every *Liberal Art*, is far enough above Contempt and Ridicule. The only Reaſon is, the Ambition of having the Honour of a Patron in ſo *great*, ſo *wiſe*, and ſo *good* a Man, as My Lord Chief Justice *LEE*.

It was with the higheſt Satisfaction and Pleaſure, that I had the Honour to obſerve, how nicely the *ſublime Speculations* and *exalted Divertiſements* of *Philoſophy* were ſuited to Your Lordship's Taſte and Turn of Mind; and how ſtrongly were confeſs'd in each conſcious Lineament the Pleaſure and Delight that aroſe from a near Inſpection of the manifold Inſtances of *Creating Power and Wiſdom*.

'Tis true, My Lord, Knowledge is now become not only *reputable*, but even *faſhionable*, among the *Great*. Many now think it no Diſgrace to be wiſe and learned: We can now look up into High Life, and view *Patriots* and *Patrons*, *Miniſters* and *Mecænas's*, in the ſame Perſons: We can now boaſt our Chesterfields and Richmonds, our Orreries and Macclesfields, and many other illuſtrious Perſonages, among the *Nobles*, the *Judges*, and *Rulers* of the Land, who are nevertheleſs the Fautors *of Learning* in general, and in particular the Aid *and* Ornament *of Philoſophy*.

Yet, after all, there is but too much Reaſon to wiſh theſe great

Exemplars were more common. 'Tis ſad to obſerve how few, like YOUR LORDSHIP, live up to the Dignity of Human Nature, and their exalted Stations in Life; that ſoar above the ſordid Pleaſures of mere *Animal Senſation*, and the *low Amuſements of Vulgar Minds*; that can reliſh and be delighted with the Maxims and Dictates of *Philoſophy*, *Religion*, and *Reaſon*; and, laſtly, that have Pleaſure in *ſearching out the Works of the Lord*, and *contemplating the Operations of his Hands*.

THAT *Health* and *Longevity* may compleat the Bleſſings of Wiſdom to YOUR LORDSHIP and your auſpicious Off-ſpring, (whoſe peculiar Felicity it is to enjoy not only Riches and Honour, but Wiſdom and Virtue by Inheritance) is the ſincere Prayer of

May it pleaſe YOUR LORDSHIP,

YOUR LORDSHIP'S *moſt dutiful*

and moſt obedient Servant,

B. MARTIN.

PREFACE.

HAVING read and conſider'd the Deſign of the ſeveral Books hitherto publiſh'd for the Explanation of the NEWTONIAN PHILOSOPHY, under the Titles of *Commentaries*, *Courſes*, *Eſſays*, *Elements*, *Syſtems*, &c. I obſerved not one of them all could be juſtly eſteem'd a TRUE SYSTEM, or COMPLEAT BODY of this Science.

I WOULD not here be underſtood to have any Deſign of depreciating the Labours of thoſe Gentlemen who have gone before me on this Subject; I only would intimate that their Views and Deſigns were very different from mine, which is to preſent the World with a PHYSICO-MATHEMATICAL TREASURY of NATURAL KNOWLEDGE, upon the Principles of the *Newtonian Philoſophy*; which I have been chiefly enabled to effect from the Abundance of Materials ſupplied by the Authors of the Treatiſes in the following Catalogue, each of which is excellently good in his Way.

SO that if the Caſe be truly ſtated, my Book may be look'd upon as a *General Compendium*

pendium or *Abridgment* of all theirs. For as they have all lain before me, I have conſtantly collated them upon every particular Subject, as I proceeded through the Work, and ſelected the beſt of every Thing I could find for my own Compoſition.

Hence it is, that where ſome of them have been *very prolix*, my Labour has been to abridge or reduce them to a Compendium of Principles and Eſſential Propoſitions only. Thus *Keill* on the Subject of *Motion* and *Aſtronomy*, *Muſchenbroek* on *Cohesion*, Sir *Iſaac Newton* on *Optics*, &c. have all been treated.

As Redundancy on the one Hand has been avoided, ſo their Deficiencies on the other have been ſupplied. Thus *Keill*'s Introduction to *Philoſophy* relates to the Mechanical Part only. Profeſſor *Muſchenbroek* has wholly omitted the grand Article of *Aſtronomy*, and the *Phyſical Cauſes* of the *Celeſtial Motions* and *Phænomena*. Dr. *Deſaguliers* has thought fit to omit the moſt eſſential Science of *Optics*, and is almoſt wholly taken up on Mechanical and Hydroſtatical Subjects. Yea, the beſt and moſt general Writer *S'Graveſande*, in his laſt large Edition of his *Elements*, falls far ſhort of a general Syſtem or Body of *Philoſophy*, in various important Subjects, as will eaſily appear, by comparing his Table of Contents with mine.

I TAKE no notice of leſſer Tracts, ſince few of them pretend to any Thing ſyſtematically; and they who do, propoſe it with reſtricted Views, either to the Theory only, or elſe to ſome principal Parts or Branches of the Science. Thus *Worſter*'s *Principles of Philoſophy* amount to little more than a Rehearſal of *Theſes*, with ſcarce any Demonſtrations throughout. *Helſham*'s *Lectures* are admirably good as far as they go. And the Lectures, lately publiſh'd by the *Abbé Nollet*, are but a Part of his Work, and are indeed excellent Philoſophical Sermons or Declamations, and ſtrictly in the *French Style and Taſte*[*].

BUT what others have done is not ſo much the Deſign of this Preface to relate, as what I have done Myſelf in this Treatiſe, which I ſhall ſpecify under the following Heads.

FIRST, I have endeavour'd to compile a *Compleat Syſtem of Philoſophy*, from the great Fund of Materials which the Writings and Inventions of the Moderns have ſupplied me with. Under this Head I have had regard only to the Choice and Propriety of the Subjects; and their Order and Diſpoſition, or due Arrangement in the Body of the Work. Hence it is that I have admitted nothing that is not of a Phyſical Nature, and omitted nothing that is ſo, as far as the Limits of my preſcribed Form would admit me to go. Nor are theſe ſo ſtrait, as not to be comprehenſive of every material Branch of natural Science, or conſiderable Phænomenon, as will eaſily appear, from a View of the *Contents* and *Index*.

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[*] The Lord *Roſcommon* obſerves,

The weighty *Bullion* of *one Sterling Line*,
Drawn to *French Wire*, would thro' *whole Pages ſhine*.

SECONDLY, What has been wrote upon each Subject in every Part of the Science has been carefully examin'd, and ſuch an Extract made as was judged to give a ſufficient Idea or Notion thereof to the Reader. And where I thought a more extenſive View could be of Service, I have conſtantly referr'd to the beſt Authors on the Subject.

THIRDLY, If from the whole Account, given by others, of any particular Branch of Science, I could not collect what ſufficed for an entire Sketch, I have endeavour'd to ſupply thoſe Deficiencies, in order to render every Part compleat, and ſave the Reader the Trouble of turning to ſeveral Authors for that Purpoſe. Remarkable Inſtances of this may be found under the Articles of *Wheel-Carriages*, *Hydraulics*, *Optics*, &c.

FOURTHLY, If the Method of repreſenting or demonſtrating any Subject or Propoſition by others has not been judged ſo clear, conciſe, or natural as was neceſſary, I have propoſed it in my own way, to anſwer this important End; For the greateſt Excellency of a Demonſtration is Conciſeneſs and Perſpicuity. The Want of this is a great Diſcouragement to Phyſical Enquiries, as I am but too well convinced by my own Experience, and that of many others.

Fifthly, According to the different Nature of the Subject, a various Proceſs has been made uſe of to explain or illuſtrate the ſame. Thus ſome Things require a *Phyſical*, others a *Mechanical*, ſome a *Geometrical*, and ſome an *Experimental Proof*, which have been accordingly applied, ſometimes ſingly, ſometimes two or more, and ſometimes all together. Thus from *Phænomena* we inſtitute a Ratiocination concerning the Materiality, the Velocity, different Magnitudes, Colours, and other Affections of the Particles of Light. Then the Laws of Reflection and Refraction are explained by *Mechanical Principles* relating to the Motion and Action of Bodies one upon another. Again, The *Principles of Geometry* are call'd in to explain and account for the Effects of Mirrours, Lenſes, Optic Inſtruments, the Rainbow, *&c.* And, Laſtly, thoſe various Phænomena are repreſented, and the Manner how they happen explain'd by *Experiments.*

Sixthly, I have all along applied the *Newtonian Geometry*, that is, the *Fluxionary Calculus*, for the Demonſtrations, which I think is with the greateſt Propriety and

Juſtneſs in Phyſical Matters, becauſe the Principles of this Geometry are in themſelves ſtrictly Philoſophical, as I have ſhewn in its proper Place. Beſides, the Proceſs is not only render'd by this means much more congruous and natural, but more evident and conciſe than by the common Methods of Demonſtration it could be. Of this we have a ſignal Inſtance in the Calculation of the Angle, which the incident Ray makes with the laſt refracted Ray in a Drop of Water, as will appear by comparing what is here contain'd in *Scholium* to *Annot.* CXXIV. with what you find on this Head in other Authors.

Seventhly, As the Bounds of this Science have been of late Years vaſtly extended, ſo I have been very careful to treaſure up every uſeful Invention, and given a ſhort hiſtorical Account of the ſame, and the Names of the Authors. If in any of them I found an Improvement could be made, I have done it, as in the *Air-Pump*, the *Microſcope*, &c. or hinted it to the Reader, as in caſe of the *Air* or *Steam-Gun.*

Eighthly, I find by common Experience nothing tends more to the Conception and enlivening an Idea, and fixing it in the Memory, than a good Delineation of the Figure or Form thereof. Hence it is that the Reader will here find a greater Number of Copper-Plate Figures than in any other Treatiſe of this Kind, of its Size.

And if I tell him they were all done by the ingenious Mr. *Bowen* (Geographer to his Majeſty) it may likewiſe prepoſſeſs him in Favour of their Goodneſs; which though an indirect and collateral, is yet no inconſiderable Argument to recommend a Book of this Kind.

NINTHLY, I have deliver'd the Whole by way of LECTURES and ANNOTATIONS thereto, (a Method of late Years the moſt uſed and approved of) not only becauſe the Reader is by this Means led through a Syſtem of Philoſophy, without being confin'd to the Irkſomeneſs of a preciſe and regular Path, and a ſimultaneous Proſpect or View of the Whole at once; but alſo becauſe that vaſt Variety of Things, which here preſent themſelves, may have the Pleaſure that attends Variety heighten'd and increaſed by emerging freſh to his View as he paſſes along, and regaling him with ſomething ſtill novel and unexpected. *Annotations* ſeem to anſwer the End of *Scenes* in a Play; they preſent the Whole moſt agreeably in Parts, which thus more immediately affects us, and gives us the greateſt Pleaſure and Entertainment. I need not here mention the Conveniency of this Method alſo to thoſe Gentlemen and Ladies who attend my COURSES OF LECTURES.

TENTHLY, With Reſpect to the Size of the Print, it is ſmall, yet fair and legible to a common Eye; a larger Print was incompatible with our View, which was to compriſe a Body of Philoſophy in the Compaſs of two Pocket Volumes. And the Reader will not be diſagreeably ſurprized when he is told, that he buys more in two Volumes 8vo now, then he uſed to have in two large Volumes in 4to, and at leſs than half the Price. I believe few will blame me for this Contrivance, beſides thoſe who buy Books more on account of their Size and Appearance, than for what they contain, and who have a greater Regard to the Pomp and Magnificence of a Library, than to the Improvement of a vain and barren Mind with ſublime and uſeful Knowledge. But for ſuch I write not, nor regard their Cenſure.

THUS much for the Oeconomy of the Work; and as it is manifeſtly calculated for a general Good, ſo I believe none will find fault with the Deſign. As to the Manner of executing it, I have done it as well as I could; and if any one thinks me deficient in any thing, let him qualify his Criticiſm by conſidering that I pretend not to Infallibility, but only where I give a plain and clear Demonſtration. In this, if I am not underſtood ſometimes, I am not perhaps to be blamed; for I'll appeal to Envy itſelf, if the Manner of the Demonſtration, where moſt difficult, be not as plain and as obvious as the Nature of the Thing will admit. And here it will be proper to obſerve, that although I have made uſe of *Geometry*, *Algebra*, and *Fluxions*, to inveſtigate the Truth of all the principal Propoſitions in Philoſophy, yet as I cannot ſuppoſe Readers in general qualified to underſtand it in that Manner, I have been careful to expreſs the Reſult of every Proceſs in plain *Engliſh*; ſo that every one may know what is the Truth, though he cannot be ſo properly ſaid to underſtand it. For Inſtances of what I now ſpeak of ſee the ſeveral Articles of *Annotat.* XXVII, XXXIV, and XLVIII.

I DO not know that I have advanced any thing here, which I have not one Way or another proved to be true; however, I deſire that nothing but what is demonſtrated may be accounted to deſerve the Title of NEWTONIAN PHILOSOPHY. Our Great Author ſeem'd quite averſe to *Hypotheſes*; and though he indulged them to others under proper Reſtrictions, yet did himſelf never make uſe of this *fallacious Way of Reaſoning*, though he has been unjuſtly charged therewith by ſome, who, by *National Inſtinct*, ſcorn to ſubſcribe to any Syſtem but of their own making.' Hence it is we ſo often hear them babbling about *Newton's Hypotheſis*, &c. and making unſufferable Compariſons between him and *Deſcartes* and *Leibnitz*. One would think they who knew but little of Philoſophy would yet ſee no Reaſon for a Parallel between a Syſtem

of

of Philoſophy that is founded on nothing but *Demonſtration*, and is every Way comportant with *Phænomena*, and one that is wholly *hypothetical*, and, inſtead of accounting for, runs counter to every thing that appears! I think it is little Glory, (not to ſay *Vain-Glory)* for a Man to boaſt that his Philoſophy *is not that of Newton* *, till he has firſt ſatisfied the World his Abilities and Diſcernment into the Nature of Things are greater than thoſe of Sir *Iſaac*. This I am well aſſured of, that the greater Share of *Mathematical Knowledge* any Man has, the more readily he embraces Sir *Iſaac*'s Principles; and I ſcarce ever found any that oppugn'd Sir *Iſaac*, but who were either groſsly ignorant of what they deſpiſed, or were actuated by ſome particular Views. In ſhort, nothing but *Pride*, *Prejudice*, and *Ignorance*, will ever ſtir up Enemies to the *Newtonian Philoſophy*.

THEY who depreciate this Philoſophy by the Term *Hypotheſis*, ſeem to me not to have read (at leaſt not conſider'd) what Sir *Iſaac* has wrote, when he tells us, "That in Experimental Philoſophy, Hypotheſes are not to be regarded.—What I call ATTRACTION may be perform'd by IMPULSE, or by ſome OTHER MEANS unknown to me. I uſe that Word to ſignify *only in general* ANY FORCE *by which Bodies tend towards one*

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* See the Preface of Abbot NOLLET's *Leçons de Phyſique.*

“ *another*, whatſoever be the Cauſe.”* After our illuſtrious Author had thus expreſsly diſclaim'd all Hypotheſes, and ſo fully explain'd his Sentiments about the Power which he calls *Attraction*, is it not very wonderful to hear a Set of People *charging him with founding his Philoſophy on the Hypotheſis of Gravity?* Will theſe very ſagacious Gentlemen allow *any Power at all* is concern'd in the *Tendency* of one Body to another? If they do, is it not becauſe it is impoſſible to conſider Bodies acting at a Diſtance upon each other without? If ſo, is not the Exiſtence of ſuch a Power a Reality in Nature? And can ſuch a Force then be call'd an *Hypotheſis?* Surely not. Again, their accuſing Sir *Iſaac* for accounting for Things *by Attraction*, is meer cavilling about Words, and begging him guilty; he declares he knows not the Manner in which that Force acts, whether by *Attraction* or *Impulſion*, or otherwiſe; but as ſome Name muſt be given it, why may not he call it by that of *Attraction* or *Gravity*, as well as by any other? Not that this great Man did even this without a Reaſon; for 'tis evident from a thouſand Experiments and Phænomena, that this is much more like the Effect of one Body's *drawing* than *impelling* another towards it.

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* OPTICS, *Qu.* 31. See alſo the Definitions of his *Principia*.

But ſo far is Sir *Iſaac* from *ſuppoſing* that Bodies tend to each other by ATTRACTION, that he once condeſcended to form an *Hypotheſis* to ſhew that it might be done by IMPULSION, and that of a very ſubtle Elaſtic Spirit or Æther. But as he was not able to prove the Exiſtence of any ſuch Thing, he ſeem'd not at all delighted with the Thought, nor ever laid any Streſs upon it; and accordingly we have not admitted it as a Principle of the *Newtonian* Philoſophy. Beſides, ſuppoſe this could be proved, we ſhould ſtill be at a Loſs to account for the *material Cauſe* of this very great Elaſtic Power between the Particles of this Æther; we ſhould indeed advance three or four Steps higher on the infinite Ladder, but ſhould be equally *non-plus'd* with the mechanical Cauſe of *Elaſticity* then, as we are at preſent with that of *Gravity*. But after all, 'tis evident by Experiments that this Power which actuates Bodies, whatever it be, is not either *Attractive* or *Repulſive* ſolely; but it produces both the different Effects in *different Circumſtances of Diſtances*. Thus the Loadſtone at one Diſtance moves the Needle *towards itſelf*, at another Diſtance it cauſes it *to recede* or move *the contrary Way*. This Power in Iron is attractive, if touch'd upon the Stone in one Direction; but if in a contrary, it becomes repulſive. The ſame ambiguous Properties of this Power obtain in *Electricity*, and undoubtedly in all other Sorts of Attractions. Therefore, before we ſet ourſelves about finding out a Power, Spirit, or Æther, that ſhall move Bodies either by *Attraction* or *Repulſion*, it is in the firſt Place neceſſary to find out an Agent that ſhall do both, for ſuch a Principle is at preſent the *Deſideratum* to our further Advances in the Reſearch of natural Cauſes.

It is ſometimes ſurprizing, to obſerve, how very fond People ſeem of this *ſubtle Æther*; one accounts for the *Cauſe of Gravity* thereby, another for *Muſcular Motion*, a third for *Electricity*, a fourth from hence derives *Animal Spirits*, a fifth *Elementary Fire*: And, in ſhort, we want nothing but the *Proof of ſuch an Æther* to account for almoſt every Thing by it. But is it not a prepoſterous Proceeding to account for any Thing by a Principle in itſelf unaccountable, and utterly unknown?

However, we are arriv'd at great Dexterity ſince Sir *Iſaac*'s Time; for we can now almoſt prove the Exiſtence of this *Æther* by the Phænomena of Electricity; and then we find it very eaſy to prove that Electricity is nothing but this very Æther condenſed and made to ſhine. But, I believe, when we enquire into the Nature and Properties of this Æther and Electricity, we ſhall find them ſo very different and diſſimilar, that we cannot eaſily conceive how they ſhould thus mutually prove each other. For according

cording to Sir *Iſaac*, this Æther is the moſt ſubtle of all Bodies, and conſiſts of Particles (of courſe) very much ſmaller than thoſe of Light, and which have a much greater Velocity; this Æther therefore pervades with the greateſt Freedom all Bodies, actuates their Parts, *&c.* But on the other hand, we find by Experiments that the Fire of Electricity is a very groſs Subſtance; for it daſhes againſt the Surface of Glaſs like ſo much Water, and does not enter the Pores, as Light does, that is ſo much groſſer than Æther. This is ſtill made more plain and demonſtrable by electrifying a Wire in an exhauſted Receiver, where the Electricity iſſues out in much greater Abundance, and in divers Jets, ſtriking againſt the Sides of the Glaſs, and appearing to run down by the Sides in all Reſpects like *Liquid Phoſphorus*; and is no ways reflected or refracted like common Light, or even like the Light of a Candle: For if a Candle be view'd through a Priſm, the Light coming from every Part thereof is refracted, and the whole Flame is colour'd and diſtorted; but it is quite otherwiſe with the *Flame of Electrical Fire*, for I often made that much larger than the Flame of a Candle, and view'd it through the Priſm, but no Colours were thereby produced, no different Form of Flame, but every way the ſame as it appears to the naked Eye. Indeed the denſer Part, juſt upon the Tip of the Iron whence the Blaze diverged, appear'd a little tinged with Colours; which I take to proceed from ſome common *Solar Light* intermix'd with that of Electricity. From hence (and for many other Reaſons I could name) I ſee no Cauſe to believe that the Matter of Electricity is any thing like the Idea we ought to have of the *Spiritus ſubtiliſſimus* of Sir *Iſaac*. Nor indeed can it be the ſame with the Matter of common Light, which is differently reflexible and refrangible; whereas this appears not to be ſo, but is a particular Sort of Light, and nearly the ſame as *Phoſphorus*, which I could never obſerve to exhibit the leaſt Appearance of Colours through the Priſm. The Smell alſo of *Electrical Fire* is ſo very much like that of Phoſphorus, that one may be eaſily induced to believe a great Part of the Compoſition of both is the ſame.

As to the Doctrine of ELEMENTS and *Elementary Fire*, we leave that to others, and expatiate only on the noble *Theory of Light and Fire* left us by our incomparable Author, who juſtly conſiders them as they are in Nature, illuſtrates all their Properties and Qualities, and applies them to the Explication of a great Number of the moſt intereſting and important Phænomena of Nature. The *Newtonian* Doctrine of *Elements* is in the *Singular Number: One Sort of Matter only* makes the Subſtance of all the infinite Variety of Bodies we behold.

Thus our Author ſpeaks: "It ſeems probable to me, that GOD in the Beginning form'd Matter in ſolid, maſſy, hard, impenetrable, moveable Particles, of ſuch Sizes and Figures, and with ſuch other Properties, and in ſuch Proportion to Space, as moſt conduced to the End for which he form'd them.—Now by Virtue of certain active Principles or Powers all material Things ſeem to have been compoſed of the above-mention'd Particles, variouſly aſſociated in the firſt Creation, by the Counſel of an intelligent Agent."

To conclude: It was an Obſervation worthy of ſo great a Genius as Mr. *Cotes*, that the *Newtonian Philoſophy* was our *(munitiſſimum præſidium adverſus Atheorum impetus)* beſt Defence againſt the Efforts and Aſſaults of Atheiſts.—That herein we more nearly view the Majeſty of Nature, and enjoy it in the moſt grateful Contemplation; at the ſame time we are excited more intenſely to worſhip and revere the CREATOR and LORD of all Things. He ought to be reputed *blind*, who cannot immediately behold, in the beſt and wiſeſt Structures of Things, the infinite Goodneſs and Wiſdom of the Omnipotent Architect; and *mad*, who will not confeſs it.

A CATA-

A
CATALOGUE
OF THE
Principal BOOKS
Made uſe of in compiling the following Work.

1. *Philos. Nat. Principia Mathematica,* Auctore I. Newtono, *Eq; Aurato,* Ed. Tertia.
2. *Eadem Principia illuſtrata Perpetuis Commentariis, communi Studio* PP. Thom. Le Seur & Fran. Jacquier. *Genevæ,* 1739.
3. *A View of Sir* I. Newton's *Philoſophy,* by *Dr.* Pemberton. *Lond.* 1728.
4. *Phyſices Elementa Mathematica, Experimentis confirmata.* Auctore Jacobo s'Gravesande, Tomis Duobus, Edit. Tertia. *Leidæ,* 1742.
5. *Eſſai de Phyſique,* par M. P. van Muschenbroek, Tome i. & ii. *Leyden,* 1739.
6. *A Courſe of Experimental Philoſophy,* by Dr. Desaguliers, 2 Vols. *London,* 1734 and 1744.

7. *Phoronomia, ſive De Viribus & Motibus Corporum Libri Duo.* Auctore JACOBO HERMANNO. *Amſtelædami*, 1716.
8. *The Philoſophical Tranſactions abridged*, by Mr. LOWTHORP, JONES, MARTYN, EAMES, &c. Seven Vols.
9. *A Compleat Syſtem of Optics*, by Dr. SMITH, Two Vols. *Cambridge*, 1738.
10. *Lectiones Opticæ*, Auctore I. NEWTONO, *Eq; Aurato*.
11. *Optics*, or a *Treatiſe of Reflections, Refractions, Inflections, and Colours of Light*. By Sir I. NEWTON, *Knt*. *Lond*. 1718.
12. *An Eſſay on Diſtinct and Indiſtinct Viſion.* By Dr. JURIN.
13. *Architecture Hydraulique*, par M. BELIDOR, Tom. i. & ii. A *Paris*.
14. *Hydroſtatical and Pneumatical Lectures*, by Mr. COTES, with Notes by Dr. SMITH. *London*, 1738.
15. *The Motion of Fluids Natural and Artificial*, particularly *Air* and *Water*. By Mr. CLARE. *London*, 1737.
16. *Aſtronomy*, in five Books. By Dr. LONG. *Cambridge*, 1742.
17. *An Introduction to the True Aſtronomy.* By Dr. JOHN KEILL. *London*, 1721.
18. *Aſtronomical Lectures.* By Mr. W. WHISTON. *London*, 1728.
19. *The Chronology of Ancient Kingdoms amended.* By Sir I. NEWTON. *London*, 1728.

20. *The Elements of Algebra*, in Ten Books. By Dr. SAUNDERSON. *Cambridge*, 1740.

21. *Sir* I. NEWTON's *Two Treatiſes of the Quadrature of Curves and Analyſis, by Equations of an infinite Number of Terms explain'd.* By Mr. J. STEWART. *London*, 1745.

22. ROHAULT's *Phyſics*, with Dr. CLARKE's Annotations. *London*, 1710.

23. *A Compendious Syſtem of Natural Philoſophy, with Notes.* By J. ROWNING, M. A. *Cambridge*, 1735.

24. *A Syſtem of the Conic-Sections, with the Doctrine of Fluxions.* By JOHN MULLER. *London*, 1736.

25. *A Treatiſe of Fluxions*, in Two Books. By COL. MAC LAURIN. *Edinburgh*, 1742.

26. *A Treatiſe of Fluxions*, by the Marquis DE L'HOSPITAL, *with Additions* by Mr. STONE. *London*, 1730.

27. *A New Treatiſe of Fluxions.* By Mr. THOMAS SIMPSON. *London*, 1737.

28. *An Introduction to Natural Philoſophy.* By Dr. JOHN KEILL. *London*, 1720.

29. *A Compendious and Methodical Account of the Principles of Natural Philoſophy.* By Mr. BENJAMIN WORSTER. *London*, 1730.

30. *A Courſe of Lectures in Natural Philoſophy.* By Dr. HELSHAM, with Additions by Dr. ROBINSON. *London*, 1739.

31. *Philoſophiæ Mathematicæ Newtonianæ illuſtratæ Tomi Duo.* A' GEOR. PETRO DOMCKIO. *Londini*, 1730.

32. *Commercium Epiſtolicum de varia re Mathematica. Londini*, 1725.

33. *The Figure of the Earth determined from Obſervations at the Polar Circle.* By Mr. DE MAUPERTUIS, *&c. London*, 1738.

34. *Sidereus Nuncius.* A GALILEO. *Londini*, 1653.

35. *Vegetable Statics.* By Dr. HALES. *London*, 1738.

36. *Hæmaſtatics.* By the ſame Author. *London*, 1733.

37. *The Elements of Aſtronomy, Phyſical and Geometrical.* By Dr. GREGORY, 2 Vols. *London*, 1715.

38. *An Inſtitution of Fluxions.* By Mr. DITTON. *London.*

39. *A Synopſis of the Aſtronomy of Comets.* By Dr. HALLEY.

40. *Miſcellanea Curioſa.* Three Vols. 8vo, *London.*

41. *Inſtitutiones Medicæ.* Ab HERM. BOERHAAVE. *Lugd. Batavorum*, 1720.

42. *A Courſe of Lectures in Chemiſtry.* By Dr. SHAW.

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LECTURE I.

Of EXPERIMENTAL PHILOSOPHY *in general. Of the* NEWTONIAN METHOD *and* RULES *of Philoſophizing. Of* MATTER *in general, and its Eſſential Properties. Of the* CHEMICAL ELEMENTS *or* PRINCIPLES *of Natural Bodies. Of* ATTRACTION *and* REPULSION. *The* Attraction *of* COHESION. *Of the* COHESION *of various Sorts of Bodies. The Phænomena of* CAPILLARY TUBES. *Of a* VACUUM. *The* Rationale *of various Proceſſes of* CHEMISTRY, *and other Arts. Of the principal Properties of* ELECTRICAL Attraction *and* Repulſion. *Of* MAGNETISM *and its Laws; of the various Properties of the* MAGNET *in relation to the Needle; its* VARIATION *and* INCLINATION.

THE Buſineſs of Experimental Philoſophy (the Subject of this Courſe of Lectures,) is to enquire into and inveſtigate the Reaſon and Cauſes of the various Appearances (or *Phænomena*) of Nature; and to make the Truth or Probability thereof obvious and evident to the Senſes, by *plain, undeniable, and adequate Experiments*, repreſenting the ſeveral Parts of the grand Machinery and Agency of Nature.

IN our Enquiries into Nature we are to be conducted by thoſe Rules and Maxims which are found to be genuine, and conſonant to a juſt Method of Phyſical Reaſoning; and theſe Rules of Philoſophizing are, by the greateſt Maſter in this Science, (the incomparable Sir *Iſaac Newton*) reckon'd Four; which I ſhall give, from his *Principia*, as follows:

RULE I. *More Cauſes of natural Things are not to be admitted, than are both true and ſufficient to explain the Phænomena.* For Nature does nothing in vain, but is ſimple, and delights not in ſuperfluous Cauſes of Things.

RULE II. *And therefore of natural Effects of the ſame Kind the ſame Cauſes are to be aſſign'd, as far as it can be done.* As of Reſpiration in Man and Beaſts: Of the Deſcent of Stones in *Europe* and *America:* Of Light in a culinary Fire and in the Sun: And of the Reflection of Light in the Earth and in the Planets.

RULE III. *The Qualities of natural Bodies which cannot be increaſed or diminiſh'd, and agree to all Bodies in which Experiments can be made, are to be reckon'd as the Qualities of all Bodies whatſoever.* Thus, becauſe Extenſion, Diviſibility, Hardneſs, Impenetrability, Mobility, the *Vis Inertiæ*, and Gravity, are found in all Bodies which fall under our Cognizance or Inſpection, we may juſtly conclude they belong to all Bodies whatſoever; and are therefore to be eſteemed the original and univerſal Properties of all natural Bodies.

RULE IV. *In Experimental Philoſophy, Propoſitions collected from the Phænomena by Induction are to be deem'd (notwithſtanding contrary Hypotheſes) either exactly or very nearly true; till other Phænomena occur by which they may be render'd either more accurate, or liable to Exception.* This ought to be done, leſt Arguments of Induction ſhould be deſtroy'd by Hypotheſes.

If according to theſe Rules we take a Survey of the viſible World, and ſtrictly examine the Nature of particular Bodies, we ſhall find Reaſon to conclude, that they all conſiſt of *one and the ſame Sort of Matter or Subſtance*; and that all the Diverſity or Difference we obſerve among them ariſes only from the various Modifications and different Connection or Adheſion of the ſame primigenial Particles of Matter (I).

(I) Theſe four Rules of Philoſophizing are premiſed by Sir *Iſaac Newton* to his third Book of the *Principia*; and more particularly explain'd by him in his *Optics*, where he exhibits the Method of proceeding in Philoſophy in the following Words:

1. As in Mathematics, ſo in Natural Philoſophy, the Inveſtigation of difficult Things by way of *Analyſis*, ought always to precede the Method of *Compoſition*. This Analyſis conſiſts in making Experiments and Obſervations, and in drawing general Concluſions from them by Induction (*i. e.* Reaſoning from the Analogy of Things by natural Conſequence,) and admitting no Objections againſt the Concluſions but what are taken from Experiments, or other certain Truths. And altho' the arguing from Experiments and Obſervation by Induction be no Demonſtration of general Concluſions; yet it is the beſt way of arguing which the Nature of Things admits of, and may be looked upon as ſo much the ſtronger by how much the Induction is more general. And if no Exception occur from Phænomena, the Concluſion may be pronounced

MATTER, thus variouſly modified and configurated, conſtitutes an infinite Variety of Bodies,

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generally. But if at any Time afterwards, any Exceptions ſhall occur from Experiments, it may then be pronounced with ſuch Exceptions. By this way of *Analyſis* we may proceed from Compounds to Ingredients, and from Motions to the Cauſes producing them; and in general from Effects to their Cauſes; and from particular Cauſes to more general ones, till the Argument ends in the moſt general. This is the Method of Analyſis. And that of *Syntheſis* (or Compoſition) conſiſts in aſſuming Cauſes diſcover'd and eſtabliſh'd as Principles, and by them explaining the *Phænomena* proceeding from them, and proving the Explanations.

2. That Matter or Subſtance is *one and the ſame* in all Bodies, and that all the Variety we obſerve ariſes from the various Forms and Shapes which it puts on, is I think very probable, and may be concluded from a general Obſervation of the Procedure of Nature in the Generation and Deſtruction of Bodies. Thus, for Inſtance, *Water* rarified by Heat, becomes *Vapour*; great Collections of Vapours form *Clouds*; theſe condenſed deſcend in form of *Rain*, or *Hail*; Part of this collected on the Earth conſtitutes *Rivers*; another Part mixing with the Earth enters into the Roots of Plants, and ſupplies Matter to, and expands itſelf into various Species of *Vegetables*. In each Vegetable it appears in one Shape in the *Root*, another in the *Stalk*, another in the *Flowers*, another in the *Seeds*, &c. From hence various Bodies proceed; from the Oak, *Houſes*, *Ships*, &c. from Hemp and Flax, we have *Thread*; from thence our various Kinds of *Linen*; from thence *Garments*; theſe degenerate into *Rags*, which receives from the Mill the various Forms of *Paper*; hence our *Books*; which by Fire are converted partly into Water, partly into Oil, another Part into Air, a fourth Part into Salt, and a fifth into Earth; which are call'd the Elements of Bodies, and which mix'd with common Earth, are again reſuſcitated in various Forms of Bodies.

3. The ELEMENTS, or Principles to which all Bodies are ultimately reduced, are the five abovemention'd, *viz.* (1.) WATER, or *Phlegm* (as 'tis call'd) which generally riſes and goes off firſt, as in the Chemical Analyſis of a Plant in a Retort by Fire. (2.) AIR, which eſcapes unſeen in great Quantities from all Bodies; and tho' it has not till lately been known to make a Part of natural Bodies in a fix'd State (and

all which are found to have the following *common Properties*, viz.

EXTENSION, or that by which it poſſeſſes or takes up ſome Part of univerſal Space; which Space is call'd the PLACE of that Body. For all Bodies are *extended* either (1.) into *Length* only, and then it is called a LINE; or (2.) into *Length* and *Breadth*, which is call'd a SUPERFICIES; or (3.) into *Length*, *Breadth*, and *Depth*, which then is call'd a SOLID. Theſe are the *three Dimenſions*, according to the Quantity of which the *Magnitude* or Bulks of Bodies are eſtimated (II).

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therefore never taken Notice of as an Element of natural Bodies) yet that it is ſo in a very remarkable Degree, (even ſo far as to make half the Subſtance of ſome Bodies) I ſhall give ſufficient Proof when I come to treat of artificial or factitious Air. (3.) OIL, which appears ſwimming on the Top of the Water. (4.) SALT, which is either *Volatile*, or riſes in the Still, as that of Animal Subſtances; or *Fix'd*, as in Vegetables, and which is extracted by diſſolving them in Water from a Lixivium of the Aſhes, and afterwards by evaporating the Moiſture to a Pellicle, and ſetting the Salt to ſhoot into *Cryſtals*. (5.) EARTH, or what is call'd the *Caput-Mortuum*, being what remains of the Aſhes after the Salt is extracted, which can be no farther alter'd by any Art whatſoever.

(II) The MAGNITUDE of Bodies is the Quantity of their Dimenſions expreſs'd in ſome common or *ſtandard* Meaſure, as an *Inch*, a *Foot*, a *Yard*, &c. and it is thus eſtimated:

Plate I. Fig. 1.

1. When Bodies have but one Dimenſion, as Lines, then it is expreſs'd by the Number of Inches and Parts of an Inch contain'd in their Length: Thus the Dimenſion of the Line A B is 3 Inches; of C B 2 Inches; and the Compariſon of the Length of A B to C B is call'd the Proportion or *Ratio*; thus the Ratio of A B to C B is that of 3 to 2, or (as it is uſually expreſs'd) A B : C B :: 3 : 2. And becauſe this Compariſon conſiſts of *only one Ratio*, it is ſaid to be a *ſimple Ratio*.

2. Thoſe Bodies which have two Dimenſions, as Superficies, have their Magnitude expreſs'd by the Rectangle under both, or the Product of their Length by their Breadth; thus

SOLIDITY, ſometimes called the Impenetrability of Matter, is that Property by which a Body excludes all others from the Place which itſelf poſſeſſeth; for it would be abſurd to ſuppoſe two Bodies could poſſeſs one and the ſame Place at the ſame Time. From this Defi-

Plate I. Fig. 2. if any Surface ABCD has its Length $AB = 4$, and its Breadth $BC = 2$; then its Dimenſion (which in this Caſe is call'd the *Area*, or *Superficial Content*) is thus expreſs'd, $AB \times BC = 4 \times 2 = 8$; that is, there are eight ſmall Squares, which are ſquare Inches, ſquare Feet, *&c.* according to the Meaſure of the Sides.

3. If this Surface be compared with any other, as EG, Plate I. Fig. 3. whoſe Length is $EF = 3$, and Breadth $FG = 2$; then their Magnitudes will be to each other as $AB \times BC$ to $EF \times FG$, that is, as 8 to 6, or as 4 to 3. And becauſe in this Compariſon each Term conſiſts of two Parts, or there is a twofold Ratio of Length to Length, and Breadth to Breadth, therefore this is ſaid to be a *Duplicate Ratio*; and ſo all Surfaces are to each other in the *Duplicate Ratio* of their Sides.

4. In like Manner all Solid Bodies, which have three Dimenſions, Plate I. Fig. 4, 5. have their Magnitudes expreſs'd by the Product of their *Length*, *Breadth*, and *Depth* together. Thus if there be one Solid AG whoſe Length $AB = 4$, its Breadth $AE = 2$, and Depth $AD = 3$; and another Solid HO, whoſe Length $HI = 3$, Breadth $HM = 1$, and Depth $HL = 2$; then will their Magnitudes be to each other as $AB \times BE \times AD$ to $HI \times HM \times HL$, that is $4 \times 2 \times 3 = 24$ to $3 \times 1 \times 2 = 6$, or as 24 to 6. And therefore Solids are ſaid to be in a triplicate Ratio, *viz.* of their Length, Breadth, and Depth, or Thickneſs; and the Standard Meaſure in this Caſe is call'd a *Cubic Inch*, *Foot*, &c. becauſe of its being in the Form of a Dye, or *Cube*, which Figure is contain'd under 6 equal and rectangular Planes.

5. And here it will be neceſſary to advertiſe the Reader, that any Quantity is generally denoted by a ſingle Letter, as A, B, *&c.* and the Square of that Quantity by $A \times A$ or AA, or A^2, B^2, *&c.* and the Cube by A^3 or B^3. And when we expreſs the Ratio of two Quantities A and B, it is uſually thus, $A : B$; and when we compare this Ratio with any other, as of C to D, we expreſs it thus $A : B :: C : D$. If both Terms of the Ratio increaſe or decreaſe together, the Ratio is ſaid to

nition it follows, that the *ſofteſt Bodies* are equally *ſolid* with the *hardeſt* (III).

be direct, and expreſs'd as before: But if one Quantity increaſes while the other decreaſes, and *vice verſâ*, in each Ratio; then they are expreſs'd in a different Manner, *viz.* $A : \frac{1}{B} :: C : \frac{1}{D}$, or $A : C :: \frac{1}{B} : \frac{1}{D}$. And the Addition of one Ratio $A : B$ to another $C : D$ is performed by multiplying the firſt Terms in each Ratio together, and alſo the laſt; and the Ratio of thoſe Products $AC : BD$ is the Sum of both the other. To ſquare any Ratio is to multiply it by itſelf $A : B \times A : B = A^2 : B^2$, and ſo the Cube of any Ratio, $A : B$ is $A^3 : B^3$. If any would know more of the Nature of Ratios, they may conſult Dr. *Sanderſon*'s Algebra, or my *Logarithmologia*.

(III) FIGURABILITY is as neceſſary to Matter as any of the Properties abovemention'd; for ſince Matter is not infinite, it muſt be circumſcribed within certain Limits and Bounds on every Part, which conſtitute the Figure of the Body; and as the Particles of Matter may exiſt together in any Manner of Situation, ſo the Figures or Form of Bodies, which they compoſe, may be infinitely various and different from each other.

2. On this Property ſeveral Things of Moment depend; thus according to the ſeveral Figures of the Corpuſcles, they will touch by a greater or leſſer Quantity of Surface, and ſo will cohere more or leſs firmly together; from hence will ariſe various Qualities of Bodies, which are the Foundation of moſt of the conſiderable Phænomena of Nature, as will be hereafter taken Notice of.

3. Hence we may obſerve likewiſe, that Bodies of different Figures contain, under the ſame Quantity of Surface, different Magnitudes or Bulks; thus a *Circle* contains a greater Area under the ſame Length of Periphery, than any other figured Superficies; and a *Sphere*, under the ſame Quantity of Surfaces, contains a greater Bulk or Space, than any other Solid whatſoever.

4. Of all the infinite Variety of Forms or Figures, which Matter is liable to, there are only *five* which will admit the Particles of Matter, placed together, to fill the Space between them compleatly, or ſo as to leave no Pore, Vacuity, or Interſtice between them; and Bodies of theſe Forms have been known by the general Name of the *five Platonic Bodies*, and are as follows:

DIVISIBILITY is that Property by which the Particles of Matter in all Bodies are capable of a Separation or Diſunion from each other. Hence the *Reſolution* or *Diſſolution* of Bodies into their conſtituent Corpuſcles, as in many Operations of Chemiſtry. How far this may actually obtain in Nature is not eaſy to ſay: But that Matter is infinitely diviſible in a mathematical Conſideration, is demonſtrable various Ways (IV).

1. *Tetrahedron*, which has *four equal* triangular Sides.
2. *Hexahedron*, of *ſix equal* ſquare Sides, *viz.* a Cube.
3. *Octahedron*, of *eight equal* pentagonal Sides.
4. *Dodecahedron*, of *twelve equal* triangular Sides.
5. *Icoſahedron*, of *twenty equal* triangular Sides.

9. Hence the Aſſerters of a *Plenum* are neceſſitated to prove, that tho' Matter is liable to an Infinity of Forms indifferently, yet all its Particles muſt have but ſome one of the five abovemention'd; otherwiſe it will be as evident as an *Axiom*, that the Particles of Matter have a different Form, and muſt therefore admit Vacuities or Pores in the Compoſition of every kind of Bodies. Tho' this Argument for a Vacuum has been little obſerved, yet perhaps there is none more evident, geometrical and concluſive.

Plate I. Fig. 6.

(IV) 1. The eaſieſt Way to demonſtrate the infinite Diviſibility of Matter, I think, is the following. Let A B be the Length of a Particle to be divided; thro' each Extreme, A and B, let there be drawn the Lines C D, E F parallel to each other: In the Line C D, between A and C, let there be taken any Point G; and in the Line E F, from B towards F, let there be taken any finite Number of Points H, I, K, L, &c. If now from the Point G we draw the Lines G H, G I, G K, G L, &c. they will each of them cut off a Part of the given Particle A B; and yet after all a Part P A will remain; and ſince this will be the Caſe for any finite Number, 'tis plain the Particle A B contains a Number of Parts greater than any *finite Number*, and therefore *infinite*.

Plate I. Fig. 7.

2. Another Demonſtration equally obvious is the following, *viz.* Let A B be the Length of the given Particle; thro' A draw C D, and in the Point E, at right Angles, draw the indefinite right Line E F, in which let there be any finite

MOBILITY is that Property which all Bodies have, of being moveable or capable of changing their Situations or Places. This Property of Matter is evident to all our Senſes; and what nobody has confeſſedly deny'd (V).

Number of Points taken, as G, H, I, K, *&c.* on theſe, as Centers, deſcribe the ſeveral Arches of Circles EL, EM, EN, EO, *&c.* to cut the Particle AB, each of theſe will take off a Part, and yet there will a Part AP remain; therefore, *&c.* as before.

3. Tho' we have no Poſſibility of an Inſtance of this infinite Diviſibility of Matter, yet Nature proceeds towards it to inconceivable Lengths in many of her wonderful (tho' common) Operations. How minutely is Matter divided by Fire, as a Piece of Tallow, for Inſtance, in a lighted Candle! What a prodigious Sphere or Space does the leaſt Flame fill with Particles of Light, or a Grain of *Aſſa fœtida* with odorous Particles! How numerous and ſmall the Particles of Vapours ariſing from the Surface of Fluids! Or the Particles of Copper, which from a ſingle Grain ſhall be diffuſed thro' the Subſtance of many Millions of Times its own Quantity of Water, and tinge it of a different Colour! But Inſtances of this Kind are endleſs.

(V) *Des Cartes* and his Followers, who deny a *Vacuum*, do virtually deny Motion; for ſince Motion is the Tranſlation of a Body from one Part of Space to another, if there be no *void Space*, ſuch a Tranſlation muſt be made thro' a Space abſolutely fill'd with Matter, or not at all; if the firſt be aſſerted, it will follow, that two Bodies may be in one and the ſame Place at once, which is abſurd; therefore there can be no Motion at all, upon their Principles. To ſay a Body may move in a *Plenum*, as a Bird moves thro' a Body of Air, or a Fiſh thro' Water, is ſaying nothing to the Purpoſe; becauſe both Air and Water abound with Vacuities, (as is well known from the Nature of each,) whereas a *Plenum* admits of no vacuous Space, and therefore of no Motion of the Particles, nor conſequently of any Body between them. Beſides, neither the Bird nor Fiſh would be able to continue in the Air or Water, were they not ſpecifically heavier than thoſe Fluids; but upon the Suppoſition of a *Plenum*, all Specific Gravity is taken away (as will be ſhewn,) and ſo the Caſe is not parallel, and therefore the Argument fallacious.

Plate I, Fig. 8.

THE VIS INERTIÆ, (as Sir *Iſaac* called it) or the *Inactivity* of Matter, is that Property of it by which it endeavours to continue in its State either of *Motion* or *Reſt*, or by which it reſiſts the Actions and Impreſſions of all other Bodies which tend to generate or deſtroy Motion therein (VI).

ATTRACTION is a Property that we find all Matter endued with in a greater or a leſſer Degree: By this Property the Particles of Matter attract each other by a Power which

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(VI) This Power or Force of Matter ariſes wholly from its Sluggiſhneſs or Inactivity, and not at all from its Gravity, as ſome (of no ſmall Name) have miſtakenly conceived of it. To make this Matter plain, ſuppoſe a Body A at reſt in the Point A, it would not of itſelf move out of that Point, unleſs by ſome external Force impreſs'd. *Firſt*, Suppoſe the Body abſolutely light, and it be required to move it from A to B, in a certain Time; to this End ſome Force muſt be applied acting in the Direction AB, by which, at the End of that Time, the Body will be found in B; and this Force once impreſs'd will continue in the Body, and in every ſuch Particle of Time will carry it thro' a Space equal to AB. *Secondly*, Suppoſe now Gravity begins to act, and give the Body Motion, and that by the Power of Gravity the Body be carried from A to D in a Line, paſſing thro' the Earth's Centre, in the ſame Time as it before moved from A to B; draw BC and DC equal and parallel to AD and AB; and it is evident all the Effect of Gravity will be only to cauſe the Body during that Time, to deſcend through Space equal to AD from the Line AB, and ſo in the End of that Time inſtead of being at B it will be in C; but ſince the Point C is as far from D as B is from A, it follows, that whether Gravity acts or not, the ſame Space is deſcribed in the ſame Time in the Direction AB, and ſo the ſame Force is required to give it the Motion, which therefore has no Relation to or Dependance on Gravity; but ariſes ſolely from the Nature of inactive Matter, and is on that Account ever proportional to the Number of Particles or Quantity of Matter in Bodies.

cauſes them mutually to accede to, or approach each other. This Tendency of one Body to another is call'd GRAVITATION, or, in the Abſtract, the GRAVITY or WEIGHT of Bodies (VII).

REPULSION ſeems to be a Property belonging to the ſmall Particles of Matter univerſally; for they do not more evidently *attract* in ſome Circumſtances, than they *repel each other* in others, as will be evident by Experiments, hereafter to be exhibited (VIII).

(VII) This Power, or Virtue, was originally communicated to all Matter by the Omnipotence of the Deity, and is no ways neceſſary to its Exiſtence; therefore I apprehend when Sir *Iſaac* ſays, he is ignorant of the Cauſe of Gravity, he can only mean the particular Manner in which it is exerted from one Body, and in which it acts upon another, and not that he was ignorant of the Source or Principle of Power whence it could be derived. But as the Manner in which it is exerted, ſo the Means by which it is propagated from one Body to another, is as yet unknown. To account for the Propagation of this Virtue, as of Light, &c. 'tis ſuppoſed that there is an exceeding fine imperceptible Medium or ethereal Spirit, diffuſed thro' all the Syſtem, and pervading the Pores of all Bodies; that this Medium is extremely elaſtic, and that by its Tremours and Undulations, which are almoſt inſtantaneous, the Emanations of the attracting Virtue, Light, &c. are propagated with an immenſe Velocity, much after the ſame Manner as Sounds are convey'd by the undulatory Motion of the Air. By Means of this moſt ſubtle Spirit, 'tis ſuppoſed, likewiſe, that electric Bodies exert their Power of attracting and repelling light Bodies at greater Diſtances; that Light is emitted, reflected, refracted, inflected, and heats Bodies; that all Senſation is excited, and animal Motion perform'd at Will; namely, by the Vibrations of this Spirit mutually propagated along the ſolid Filaments of the Nerves from the outward Organs of Senſe to the Brain, and from the Brain to the Muſcles.

(VIII) *Attraction* and *Repulſion* differ in no other Reſpects than this, that the Attracting Virtue in the firſt Caſe carries Bodies *towards* the Attracting Body; and in the latter, it carries them

THESE are the general Properties of Matter, which we muſt regard in our Explications of the various Phænomena of Nature, in the Senſe we have defined them. Theſe are the ſeveral *Data*, or fundamental Principles on which the Science of Philoſophy depends, and which will each of them afford an ample Field both in the *ſpeculative and practical Part*. We ſhall (for the ſake of Method) begin with the *Attraction* of Bodies, conſider its ſeveral Species, and prove their Exiſtence and the Properties of each by Experiments.

THE *Power of Attraction*, or Cauſe of *Gravity*, we preſume not to define, or ſay what it is, but only that it is, or does exiſt; and the Laws of its Action we ſhall endeavour to aſſign by what may be diſcover'd by Reaſon and Experiment. To this End we muſt conſider, that *any Kind of Power or Virtue, proceeding or propagated from a Body in Right Lines every way as from a Centre, muſt decreaſe in its Energy or Strength as the Squares of the Diſtances from the Body increaſe*; for 'tis evident, the Force will be every where as the Number of Particles iſſuing from the central Body on a given Space, which Number of Particles will decreaſe as the Squares of the Diſtances increaſe. Thus the Number of Parti-

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from it. In each Caſe, the Particles are moved in the ſame Manner among themſelves, by the Attracting, Electric, or Magnetic Power.

cles which at any *one Diſtance* A B, from a Point in the Body at A, falls on a Square Inch B E F G, will be *four times* as great as the Number which falls on a Square Inch C H I K at *twice* that Diſtance A C; and *nine times* as great as the Number which falls on the ſaid Square Inch D L M N at *three times* that Diſtance A D; and ſo on, as is evident from the Diagram.

Plate III. Fig. 1.

Hence, ſince we have no Reaſon to doubt but that all Kinds of Attraction conſiſt in fine imperceptible Particles or inviſible Effluvia, which proceed from every Point in the Surface of the attracting Body, in all right-lined Directions every way, which in their Progreſs lighting on other Bodies urge and ſollicit them towards the ſuperior attracting Body; therefore the Force or Intenſity of the attracting Power in general muſt always decreaſe as the Squares of the Diſtances increaſe.

Hence alſo we may obſerve, by the way, that Light and Heat, Odours and Perfumes, which conſiſt of Particles or Effluvia that proceed every way from luminous, heated, and odoriferous Bodies, as from a Centre, have always their Forces abated according to the above Law. The Force of Sounds alſo decreaſes in the ſame Proportion, for Reaſons that will be hereafter aſſign'd.

By virtue of attracting Power, the grand Machinery of the Solar Syſtem, and doubtleſs of all the others in the Univerſe, is effected, eſtabliſhed, and conſerved. It is therefore of the greateſt Conſequence, to be acquainted with the different

Species of this univerſal Power or Agent, and to learn by Experiments the peculiar Nature, Laws, or Manner of Action in each. Naturaliſts generally reckon *Four* different Sorts of Attraction, *viz.*

I. THE *Attraction of* COHESION, which is peculiar to the Corpuſcles or primigenial Particles of Matter, of which larger Bodies are compoſed, by the Accretion and firm Adheſion of thoſe Particles, ariſing from their ſtrong attractive Power.

II. THE *Attraction of* ELECTRICITY, which is peculiar to ſome Kinds of Bodies, as Glaſs, Amber, Sealing-Wax, &c. which are therefore call'd *Electrical*.

III. THE *Attraction of* MAGNETISM, or of the Loadſtone, which is peculiar to, and mutual between the Loadſtone and Iron.

IV. THE *Attraction of* GRAVITATION, which is obſervable only in the larger Compoſitions and Syſtems of Matter; as in the *Earth* and *Moon*; and the *Sun* and *Planetary Bodies* which compoſe the Solar Syſtem. Of each of theſe Species of Attraction in Order; and firſt,

OF the *Attraction of* COHESION: The Laws and Properties of this Attraction are the following. (1.) It is very diſcernible and moſt powerful in Corpuſcles, or the ſmalleſt Particles of Matter. (2.) It is mutually exerted between thoſe Particles; or they mutually attract, and are attracted by each other. (3.) The Sphere of Attraction, or Extent of this Power, is greater in ſome Particles of Matter than in others, but very ſmall at the utmoſt: For (4.) This Power is inſenſible in ſolid Bodies in the leaſt ſenſible Diſtance, acting as it were only on Contact; and therefore, (5.) It muſt be nearly proportional to the Quantity of contiguous Surfaces; or the Parts of Bodies cohere moſt ſtrongly, whoſe touching Surfaces are largeſt. (6.) This Power muſt decreaſe as the Squares of the Diſtances increaſe; becauſe it muſt be ſuppoſed to iſſue from each Particle in right-lined Directions. (7.) Where the Sphere of Attraction ends, there a *Repelling* Power begins, by which the Particles, inſtead of *attracting*, *repel* and fly from each other. (8.) By this Power the ſmall Portions or Drops of a Fluid conform themſelves to a ſpherical Figure (IX)

(IX) To illuſtrate this Matter of Corpuſcular Attraction and Repulſion: Let A, B, (in *Fig.* 9, 10, 11, 12.) repreſent two Plate I. ſmall Bodies or Corpuſcles of Matter; and their Spheres of Attraction, (or Extent of their Attracting Power every way) be CDE, and EFG. Then it is evident,

1. That the Force with which the Bodies attract each other will be greateſt in the Caſe where they touch each other.

2. If the Bodies are of a Spherical Figure (as *Fig.* 9.) they Plate I. can touch but in a ſingle Point; and therefore the Power with which they ſtick or cohere together will be the ſlighteſt or weakeſt poſſible.

3. But if the Bodies are bounded by plain Surfaces (as *Fig.* 12.) the Power of Coheſion will be ſo much the greater, as the Number of Particles, or Area of the Surface by which they touch is larger.

4. If the Corpuſcles A, B, touch not, but are yet within the Sphere of each other's Attraction, (as *Fig.* 10.) they will then be eaſily moveable or voluble about each other, and yield to the leaſt Impreſſion of external Force; and this will be the Caſe of all Particles, whatever be their Figures or Forms.

5. If the Corpuſcles A, B, are ſeparated to ſuch a Diſtance as to be without each other's Sphere of Attraction (as in

THE firſt and ſecond of theſe Properties are made manifeſt by various Experiments; as the ſudden Union of two contiguous Drops of Mercury, Water, &c. The ſtrong Adheſion of two Leaden Balls, which touch by poliſh'd Surfaces (X);

Fig. 11.) then will they fly from, or repel each other, with a Power that extends to a conſiderable Diſtance, and is exerted every way equally from each Particle.

6. If there be a great Number of theſe Particles thus diſengaged from each other, but yet confin'd within certain Bounds, they will, by this repulſive Power, be all ſituated at equal Diſtances from each other through the whole Space, if the confining Power be on every Side equal.

7. This Power, while *attractive*, extends to but very ſmall Diſtances from the Corpuſcles, and then becomes *repulſive*, for ſtrictly ſpeaking, there are no Bodies in Nature which repel *abſolutely*, but only at certain Diſtances; thus the Loadſtone does not, at all Diſtances, repel the Needle, but only at a definite Diſtance; for either Pole will attract either End of the Needle at a very near Diſtance, or upon Contact, as is eaſy to experiment.

(X) This Adheſion of Leaden Balls is ſo very conſiderable, that with two (not weighing above a Pound each, nor touching upon more than $\frac{1}{10}$ of a ſquare Inch Surface) I have lifted above 150 *lb.* weight. In order to this, the Surfaces by which they touch muſt be finely planed with the Edge of a ſharp Pen-knife, and equally preſs'd together with a conſiderable Force, with a gentle Turn of the Hand at the ſame Time; and thus two common leaden Bullets will adhere ſo firmly together, as to require upwards of 50 *lb.* to ſeparate them.

2. In poliſh'd Surfaces that are very hard, as Glaſs, Braſs, &c. it is impoſſible to bring them into ſuch cloſe Contact as to cohere without the Interpoſition of Water, or ſomething humid to fill the Pores by expelling the Air contain'd therein, which prevents the Planes coming together while dry; the Humidity in this Caſe proves a Cement which holds the Planes together by all its Force of Attraction on either Side.

3. This Force of Attraction between the braſs Planes is greater with Oil than with Water; and greater with any Sort of Greaſe or Fat that will harden with Cold than with Oil. I never yet could meet with two Men ſtrong enough to ſeparate

as alſo of Glaſs Planes, and Cryſtal Buttons: The Aſcent of Water between Glaſs Planes, and in Capillary Tubes: The riſing of Water by the

Planes thus put together, which are but 4 $\frac{1}{4}$ Inches Diameter; and therefore they cohere with a Force much ſuperior to the Force or Preſſure of Air on ſuch a Surface, which contains about 14 ſquare Inches, and allowing 15 *lb.* for Preſſure to every Inch it will amount to but 210 *lb.* which is not near equal to the Strength of two Men pulling againſt each other to the beſt Advantage.

4. Profeſſor *Muſchenbroek* has made many Experiments to ſhew the Force of Coheſion between Planes of various Subſtances, and about 2 Inches Diameter, well poliſh'd, having firſt heated them in boiling Water, and then beſmear'd them firſt with a cold Tallow-Candle, and afterwards with boiling Greaſe, and the Weights to ſeparate them were as in the following Table:

	Cold Greaſe.	*Hot Greaſe.*
Planes of Glaſs	— 130 *lb.*	300 *lb.*
of Braſs	— 150	800
of Copper	— 200	850
of Marble	— 225	600
of Silver	— 150	250
of Iron	— 300	950

5. Theſe Planes adhere by other Sorts of Matter with Forces as in the Table below, where the Weights neceſſary to ſeparate them are ſpecified:

With Water ————	12 Ounces.
With Oil ————	18 Ounces.
With *Venice* Turpentine —	24 Ounces.
With Roſin ————	850 Pounds.
With Tallow Candle ——	800 Pounds.
With Pitch ————	1400 Pounds.

Though theſe Experiments would not always give the ſame Numbers, yet they ſufficiently ſhew the vaſt attractive Forces, and the very great Difference between them.

6. After this he gives us an Account of his Experiments made to find the Force with which Bodies cohere naturally, or what is the abſolute Force of Coheſion in various Bodies, which he eſtimates by the Weights required to pull them aſunder, drawing according to their Length; this he tried firſt in Wood, and afterwards in Metal. His Pieces of Wood were of a long ſquare Form, of which each Side was $\frac{26}{100}$ of

Sides of a Glaſs Veſſel, and into Tubes of Sand, Aſhes, Sugar, Sponge, and all porous Subſtances.

an Inch, and by Weights ſuſpended they were drawn aſunder, according to the ſeveral Sorts, as mention'd below:

Wood of Linden-Tree	1000 *lb.*
of Alder	1000
of Fir	600
of Oak	1150
of Elm	950
of Beech	1250
of Aſh	1250

7. The Tryal he made with Metals was of Weights ſuſpended to Wires of each Sort, whoſe Diameter was $\frac{1}{10}$ of a Rhinland Inch; or becauſe the Rhinland Foot is to ours as 139 to 135, the Wire was $\frac{135}{1390}$ Part of an Inch *Engliſh*. The Metals and Weights were as follow:

Of Copper	299 ¼ *lb.*
Of yellow Braſs	360
Of Gold	500
Of Silver	370
Of Iron	450
Of Tin	40 ¼
Of Lead	29 ¼

8. Theſe Experiments ſhew the *abſolute Force of Coheſion* in Bodies. That which he calls the *Relative Force* is that by which any Body reſiſts the Force of any other Body acting upon it in a Direction perpendicular to its Length; and this he eſtimated in the ſame Pieces of Wood as before, by putting one End into a ſquare Hole of a Metal Plate, and hanging Weights towards the other End ſufficient to break each Piece of Wood at the Hole. Theſe Weights and Diſtances from the Hole were in his Experiments as follow.

	Diſtance.	*Weights.*
Fir	9 Inches	40 Ounces
Oak	8 ½	48
Elm	9	44
Pine	9 ½	36 ½
Alder	9 ¼	48
Beech	7	56 ½

Such as would ſee much more on this Subject, may conſult the Author's admirable *Eſſai de Phyſique*; and his Treatiſe of the Coheſion of Bodies, altogether on the Subject.

THE third of theſe Properties is evident by the Experiments of Water riſing above the common Level, and Mercury's ſinking below it, in Capillary Tubes: By the ſticking or adhering of Water to common Subſtances, which by Mercury are left dry.

THE fourth and fifth Properties are evinced by the Experiments of the different Heights to which Fluids aſcend between Glaſs Planes unequally inclined, and in Capillary Tubes of different Bores: Alſo, by the accelerated Motion of a Drop of Oil between two inclined Planes: And likewiſe by the *Hyperbolical Curve* form'd by the Superficies of a Fluid aſcending between Glaſs Planes touching each other on one Side.

THE ſixth Property is evident. The ſeventh ſeems evident between *fat and oily Particles of Matter*, and thoſe of an *aqueous Nature:* But is moſt manifeſt from the Elaſtic Property of the Air, whoſe Particles compreſs'd together reſtore themſelves by this repellent Power to their firſt State: Alſo by the Aſcent of *Steam* or *Vapour* from humid or fluid Bodies.

THE eighth Property of this Attraction is manifeſt by Drops of Water falling on Duſt; by Drops of Dew gathering on the Tops of Graſs; and laſtly, by Quickſilver divided into ſmall Portions, which always form themſelves into perfect Spherules or Globules (XII).

(XII) 1. In order to account for the Phænomena of Glaſs Capillary Tubes, &c. it will be neceſſary firſt to premiſe, that there is a greater Attraction between the Particles of

FROM this Account of the *Attraction of Coheſion* we learn a rational Solution of ſeveral very

Glaſs and Water, than there is between the Particles of Water themſelves: For if it were not ſo, the leaſt Quantity or Drop of Water C applied to the under Side of a Glaſs Tube, in a Poſition parallel to the Horizon, as A B, would not adhere to it, but immediately fall down by its Gravity; but we ſee that it does not, till its Bulk and Gravity be ſo far increaſed as to overcome the Attraction of the Glaſs, and then it falls off. Plate I. Fig. 13.

2. Since we find ſuch a ſtrong attractive Power in the Surface of Glaſs, 'twill be eaſy to conceive how ſenſibly ſuch a Power muſt act on the Surface of a Fluid (not viſcid) as Water, contain'd within the ſmall Cavity or Bore of a Glaſs Tube; as alſo that it will be in Proportion ſtronger as the Diameter of the Bore is ſmaller; for that the Efficacy of the Power follows the inverſe Proportion of the Diameter is evident from hence, that only ſuch Particles as are in Contact with the Fluid, and thoſe immediately above the Surface, can affect it.

3. Now thoſe Particles form a Periphery, or rather a very narrow *Annulus* or Ring contiguous to the Surface, the upper Part of which attracts and raiſes the Surface, and the lower Part, which is in Contact with it, ſupports and holds it up; ſo that neither the Thickneſs nor Length of the Tube avails any thing, only the ſaid Periphery of Particles, which is always proportional to the Diameter of the Bore.

4. The Quantity of the Fluid raiſed will therefore be as the Surface of the Bore which it fills; that is, as the Diameter, ſince the Effect would not be otherwiſe proportional to the Cauſe.

5. Since the Quantities follow the Ratio of the Diameters, the Heights to which the Fluid will riſe in different Tubes will be inverſely as the Diameters, which is thus demonſtrated:

Plate I. Fig. 15.

Let $\begin{cases} Q,\ q, \text{ repreſent the Quantities of Matter raiſed;} \\ D,\ d, \text{ the Diameters of the Tubes Bores;} \\ H,\ h, \text{ the Heights to which the Fluid riſes in the Tubes.} \end{cases}$

Then ſince Q, q, repreſent the Contents of two Cylinders of the Fluid (ſuppoſe G L and I M) it will be $Q : q : DDH : ddh$, from the Nature of a Cylinder; and from the Nature of this Attraction, it is $Q : q :: D : d$ (as was juſt now ſhewn;) therefore we have, As $D^2 H : d^2 h :: D : d$; and ſo curious and ſurprizing Phænomena of Nature: As why the Parts of Bodies adhere and ſtick ſo

$D^2Hd = d^2Db$, that is $DH = db$; conſequently, $D : d :: b : H$.

6. The Velocity with which it begins to riſe in the Tube is exceeding great, but preſently abates, and ſo continues by increaſing the Weight of the Fluid that is raiſed; and it will continue to riſe till an Equilibrium be made between the Gravity and the attracting Force of the Glaſs.

7. The Preſſure of the Air neither helps nor hinders the Riſe of the Fluids, for the Effect is the ſame *in Vacuo* as in the Air. But the more viſcid the Fluid, the leſs apt it is to riſe in the Tube.

8. The Power by which the Fluid is raiſed will keep it in the Tube for any Time without Exhaling or Evaporation, as I have tried by hanging ſeveral Capillary Tubes thus charged with their Fluids for ſeven or eight Months together in the Summer Sun, whoſe Heat did not appear to diminiſh the Fluids in the leaſt ſenſible Degree.

9. By the ſame Power the Water riſes between Glaſs Planes, ſo as to form the celebrated LOGARITHMETICAL CURVE, when they are ſo placed in Water as to touch on one Side, and be ſeparate or open'd to a ſmall Diſtance on the other. Thus ſuppoſe ABCD the Surface of Water, in which are placed the two Planes *fmo* and *emo*, ſo as to touch all along the Side *mo*, and yet open on the other Part, ſo as to contain a ſmall Angle *foe*; then biſect *of*, *od*, *oc*, *ob*, in the Points *d*, *c*, *b*, *a*; then will the Diſtances of the Planes in theſe Points be as the Numbers 1, 2, 4, 8, 16, and the Heights to which the Water will riſe over theſe Points will be inverſely proportional to thoſe Numbers; that is, whatever be the Heighth at *g* above the Surface AB, it will have twice that Height at *h*, and four Times that Height at *i*, and ſo on. So that the Superficies of the Water between the Planes will form the Curve *g*, *h*, *i*, *k*, *l*, which is the very ſame as that uſed in the Conſtruction of Logarithms, whence it has its Name.

Plate I. Fig. 14.

10. If two long Glaſs Planes are firſt ſmeer'd over with Oil and then ſet together at their Ends, and inclined to each other under a very ſmall Angle, and a Drop of the Oil ſo placed between them as to touch both Planes, it will immediately begin to move towards the touching Ends, or Angles of the Planes, and that Motion will be continued with an accelerated Velocity, by reaſon of the increaſing Attraction of the Planes, on ac-

firmly together; why ſome are *hard*, others *ſoft*; ſome *fix'd*, others *fluid*; ſome *elaſtic*, others *void*

count of the decreaſing Diſtance between them, and the larger Portion of touching Surface on each Side the Drop; concerning all which a great deal has been wrote to very little Purpoſe.

11. If Glaſs be applied to any other Fluid, whoſe Particles attract each other more ſtrongly than Glaſs attracts them, all the Phænomena of ſuch a Fluid in Capillary Tubes, and between Glaſs Planes, will be juſt the Reverſe of thoſe now mention'd of Water. Now Quick-ſilver is ſuch a Fluid, and therefore it will ſtand lower within a Capillary Tube than without; the Surface convex, and not concave, as in Water; and between the Planes it will move a contrary Way, &c.

12. But if a Baſon or Diſh be made of Copper or Braſs, and poliſh'd well within Side, and then tinn'd all over, Mercury put into ſuch a Veſſel will every where unite with the Tin, and may be properly ſaid to *wet* it, as Water does Glaſs; and the Mercury, if clean, put into this *Mercurial Baſon*, will be attracted and riſe all around the Sides, and have the ſame Phænomena with Water put into a wet Glaſs, Tranſparency only excepted.

13. Here I think proper to mention an Experiment, with which I have often entertain'd myſelf and others, and that is, pouring Mercury into the abovemention'd Diſh or Baſon form'd into a true Spherical Figure, and well planiſh'd, it leaves the concave Surface ſo nicely ſilver'd, that it makes the moſt delightful Mirrour, and reflects the Light ſo perfectly, as to produce the ſtrongeſt and moſt diſtinct Image of an Object that can poſſibly be effected by Art. How far the Nature and Application of Mirrours may be improv'd by this Method, muſt be left to farther and future Experience: And alſo whether it be not practicable by this Means to make a very large and moſt effectual Burning-Glaſs, at a very eaſy and cheap Rate.

14. A late *French* Author (L'Abbé *Nollet*) has thought fit to call in queſtion the Cauſe we have here aſſign'd for the Riſe of Fluids in Capillary Tubes, and has almoſt ſneer'd Dr. *Jurin*'s learned Diſſertations on that Subject; but I ſee nothing that could oblige him to do this, from what he has ſaid to explode it; no Man can be convinced by his *ipſe dixit*, that this is a mere Hypotheſis; the Phænomena ſo plainly indicate the Cauſe to be Attraction, and agree in every Particular ſo nicely to the Theory, that 'tis not to be expected we ſhould

of any elaſtic Quality; which all ariſe from the different Figure of the Particles, and the greater

give it up till we ſee better Reaſon for it than this reverend Father has given us in his prolix Digreſſion on this Subject. See his *Leçons de Phyſique Experimentale.* Tom. II. Pag. 421, &c.

15. But though the Attraction of the Glaſs is the indubitable Cauſe of the Aſcent of Fluids, yet it muſt be allow'd that the Nature and Genius of the Fluid is to be regarded in moſt of the *Phænomena*, which are not proportion'd to the attracting Power of the Glaſs only, but to the Power conjointly with the various Diſpoſition of Fluids to yield thereto; nor is the Denſity of the Fluid of primary Conſideration, it being evident by Experiment, that ſome lighter Fluids will riſe to a leſs Height than others which are much heavier: Thus Spirit of Wine will riſe but 4½ Tenths of an Inch in a Tube, where Oil of Tartar will riſe 9, as in the Table below. Nor does it at all depend on the Viſcidity or Tenacity of Parts; for hard white Varniſh (very thick and viſcid) and Spirit of Wine aſcend nearly to the ſame Height. I am inclined to think from the Experiments which I have made, that the Aſcent of Fluids depend greatly upon the Spirit they contain, and that not on the Quantity of it, but on ſome peculiar Action or Diſpoſition thereof, relative to the attracting Power of the Glaſs. So from the Table it appears, that Spirituous Liquors riſe to a ſmall Height in general; but Water, which has none, to the greateſt; yet Spirit of Hart's-horn and Spirit of Urine riſe higher than Ale and Small-Beer, which contain but a ſmall Proportion of Spirit.

16. But to ſet this Matter in a clear Light, I have very carefully made the Experiment on the following Fluids, which were each of them often repeated, and with the ſame Succeſs; the Bore of the Tube was about $\frac{1}{10}$ of an Inch in Diameter.

	Altitude.
Common Spring Water	12 Tenths of an Inch.
Spirit of Urine	11
Tincture of Galls	11
Recent Urine	10
Spirit of Salt	9
Ol. Tar. per Deliquium	9
Vinegar	9½
Small Beer	9
Strong Spirit of Nitre	8½
Spirit of Hart's-horn	8½

or leſſer Degree of Attraction conſequent thereupon. Hence the Reaſon why Flies walk on the

	Altitude.
Cream	8 Tenths of an Inch.
Skimm'd Milk	8
Aqua fortis	7½
Red Wine	7½
White Wine	7½
Ale	7½
Ol. Sulph. per Campanam	6½
Oil of Vitriol	6½
Sweet Oil	6
Oil of Turpentine	5½
Geneva	5½
Rum	5
Brandy	5
White hard Varniſh	5
Spirit of Wine	4½
Tincture of Mars	4½

17. From other Experiments it appears that Heat and Cold are not concern'd in the Aſcent of Fluids, the hotteſt Water ſtanding at the ſame Height as the Cold. Alſo Salts of any kind diſſolved in Water makes but little Alteration in the Heights, for when Water has taken up all it can in the Solution, it will riſe to 10 or 10½, which ſmall Deficiency is owing undoubtedly to the increaſed Gravity of the Fluid.

18. There is one Phænomenon of Capillary Tubes very extraordinary, which is this; if a large bored Tube N P be at one End P drawn out into a fine Capillary as at O, ſuch a Tube fill'd with Water, and then inverted into a Veſſel of Water, as A B C D, will retain all the Water ſuſpended through its whole Length, if it does not exceed the Height to which a Capillary of the ſame Bore, as at O, will attract it. To ſee the Reaſon of this, we muſt conſider the Motion of a ſmall Quantity of Water C D in a tapering Capillary A B in an horizontal Poſition. And here we muſt aſſign two Principles of Motion in the Fluid C D; one is the *attracting Power* of the Glaſs at C and D, the other is the *Velocity* of the moving Fluid in theſe Places. For the *Momentum* of any Body, whether Solid or Fluid, is always proportion'd to the Intenſity of the moving Power, and the Velocity of Motion. (As is ſhewn *Annotat.* XXIII. 1. XXXVIII. 7, 8, 9. LXX. 1, 2.) But the moving Power, or Attraction at C is to that at D as the

Plate I. Fig. 15.

Fig. 16.

Surface of the Water, and wet not their Feet. On this Principle we account for the Manner how

Diameter of the Bore at C to the Diameter thereof at D; Or, putting a, A, for the two Attractions in D and C, and *d*, D, for the Diameters, we have $a : A :: d : D$.

19. Again, the Velocities (V and v) of the Water in thoſe Points are as the Squares of the Diameters inverſely (as will be ſhewn hereafter;) that is, $V : v :: DD : dd$. Therefore $V \times a : v \times A :: D^2 d : d^2 D :: D : d$. That is, *the whole Motion of the Water at* D *is to that at* C *as the Diameter at* C *to that at* D; *conſequently the Water will move towards the End of the Tube* B.

20. Becauſe when the Water moves in the Tube, the Velocity at D is greater than that at C, and therefore the Spaces deſcribed in equal Time by the Extremities of the Fluid unequal, it follows that the Water will move from D towards B with an accelerated Velocity, the Difference between the Diameters *d* and D continually increaſing. This likewiſe ſhews the Reaſon of the accelerated Motion of the Drop of Oil between the two inclined Glaſs Planes, as mention'd *Art.* 10.

Plate I. Fig. 17.

21. The Motion of the Water once begun at D, will continue till it has with great Rapidity reach'd the End B; and if then the End B be rais'd above the horizontal Line A F, yet will the Water be detain'd at B, and not deſcend, till the perpendicular Altitude B F is equal to the Height to which the Water would riſe in a Capillary of the ſame Bore as at B.

Plate I. Fig. 18.

22. Hence alſo if the Tube be held in an upright or perpendicular Poſition, as A B, the Water will not deſcend, unleſs its Weight be ſuch as when added to the Power acting at C towards A, the Sum ſhall be greater than the Power acting upwards at B; which Power ariſes from the Attraction of the Tubes, the Velocity of the Fluid, and the Coheſion of its Particles; for unleſs we conſider this Coheſion of the Particles, the Weight of the Water, when the Veſſel is large, will be too great to ſuffer a Suſpenſion of the Fluid by the other Forces in the Capillary Part.

23. Nor is this Coheſion of the Parts a *Petitio Principii*, but what we know by Experiment to be the Cauſe of ſuch a Phænomenon when nothing elſe can be ſo; thus we find the Particles in a Drop of Water hang together by Coheſion, till the Gravity of the whole draws it from the Part to which it coheres. Without this Coheſion there could be form'd no Drop at all.

24. Hence likewiſe it is, that a Column of Mercury well

Plants imbibe the nutritive Juices or Moiſture of the Earth by the Fibres of the Roots: Alſo for the Riſe of the Sap in Vegetables, and for the whole Oeconomy of Vegetation in general. Hence the *Rationale* of the various Secretions of Fluids by the Glands of an Animal Body, and their wonderful Circulation through the fine Capillary Veſſels. Hence alſo the Reaſon of Soldering and Gilding of Metals; alſo of Melting or Fuſion by Heat. Hence alſo the Exhalation of Vapours by the Heat of the Sun or Fire; the Aggregation of aqueous Particles in the Air, forming the Drops of Rain. We hence ſee the Reaſon of Diſtillation, Filtration, Diſſolution, Digeſtion, Sublimation, Precipitation, Cryſtallization, and all the other Operations of Chemiſtry and Pharmacy, which are no otherways to be accounted for. Laſtly, we find Sir *Iſaac Newton* (at the End of his *Optics)* gives a beautiful and clear Solution to thoſe wondrous Phænomena of ſubterranean Accenſions and Exploſions;

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purged of Air, will be ſuſpended by Coheſion to the Height of 70 or 75 Inches, when the Air alone will ſuſtain it to no greater Altitude than that of 31 Inches at moſt. Dr. *Jurin* ſuppoſes the Preſſure of an elaſtic ſubtile Medium (which can penetrate the Pores of Glaſs and Water) to have ſome Part in producing this very odd Phænomenon. But if all we can learn from Experience be thought inſufficient, I ſhall leave the Reader to amuſe himſelf with what Conjecture he ſhall think fit; aſſuring him in the laſt Place, that this wonderful Suſpenſion of the Fluid in ever ſo large a Glaſs, as A C, whoſe Top terminates in the Capillary B, is not owing to the Preſſure of the Air, becauſe the ſame thing will happen in an exhauſted Receiver, as I always ſhew among the Experiments of the Air-Pump. See *Fig.* 19. *Plate* I.

ſions; of Volcano's and Earthquakes; of Hot Springs, Damps, and ſuffocating Exhalations in Mines, *&c.* on the Principles of this Sort of Attraction and Repulſion (XIII).

(XIII) 1. The *Rationale* of the ſeveral important Particulars here mention'd will eaſily appear from the preceding Principles or Properties of Matter. Thus HARDNESS of Bodies ariſes from ſuch Figures of the Particles of Matter as permit them to touch by large Surfaces, and thus to have a great Power of Attraction; and at the ſame Time, ſo to fit and fall in with each other, that they prevent any Motion enſuing among themſelves by any external Preſſure; whereas in another Sort of Figure, the Particles may be admitted to touch by pretty large Surfaces, yet ſuch as not confining each other, will permit the Motion impreſs'd on the external Parts to proceed among the interior Particles, which will thus eaſily move among themſelves, and make what we call Softneſs in Bodies.

2. FIXITY is much the ſame as Hardneſs, and owing to the ſame Cauſe, and is oppoſed to FLUIDITY, which may in ſome Senſe be eſteem'd the greateſt Degree of Softneſs, becauſe a fluid Body yields to the leaſt Preſſure, and therefore the attractive Power, by which the Particles of a Fluid cohere, muſt be the leaſt poſſible; but this reſults from ſuch a Figure as admits of the feweſt Points of Contact poſſible, that is, a *Spherical Figure.* And hence 'tis evident that Bodies are more or leſs fluid as their Parts approach more or leſs to a globular Figure. All which follows from *Annotat.* III. 2.

3. ELASTICITY in Solids ariſes from the ſame Principle of corpuſcular Attraction; thus if a Steel Spring, or Wire, or Piece of very thin Glaſs, be bent out of its natural Poſition, the Particles on the convex Part are forced from that intimate Contact they before had; and on the concave Part they are forced nearer together, or harder upon each other, than in the natural State; in both Caſes there will be a conſiderable Reſiſtance to be overcome, and require a ſuperior Force. During this State of the Particles they may be ſaid to be under a Sort of Tenſion on one Side, and Compreſſion on the other; and ſince by this Force they are not drawn out of each other's Attraction, as ſoon as the Force is remitted or ceaſes to act, the attractive Power reduces the Particles, and unbends the Wire by reſtoring it to its *natural State and Site.* Now 'tis well known that many Subſtances are compoſed of ſuch fibrous Parts or Filaments, which reſemble fine Wire, and are in-

terwoven and diſpoſed in ſuch a Manner (as in *Sponge* for inſtance) that they cannot be compreſs'd without being bent or wreſted from their natural Poſition, whence all ſuch Bodies will in ſuch a Caſe exert a Force or Spring, and thus prove *elaſtic*. This Theory may be applied to Bodies of a cloſer Texture, as *Cork*, *Wood*, *Ivory*, *Glaſs*, &c. And ſuch Matter as wants this Texture and Diſpoſition of Parts, will not exert ſuch a Spring or Force, but take the Figure ariſing from the impreſs'd Force, and thus prove what we call *Non-Elaſtic Bodies*.

4. SOLDERING is another notable Effect of this coheſive Power; for when the Surfaces of two Pieces of Metal are made clean, or freed from all intervening Matter beſides the Solder, the Solder being then flux'd, its Particles will freely enter the Pores, or come into Contact with thoſe of the poliſh'd Surfaces of Metal, and by that Means adhere firmly to each, and therefore when the melted Solder becomes cold and fix'd, it holds the Surfaces exceeding faſt together, in the Nature of a GLUE or CEMENT, which are all of the ſame kind, but differ in the Degree of the coheſive or attracting Power of the Parts.

5. MELTING or FUSION of fixed Bodies depends upon the ſame Theory; for when the Particles of a fixed Body are ſeparated by the Action of Heat ſo far as not to touch, or but very ſlightly, yet not ſo far as to be out of each other's Attraction, they then will eaſily move by each other, and put on all the Appearances of a fluid Body, (as plainly follows, from *Annotat*. X. 4.)

6. EVAPORATION proceeds likewiſe from hence, that when the Particles are ſo far ſeparated by Heat as to be without each other's Attraction, they then begin to repel each other, and thus will ſeem to riſe from the Surface of the Fluid in Form of a Vapour, or Body of Particles, which are at equal Diſtances from each other; and becoming thus ſpecifically lighter than the ſame Bulk of airy Particles, they will riſe in the fluid Body of Air till they come to that Part of it which has the ſame Gravity, and they will there make what we call CLOUDS, which will move this Way or that, according to the Current of Air in thoſe Regions. See *Annot*. X. 5, 6.

7. The VAPOURS thus raiſed become the Original Matter of all METEORS, one Degree of Cold condenſing them into larger Globules, which fall in Drops of RAIN; a greater Degree producing a Fixedneſs or Coagulation of the Particles, which ſhoot like Salts into various curious Forms, and make the *Fleaks* of SNOW; a third and ſtill greater Degree of Cold congeals the Vapours into a harder Subſtance, greatly varie-

gated in Form and Conſiſtence, and thus produces HAIL. If the Cold condenſes the Vapour ſo that it can't riſe high above the Surface of the Earth, it will there hover about, and fill the lower Air with an obſcuring FOG or MIST: Or, if the Cold be more intenſe, it freezes the Miſt to every Twig and Blade of Graſs in form of a white Incruſtation, which we call a RIME. If the Air be warm, ſo that the Vapour therein be too fine to be viſible in the Day-time, it will yet be condenſed by the Coolneſs of the Evening ſo far as to deſcend, and ſettle upon the Tops of Graſs in the Form of DEW: But, laſtly, if the Evenings of ſuch a fine Day be cold enough to freeze, then inſtead of a Dew there will appear a WHITE FROST all over the Surface of the Ground.

8. The CAPILLARY SYPHON is a moſt curious *Phænomenon*, ariſing from the ſame Principle; for the Water being raiſed to the Flexure or bended Part, by the attracting Force of the ſhorter Leg immerſed into it, the ſame Force continuing to act carries it even into the longer Leg, and there conſpiring with the Force of Gravity, the Water precipitately deſcends and drives the Air before it, and thus would keep running or dropping out till all the Water were exhauſted to the Orifice of the immerſed Leg, if the Operation were to be continued long enough.

9. The FILTER is to be conſider'd as nothing more than a *Compound Capillary Syphon*; for the Threads or Filaments of which it conſiſts, lying very near together, make long and ſlender Vacuities or Interſtices which repreſent ſo many Capillary Syphons, which attract, raiſe, and decant of the Fluid in a conſiderable Quantity, proportional to their Number or the Largeneſs of the *Filter*.

10. After the ſame Manner it is, undoubtedly, that the Humidity of the Earth is drawn into the Subſtance of the Roots of Plants and Vegetables, which we know conſiſt of long and very minute Fibres ſo diſpoſed as to form an infinite Number of tubular Interſtices, which act in the Nature of a Filter, and imbibe the Juices and Moiſture deſtin'd for the Nutriment and Growth of the Plant.

11. Thus alſo it is reaſonable to ſuppoſe the fine Expanſion or Ramification of the *Lacteal*, the *Lymphatic*, and *Sanguiferous Veſſels* thro' all the Subſtance of the *Glands* and *Viſcera* in the fineſt Capillary Tubes, does greatly aſſiſt, if not wholly promote, the Circulation of the Blood and Juices, in order to the various Secretions of each reſpective Gland; an Effect too great for the Pulſive Force of the Heart, or larger Veſſels, when thus ſo infinitely and minutely divided, were it not to be rejoin'd by the ſtrong Attraction of thoſe Capillary Parts.

12. By this Means it is that Tallow and Oil riſe into the Wicks of Candles and Lamps: That Sponge ſo readily ſucks up and retains ſo great a Quantity of a Fluid: That Tubes of Aſhes or Sand will thus raiſe Water to the Height of ſeveral Feet. 'Tis by this Virtue that the Pen retains Ink, and the Paper draws it forth in Writing; or refuſes to admit it, if oil'd, by the Repellency of the Particles of Oil.

13. By this Power fix'd Bodies are diſſolved by proper Mediums; as *Sugar* and all Kinds of *Salts* by Water; *Silver*, *Copper*, *Tin*, &c. by *Aqua-Fortis*; Gold by *Aqua-Regia*; and other Bodies in other Menſtruums. The Reaſon is, that when the Particles of the Solid and Fluid attract one another more ſtrongly, than either thoſe of the Solid or thoſe of the Fluid attract each other ſeparately, then a Separation of the Parts or *Diſſolution* muſt enſue. For ſince, for Inſtance, if a Particle of the Fluid be attracted with a greater Force than a contiguous Particle of the Solid, that Fluid Particle will ruſh upon the other with ſo great an Impetus, as inſtantly to diſplace it, and ſeparate it from the Solid Particle to which it before adhered with a weaker Force; and thus we are to conceive of every other Particle, till the Whole be diſſolved, which will then end when each Fluid Particle has fill'd its Sphere of Attraction with thoſe of the Solid; and the Fluid is then ſaid to be *ſaturated*, and will diſſolve no more.

14. PRECIPITATION likewiſe is a Conſequence from the ſame Principle; for if to the Solution of any Solid ſome other Solid or Fluid Body be added, whoſe Particles are attracted by the Fluid with a greater Force than thoſe of the diſſolved Solid, then will the former take Place of the latter, which, being diſengaged, will fall to the Bottom and ſettle in Form of a very fine Powder; thus is Silver precipitated by Copper, which the *Aqua-Fortis* attracts more powerfully than Silver. Thus a Solution of Copper is again precipitated by Iron, and other Solutions have their reſpective Precipitants.

15. FERMENTATION is alſo an Effect of the ſame Cauſe, inaſmuch as it conſiſts in the internal Agitation and Commotion of the Parts of the fermenting Body, ariſing from their mutual Action and Attraction ſo often mention'd. By this means Bodies are either wholly diſſolv'd, as has been ſaid; or have their Parts ſo far alter'd and compounded with other Matter, that they ſeem to change their Natures, and acquire quite new Properties and Principles. Thus Malt by Fermentation yields a Spirit by Diſtillation, tho' nothing like it before; not to mention Putrefaction, and the many notable Phænomena thence ariſing.

16. The HEAT, EBULLITION, and EXPLOSION of ſundry Mixtures are owing to the violent Action and great Rapidity of Motion among the conſtituent Particles; for all Heat and Accenſion proceeds from an Attrition or Rubbing of the Particles of Matter one among another. Thus *Aqua-Fortis* and the Particles of Copper act ſo violently on each other as to cauſe a great Efferveſcence and Heat; and Steel-Filings produce a Blaſt or Exploſion inſtantly. Again Oil of Caraway-Seed, pour'd on compound Spirit of Nitre, kindles to Flame with ſuch prodigious Force, that *in Vacuo* it has blown up and burſt an exhauſted Receiver ſix Inches wide and eight Inches deep.

17. In accounting for THUNDER and LIGHTNING we have recourſe to the ſame Principles, that is, a Fermentation between the ſulphureous Steams exhaled from the Earth, and the Nitrous Acids or Salt in the Air, which take Fire, and cauſe Exploſions in the ſame Manner as in the Experiments juſt mention'd. And in ſome ſuch Manner all other *fiery Meteors* are produced.

18. Alſo that Burning-Mountains, Mineral Coruſcations and Suffocating Damps, Earthquakes, Hot Springs, *&c.* are produced by the inteſtine Commotion ariſing from the Mixture of divers Nitro-acid and Sulphureous Matters in the Bowels of the Earth, is evident from a common and eaſy Experiment, which is by taking equal Parts of Sulphur and Steel-Filings, and working them together, with a little Water, to the Conſiſtence of a Paſte; then putting it under Ground, it will preſently begin to heat, and in a few Hours take Flame and blow up the Earth about it with a conſiderable Shock, ſo as to repreſent an Earthquake, and a Sort of Volcano in Miniature.

19. That Metals diſſolved in Fluids diffuſe themſelves equally thro' all the Subſtance, and keep an equal Diſtance among themſelves, ſeems owing to a repulſive Virtue. Alſo the *Reflection* and *Inflection* of the Rays of Light from Bodies before they come to touch them as alſo the Emiſſion of Light in luminous Bodies, ſeem effected by ſuch a repulſive Power; which (as has been ſaid) commences where the Sphere of Attraction ends.

20. The Spherical Figure, which the Drops of a Fluid affect being left to themſelves, is another Conſequence of Corpuſcular Attraction; for conſidering one ſingle Particle of the Fluid, its attractive Force being every Way equally exerted to an equal Diſtance, it muſt follow that other Fluid Particles are on every Side drawn to it, and will therefore take their Places at an equal Diſtance from it, and conſequently form a

ſpherical or perfectly round Superficies; in like Manner as th Ea:th and Sea affect a ſpherical Figure from the central Ar traction being every way equally directed.

21. The ſurpriſing Phænomena of Glaſs Drops (ſometimes call'd Prince *Rupert*'s Drops) are moſt probably ſuppoſed to ariſe from hence, *viz.* That while the Glaſs is in Fuſion or melted State, the Particles are in a State of Repulſion; but being dropp'd into cold Water, it ſo condenſes the Particles in the external Parts or Superficies, that they are thereby reduced within each other's Attraction, and by that means form a Sort of hard Caſe which keeps confin'd the aforeſaid Particles in a repulſive State; but when this Caſe is broke by breaking off the Tail of the Drop, the ſaid confined Particles have then Liberty to exert their Force, which they do by burſting the Body of the Drop, and reducing it to a very peculiar Form of Powder. This Theory ſeems to be corroborated by making the Drop red hot, and letting it cool gently in the open Air, for then it has no ſuch Effect. Yet another Experiment ſeems to impugn this Hypotheſis; and that is, by grinding away any Part of the Drop on a Grind-ſtone, the remaining Part continues entire; and yet there appears no manifeſt Reaſon why it ſhould not break or burſt into Duſt, if the internal Particles be the Cauſe of it, ſince by this Means they muſt needs be ſet at Liberty in the moſt ample Manner poſſible: Unleſs it be, that in Grinding, the Vacuities between the internal Particles are fill'd up with the Matter of the Stone forced in by the hard Attrition, and by this means, fixing the Particles of the Glaſs next the Stone, and thus deſtroying their repulſive Force, conſtituting, as it were, another Sort of hard external Caſe which confines the Particles no leſs than the other. And this ſeems to be confirm'd by that Part which touch'd the Stone remaining entire, when all the reſt ~~of the Stone~~ is reduced to Duſt by breaking off the Tail or ſmall Part, as any one will find to be the Caſe on Trial.

SCHOLIUM.

By this wonderful and moſt extenſive Power of Corpuſcular *Attraction* and *Repulſion*, we account for moſt or all the Phænomena of Nature, ariſing from the heterogeneous Mixture and various Aſſociations of Particles of different Bodies; but why, in ſome Caſes, thoſe Particles ſhould readily attract, and in others as ſtrangely repel each other, there has as yet been no demonſtrative Reaſon aſſign'd. The Hypotheſis of a *Polarity*, or *Magnetic Power*, in every Particle of Matter, by which they are diſpoſed on one Part to attract, and on the the other to repel, in the Manner of the Load-ſtone,

is

THE *Second Species of Attraction* is that of ELECTRICAL BODIES, as *Glaſs*, *Amber*, *Sealing-Wax*, *Jet*, &c. the principal Properties of which are as follow. (1.) Thoſe Bodies attract others which are very light, as Feathers, Hairs, Leaf-Braſs, *&c.* (2.) The Sphere or Extent of this attracting Power is at the Diſtance of ſeveral Feet; but, (3.) It varies with the State of the Weather, being greateſt in hot and dry Weather, but weaker in warm and moiſt. (4.) It may be communicated to a great Diſtance, *viz.* ſeven or eight hundred Yards, by the Intervention of a proper Body, as Hempen Strings, *&c.* (5.) This Virtue is excited by Attrition, or hard Rubbing by the Hand or with a Piece of Cloth, but will not be produced by the Warmth of Fire. (6.) It penetrates or pervades the Pores of Glaſs; and, (7.) It may be communicated to other Bodies, ſo as to render them electrical. (8.) If the Sphere of Attraction be interrupted on any Part, it deſtroys the Efficacy of the Whole. (9.) By this Virtue Bodies are not only attacted, but alſo repelled alternately to a very ſenſible Diſtance, and with a ſurpriſing Velocity. (10.) The Body once repell'd from the Tube will not be again

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is as ſpecious as it is ingenious; but ſince the *Newtonian Philoſophy*, in fundamental Principles eſpecially, is founded entirely on the Baſis of experimental Proof, or Demonſtration from Geometry, we ought to be careful how we introduce any Conjecture for Truth, unleſs all Circumſtances conclude and urge the ſame, which I believe is not the Caſe of that above-mention'd.

attracted by it, till it has firſt touch'd ſome other Body. (11.) Bodies attracted, and ſticking to the Tube, will be then attracted by other Bodies not electrical; as the Finger, *&c.* (12.) This Virtue will exert itſelf *in Vacuo*, as well as in open Air. (13.) It appears lucid, and ſparkles like Fire, in a dark Room. (14.) It is alſo ſenſible to the Ear by a crackling Noiſe, like a green Leaf in the Fire (XIV).

(XIV). 1. To the Properties of Electrical Bodies, here enumerated, we ſhall add the following.—This Quality is of two Sorts, *viz.* (1.) *Vitreous Electricity*, or that which belongs to *Glaſs*; and (2.) *Reſinous Electricity*, or that which belongs to *Amber*, *Roſin*, *Wax*, *Gums*, and ſuch like Subſtances. They are different Kinds, becauſe their Effects are contrary; for the light Bodies repell'd by one will be attracted by the other, tho' excited, which cannot happen when both Bodies have the ſame attractive Power.

2. All Vegetable Subſtances differ from Animal ones in this reſpect alſo; for an Hempen or Flaxen Thread or String will receive the Electricity of Glaſs, and carry it inſtantaneouſly to an incredible Diſtance, to ſome non-electric Body, but will not impart it to a Silk or Hair Line by which it is ſuſpended. This is the Reaſon why ſeveral Men ſtanding on Cakes of Roſin or Wax (which will not receive the *Vitreous Electricity*) convey that Virtue from one to the other in a moſt ſurprizing Manner. Thus alſo it is that a Feather ſuſpended by a Flaxen Thread will be alternately attracted and repell'd, but if hung by a Silken String will not be again attracted after it is once repell'd; and *vice verſa*, with reſpect to Amber or Sealing-Wax.

3. Hitherto we have not diſcover'd the Uſes of this wondrous Quality; but Mr. *Gray*, juſt before he died, hit upon an Experiment of this Kind, which ſeem'd to indicate that the Attractive Power, which regulates the Motions of the Heavenly Bodies, is of the Electric Kind. The Experiment was thus; he fix'd a large round Iron Ball upon the Middle of a large Cake of Roſin and Wax, and exciting the Virtue ſtrongly in the Cake, a fine Feather ſuſpended by a Thread, and held near the Iron Ball, was carried round it by the Ef-

THE *Third Species of Attraction* is that of the MAGNET, or LOADSTONE; the primary Properties whereof are the following. (1.) Every Loadſtone has two Points call'd *Poles*, which emit the Magnetic Virtue. (2.) One of thoſe Poles attracts, the other repels *Iron*, but no other Body. (3.) This Virtue is communicated to Iron very copiouſly by the Touch, which renders it ſtrongly Magnetic. (4.) A Piece of Iron ſo touch'd by the Loadſtone, and nicely ſuſpended on a ſharp Point, will be determined to ſettle itſelf in a Direction nearly *North* and *South*. (5.) The End of the Needle touch'd by the *South* Pole of the Stone will point *Northwards*; and the contrary. (6.) Needles touch'd by the Stone will dip below the Horizon, or be directed on the touch'd Part to a Point within the Earth's Surface. This is call'd the *Dipping Needle*. (7.) This Virtue is alſo to be communicated to Iron, by a ſtrong Attrition all one way; whence Files, Drills, *&c.* are always found to be Magnetical. (8.) Iron Rods or Bars acquire a Magnetic Virtue by ſtanding long in one Poſition. (9.) Fire totally deſtroys this Virtue by making the Stone or Iron red hot. (10.) This Power is exerted ſenſibly to the Diſtance of ſeveral Feet. (11.) It is ſenſibly continued through the Subſtance of ſeveral contiguous Bodies or Pieces of Iron, as

fluvia in a circular Manner, and perform'd ſeveral Revolutions; it moved the ſame Way with the Planets, from Weſt to Eaſt, and its Motion (like theirs) was not quite *circular* but a little *elliptical*. See the whole Account in *Philoſ. Tranſ.* N°. 444.

Keys, &c. (12.) It pervades the Pores of the hardeſt Bodies; and (13.) Equally attracts the Iron *in Vacuo*, as in open Air (XV.)

(XV.) 1. To the above Properties of the Loadſtone I ſhall add the following, viz. *That the ſame Pole of the Stone will communicate to a Piece of Iron the Power of attracting or repelling the ſame End of a touch'd Needle, by drawing it different Ways thereon.* Or thus, If a Piece of Iron be drawn to the Right Hand and attracts, it will repel if drawn to the Left, which is not a little wonderful.

2. I have oftentimes, by a ſmart Stroke of a Hammer on the untouch'd End of the Dipping-Needle, cauſed the Magnetic Virtue to come all to that End from the other, ſo as to make it dip on that Side as much as it did on the other Side before: And on the contrary, I have by ſuch a Stroke ſometimes made it dip much more on the touch'd End than before. Again, ſometimes by ſtriking it, the Needle, which dipp'd before, will be reſtor'd to its Equilibrium, as if the Virtue had made its Eſcape, or were uniformly diffus'd over all the Needle. So capricious are the Phænomena of this amazing Power.

3. There is a very curious Method of rendering viſible the Directions which the Magnetic Effluvia take in going out of the Stone: Thus, let AB, CD, be the Poles of the Stone; Plate II. Fig. 1. about every Side gently ſtrew ſome Steel-Filings on a Sheet of white Paper; theſe ſmall Particles will be affected by the Effluvia of the Stone, and ſo poſited as to ſhew the Courſe and Direction of the Magnetic Particles on every Part. Thus in the Middle of each Pole between AB, and DC, it appears to go nearly ſtrait on; towards the Sides it proceeds in Lines more and more curved, till at laſt the Curve-Lines from both Poles exactly meeting and coinciding, form numberleſs Curves on each Side, nearly of a circular Figure, as repreſented in the Diagram. This ſeems to ſhew that the Magnetic Virtue emitted from one Pole circulates to and enters the other.

4. The Cauſe of the Variation of the Needle has remain'd hitherto without any demonſtrative Diſcovery; yet ſince its Declination, and Inclination, (or Dipping) do both of them manifeſtly indicate the Cauſe to be ſomewhere in the Earth, it has given Occaſion to Philoſophers to frame Hypotheſes for a Solution, which make the Earth a large and general *Magnet* or Loadſtone, of which all the leſſer ones

THESE, and many others are the Properties of a Body, not more wonderful than uſeful to Mankind.

are but ſo many Parts or Fragments, and, being poſſeſs'd of the ſame Virtue, will, when left to move freely, have the ſame Diſpoſition, and Similarity of Poſition, and other Circumſtances.

5. The moſt conſiderable of theſe Hypotheſes is that of the late ſagacious Dr. *Halley*, which is this; *The Globe of the Earth is one great Magnet, having four Magnetical Poles or Points of Attraction, near each Pole of the Equator two; and that in thoſe Parts of the World which lie near adjacent to any one of thoſe Magnetical Poles, the Needle is chiefly govern'd thereby, the neareſt Pole being always predominant over the more remote one.* Of the North Poles, that which is neareſt to us, he ſuppoſes to be in the Meridian of the Land's End, and about ſeven Degrees from the North Pole of the World, which governs the Variations in *Europe*, *Tartary*, and the *North* Sea; the other he places in a Meridian paſſing thro' *California* about fifteen Degrees from the North Pole of the World, which governs the Needle in North *America*, and the Oceans on either Side. In like Manner he accounts for the Variations in the Southern Hemiſphere. See *Philoſ. Tranſ.* N°. 148.

6. The Variation of the Needle from the North and South Points of the Horizon not being the ſame, but variable in different Years and in a diverſe Manner in different Parts of the Earth, made the *Doctor* farther conjecture, that two of the Magnetic Poles were fix'd, and two moveable; and in order to make this out, he ſuppoſes the external Part of the Earth to be a Shell or *Cortex*, containing within it a Magnetic moveable *Nucleus*, of a globular Form, whoſe Centre of Gravity is the ſame with that of the Earth, and moveable about the ſame Axis.

7. Now if the Motions of both the Shell and the *Nucleus* were the ſame, the Poles of each would always have the ſame Poſition to each other; but he ſuppoſes the Motion of the *Nucleus* to be a very ſmall Matter leſs than that of the Shell, which yet is ſcarce ſenſible in 365 Revolutions; and if ſo, the Magnetic Poles of the *Nucleus* will by ſlow Degrees change their Diſtance from the Magnetic Poles of the Shell, and thus cauſe a Variation in that Needle's Variation, which is govern'd by the moveable Pole of the *Nucleus*; while that Variation which reſpects the fix'd Poles of the Magnetic Shell

remains more conſtant; as in *Hudſon*'s Bay the Change is not obſerved to be near ſo faſt as in theſe Parts of *Europe*.

Plate I. Fig. 20.

8. What ſeems a little ſtrange is, that the *Doctor* has no where, that I know of, undertook to account for the Dipping of the Needle by this Hypotheſis; tho' the Invention of this (by Mr. *Blagrave*) was before that of the Change of the Needle's Variation (by Mr *Gellibrand*); nor do I ſee at preſent which way this Phænomenon is explicable by it. But we have not yet ſo many accurate Obſervations of the Needle's Inclination as we have of its Variation, which is its only uſeful Property. By ſeveral Experiments Mr. *Graham* has very accurately made it appear, that the Quantity of the Needle's Inclination to the horizontal Line was an Angle of about 74 or 75 Degrees: That is, ſuppoſe AB a touch'd Needle, ſupported on the Point C of the Pin CD, it will contain an Angle ACH or BCO with the horizontal Line HO of 74 or 75 Degrees.

Plate II. Fig. 2.

9. The Variation of the Needle has within a Century paſt undergone a remarkable Alteration; for at *London* it was obſerv'd by Mr. *Burrows* in the Years 1580 to be 11°. 15'. Eaſt; that is, if N, S repreſent the North and South Points of the Horizon, and E, W, the Eaſt and Weſt Points, the Needle then had the Situation AB, ſo that the Arch NB = 11°. 15'. After that, in the Year 1622, it was obſerv'd by Mr. *Gunter* to be but 6° Eaſt. In the Year 1634 Mr. *Gellibrand* obſerved it to be 4°. 5'. Eaſt. In 1657, it was obſerv'd by Mr. *Bond* to be nothing at all, that is, the Needle placed itſelf then in the Situation SN, and pointed directly to the North. After this, in the Year 1672, Dr. *Halley* obſerv'd it to be 2°. 30'. Weſtward; and again in the Year 1692, he found it 6°. Weſt. Since then, in the Year 1722, Mr. *Graham*, by moſt accurate Experiments, found it to be 14°. 13'. And at preſent it is between 15 and 16 Degrees; and in ſome Places I have found it 18 Degrees Weſtward.

10. The Variation of the Declination and Inclination of the Needle is variable and ſubject to no regular Computation. What the Quantity of both Sorts of Variation is in the ſeveral Parts of the World, will be ſhewn in Dr. *Halley*'s Map of the World, improved from the Obſervations of Mr. *Pound*, which is inſerted at the End of the Lecture on Wind and Sound.

11. I think the Law of Magnetic Attraction is not yet aſcertain'd; Sir *Iſaac Newton* ſuppoſes it to decreaſe nearly in the *Triplicate Ratio* of the Diſtance: But Dr. *Helſham*, trying the Experiment with his Loadſtone, found it to be as the *Squares of the Diſtances* inverſely. And the Power of my

Loadſtone decreaſes in a different Manner from either, it being in the Seſquiplicate Ratio of the Diſtances inverſely. For Exactneſs, I made a ſquare Bar of Iron juſt ¼ of an Inch thick, and then provided three Pieces of Wood of the ſame Form and Thickneſs preciſely. Then poiſing the Loadſtone very nicely at the End of a Balance which would turn with leſs then a Grain, I placed under it the Iron with firſt one Piece of Wood, then two Pieces, and laſtly all three Pieces upon it, by which means the Steel Points of the Pole were kept at ¼, ½, ¾ of an Inch from the Iron, and in thoſe Diſtances the Weights put into the oppoſite Scale to raiſe the Loadſtone from the Wood (which it touch'd while the Beam was horizontal) were as follows:

Diſtances	*Grains.*	*Ratio of the Squares.*	*Ratio of the Cubes.*	*Seſquip. Ratio.*
¼	156	156	156	156
½	58	39	19	56
¾	28	17	6	30

12. Whence it appears, that the Numbers of Grains to counter-act the Power of the Loadſtone in theſe Diſtances approach very near, and are almoſt the ſame with thoſe which are in the *Seſquiplicate Ratio*, but are widely different from thoſe which are in the *Duplicate* or *Triplicate Ratio*; and this Experiment I tried ſeveral times for each Diſtance, and with ſcarce any Variation in the Succeſs.

13. The Power or Force of Magnets is generally greater in ſmall than in large ones, in Proportion to their Bulk. It is very rare that very large ones will take up more than 3 or 4 times their own Weight, but a ſmall one is but tolerably good that will take up no more than 8, 10, or 12 times its Weight. The Honourable Mr. *Berkeley*, at *Bruton*, has one whoſe Weight (with the Armature) is but 43 Grains, which will take up 1032 Grains, which is 24 times its Weight. But that of Mr. *Newton* (which he wears in his Ring, inſtead of a Diamond of leſs Value) which weighs ſcarce THREE *Grains*, will take up 746 Grains, which is 250 times its Weight, which is by far the ſtrongeſt and beſt of any I have ſeen; and therefore, as a great Curioſity, I have given a Cut of the Ring and Stone with the Iron hanging to it. Fig. 3.

14. Iron Bars by ſtanding long in the ſame Poſition will acquire the Magnetic Virtue to a ſurprizing Degree. I have ſeen one about 10 Feet long and 3 Inches thick, (ſupporting the Summer Beam of a Room) which has been able to turn the Needle at 8 or 10 Feet diſtance, and exceeded a Loadſtone of 3 ½ Pounds Weight. From the Middle Point upwards it was a North Pole, and downwards a South Pole; and the

I have by me an Iron Rod which has ſeveral Poles or Parts which change the Poſition of the Magnetic Needle: It is the Beam of a large Steelyard, which let A E repreſent; then at A, and from thence to B, the North End of the Needle is attracted; at B, and from thence to C, the North End is repell'd and the South End attracted; from C to D, the North End is again attracted and the South End repell'd, and from D to E the South End is attracted, and the North End repell'd; laſtly, at the End E begins another Pole, and there the Needle ſtands nearly parallel with the Beam, with neither End attracted. Fig. 7.

15. It will be proper here to acquaint the Reader with the ſeveral Methods of communicating the Magnetic Virtue to Iron, in ſo great a Degree, that ſuch a Piece of Iron or Steel is deſervedly call'd an *Artificial Magnet* or *Loadſtone*; and as it poſſeſſes all the Properties, ſo it is uſed to all the Purpoſes of the natural Loadſtone.

16. The firſt Way of making an Artificial Magnet is by preparing ſeveral Steel *Laminæ*, or thin Plates, about 6 or 8 Inches long, and ½ an Inch broad; theſe well poliſhed, and properly touch'd on a very ſtrong Loadſtone, at each End, and then put together and arm'd with Steel Points like the common Stone, will become an Artificial Magnet.

17. A ſecond Way is by preparing ſolid Pieces of Steel, and having wrought them to a proper Figure, poliſh'd, and arm'd them with Steel Armour; if then one of them be properly held in the Magnetic Meridian Line, and the other which is arm'd be drawn ſeveral Times along it, it will conceive a great Degree of Magnetiſm; if then the Armour be removed from this and placed on the other, and that in like Manner touch'd on this, it will become Magnetical. But to deſcribe the particular Manner, with all its Circumſtances, would be too tedious; I ſhall therefore refer the Reader to the Account given by the Inventor, Mr. *Savery*, in *Philoſ. Tranſ.* N°. 414.

18. A third Way is to provide a Piece of Steel, and continue rubbing it *very hard all one Way* with a hard poliſh'd Steel Inſtrument, and it will by this Sort of Friction conceive ſo great a Degree of Virtue, as to become an *Artificial Magnet*, of greater Force, very often, then the natural ones. A ſmall Piece of Wire thus order'd will be as tender, and point as nicely Northwards, as any one touch'd by the Natural Magnet.

19. A fourth Way is to heat a Piece of Steel red hot, and then quench it in Water, being held therein in ſuch a Poſition as the Dipping-Needle takes when ſuſpended on a ſharp Point. This Method will ſucceed only in the Hand of thoſe who by frequent Trials have acquired a peculiar Sagacity this

Way; and indeed the ſame may be ſaid of all the others. 'Tis Experience only that gives a Perſon the *Encheireſis* neceſſary for ſucceeding in all ſuch Affairs.

20. A fifth Way is by ſetting a Bar of Iron or Steel in a proper Place, and there letting it ſtand for a long time in a Poſition unalter'd, and it will acquire a very great Magnetic Virtue, as has been already taken Notice of; a perpendicular Poſition is beſt, and the larger the Bar, and the longer it ſtands, the better or ſtronger it will be.

21. A ſixth Method of giving this Virtue to Iron in a ſurprizing Degree, and alſo to the Loadſtone itſelf, yea of altering or reverſing its Poles, and diſpoſſeſſing it of its Virtue, almoſt in any Manner, is yet a Secret in the Hands of a Gentleman in *London*; who, 'tis hoped, will oblige the World with a better Account of the curious Art he poſſeſſes than what can be got at from common Report.

22. As ſeveral Ways have been propoſed for recovering the decay'd Virtue of Loadſtones, and but to little purpoſe, (eſpecially that of keeping the Stone conſtantly in Steel Filings) I ſhall here relate the remarkable Experiment of Mr. *Haac* for this Purpoſe, as it was attended with ſuch great Succeſs.

23. This Gentleman had a Magnet weighing about 14$\frac{1}{2}$ Ounces arm'd, which would take up 16 times its own Weight, *viz.* 14 *lb.* 8 *oz.* but having laid it by for the Space of ſome Years, unuſed, it had loſt $\frac{1}{4}$ Part of its Virtue and more, for it would now take up no more than 3 Pounds. This put him on thinking of ſeveral Ways to recover it, but without Succeſs; till at laſt he reflected that by Uſe and Exerciſe the Powers of Animal Bodies were often improv'd, and it might poſſibly happen ſo with reſpect to ſome Properties or Powers of inanimate Bodies. Upon this Thought, he hung as much Weight to the Stone as it would ſuſtain, and ſo left it for the Space of ſome Weeks. Then returning, he applied more Weight to the former, which it very eaſily bore, and thus repeating the Addition of more Weight at ſeveral Periods in the Space of about 2 Years, he at laſt found that the Stone had not only recover'd its former Strength, but increaſed it ſo far, that it would now take up more than 20 *lb.* whereas at firſt it would take up but 16 *lb.*

LECTURE II.

Of the ATTRACTION *of* GRAVITATION, *and its Laws. Of* MOTION *and* REST, *abſolute and relative; equable, accelerated, and retarded* MOTION. *Of the Spaces, Times, and Velocities of* BODIES *in* MOTION. *Of the* COLLISION *of* BODIES *Elaſtic and Non-Elaſtic. Of the* MOMENTUM *or* FORCE *of* STRIKING BODIES. *Of the* LAWS *of* MOTION. *Of the* COMPOSITION *and* RESOLUTION *of* MOTION. *Of* ACTION *and* RE-ACTION. *Of the* DESCENT *of heavy Bodies. Of the* DESCENT *of Bodies on the Inclined Planes. Of the* DOCTRINE *of* PENDULUMS. *Of the* CYCLOID, *its* PROPERTIES *and* USE; *the Curve of quickeſt Deſcent. Of the Centre of* OSCILLATION *and* PERCUSSION. *The various Uſes of the* PENDULUM, *as a* CHRONOMETER, *&c. Of a new conſtructed* PYROMETER, *and the* EXPANSION *of* METALLINE BODIES. *Of the* DOCTRINE *of* PROJECTILES, *or* ART *of* GUNNERY. *Of* CENTRAL FORCES, *and the various Laws and Properties of that Sort of Motion. Of the General Law of the* PLANETARY MOTIONS. *Of the* FIGURE *of the* EARTH, *and how determined.*

THE *Fourth and laſt Species of Attraction* is that of GRAVITATION, which is evident only between large Bodies, as the

Earth and Moon, the Sun and its Planetary Attendants. The Laws of this Attraction are as follows, *viz.* (1.) It is common to all Bodies, and mutual between them. (2.) It is proportional to the Quantity of Matter in Bodies. (3.) It is exerted every way from the Centre of the attracting Body in Right-lined Directions. (4.) It decreaſes as the Squares of the Diſtances increaſes. That is, if a Body at A, on the Earth's Surface, diſtant *one Semidiameter* from the Centre C, weighs 3600 Pounds, it will at the Diſtance of 2, 3, 4, 5, 6, Semidiameters weigh 900, 400, 225, 145, 100 Pounds; which Numbers decreaſe as the Squares of the Diſtances 4, 9, 16, 25, 36, increaſe (XVI).

Plate III. Fig. 2.

(XVI.) 1. Though we reckon this Species of Attraction different from that of Coheſion, yet, when well conſider'd, it may be found perhaps to differ no otherwiſe than as the *Whole from the Parts*; for the Gravity of large Bodies may be only the Reſult or Aggregate of the particular Powers of the conſtituent Particles, which ſingly act only upon Contact, and in ſmall Diſtances; but with their joint Forces, in vaſt Quantities, produce a mighty Power, whoſe Efficacy extends to very great Diſtances, proportional to the Magnitude of Bodies.

2. This Force of Gravity is, to Senſe, the ſame for any Diſtances near the Earth's Surface; ſince a ſmall Diſtance from the Surface of the Earth does not ſenſibly alter the Diſtance from the Centre, which is near 4000 Miles: At the Height therefore of one Mile, for Inſtance, the Diſtances from the Centre will be as the Numbers 4000 to 4001, and the Powers of Gravity inverſely as their Squares, *viz.* as 16008001 to 16000000, or as 16008 to 16000, which are ſo near an Equality as not to be ſenſibly different from each other.

3. But when the Diſtance is ſo great as to cauſe a Diſproportion between that and the Semi-diameter of the Earth, then will the Power of Gravity decreaſe very ſenſibly, according to the Law above laid down: Thus at the Diſtance of the Moon, which is, at a Mean, about 60 Semidiameters

HENCE we learn that all Bodies have WEIGHT, or are *heavy*; and that there is no ſuch Thing as *abſolute* LEVITY in Nature: And by the ſecond Law, the Weight of all Bodies is proportional to the Quantity of Matter they contain; and hence, ſince Bodies of equal Bulks are found to have unequal Quantities of Matter, it evidently follows, that a VACUUM, or Space void of Matter, muſt neceſſarily exiſt, and that an abſolute PLENUM is a Doctrine unphiloſophical, and equally falſe and abſurd (XVII).

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of the Earth, the Power of Gravity is to that on the Earth's Surface as 1 to 3600, as will be ſhewn under the Article of *Central Forces* with ſufficient Demonſtration.

(XVII.) 1. Since there is no Sort of Matter that we have any Knowledge of but what will gravitate, to ſuppoſe there is any Sort that will not, is arbitrary, and contrary to the Rules of Philoſophizing, and all ſound Reaſoning; by which we are confined to the Phænomena of real, and not imaginary Exiſtence.

2. The Weight or Quantity of Matter in Bodies may be expreſs'd by their Bulks multiplied by their Denſities. By the DENSITY of a Body is meant the greater or leſſer Quantity of Matter under the ſame Bulk; and therefore the Denſity (D) of any Body A, is to the Denſity (D) of any other Body B of equal Bulk, as the Quantity of Matter (M) in the former is to the Quantity of Matter (m) in the latter. That is, $\mathrm{D} : D :: \mathrm{M} : m$; $\because \mathrm{D}m = D\mathrm{M}$.

3. But if the Bulks are unequal, and the Quantities of Matter the ſame, or equal in any two Bodies B and C; then will the Denſity (D) of the Body B be to the Denſity (d) of the Body C, as the Bulk (b) of the latter, to the Bulk (B) of the former; *viz.* $D : d :: b : \mathrm{B}$; $\because D\mathrm{B} = db$.

4. Hence $D = \frac{db}{\mathrm{B}} = \frac{\mathrm{D}m}{\mathrm{M}}$; and ſo $\mathrm{M}db = m\mathrm{BD}$; and conſequently $\mathrm{M} : m :: \mathrm{BD} : bd$; that is, *The Quantity of Matter in any two Bodies* A *and* C, *which differ in Bulk and Weight, are to each other as the Products of the Denſities by their Bulks, as was above aſſerted.*

5. Hence alſo in ſuch Bodies, $\mathrm{D} : d :: \mathrm{M}b : m\mathrm{B}$; that

ALSO, from the third Law it follows, that all Bodies deſcending freely by their Gravity tend towards the Earth in Right Lines perpendicular to its Surface; and with equal Velocities, (abating for the Reſiſtance of the Air) as is evident by the ſecond Law above, and what will immediately follow in *Mechanics* (XVIII).

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is, *the Denſities of the two Bodies* A *and* C *are directly as the Quantities of Matter, and inverſely as their Bulks.*

6. Laſtly, *The Bulks or Magnitudes of two Bodies* A *and* C *are directly as their Quantities of Matter or Weights, and inverſely as their Denſities*; for we have B : *b* :: M*d* : *m*D.

7. What has been hitherto ſaid relates to the abſolute Weight of Bodies; but when the Weight or Gravities of Bodies of equal Bulk are conſider'd and compared together, they are then call'd the Relative Weights or Specific Gravities of thoſe Bodies; and ſince they are likewiſe proportional to the Quantities of Matter, 'tis evident, *that the Denſities and Specific Gravities are proportional to each other*, in all Bodies; and therefore what has been ſaid of the one is equally applicable to the other.

8. As a Corollary to this Article, we may obſerve with Sir *Iſaac Newton*, that all Space cannot be fill'd with Matter, as the *Pleniſts* aſſert, but that there muſt neceſſarily be Vacuities or Interſtices void of Matter in the Compoſition of natural Bodies; for if there were not, then Bodies of equal Bulk would ever be of the ſame Weight and Denſity, or, in other Words, the Denſity and Specific Gravity of *Cork* or *Air* would be the ſame with that of *Quickſilver* or *Gold.* And 'tis ſurpriſing to think, that ſuch Abſurdities are not groſs enough to be heeded by thoſe who heſitate about a *Vacuum.*

(XVIII.) The Reaſon why all Bodies deſcend with an equal Velocity towards the Centre of the Earth *(in Vacuo)* is becauſe the Force of Gravity in each is proportional to the Quantity of Matter, or Number of Particles only, independent of any other Circumſtance or Conſideration whatſoever. Thus ſuppoſe the Body A has only one Particle of Matter, and the Body B conſiſts of one hundred, then will the Force of Gravity in A be to that in B as 1 to 100; and therefore the Force acting on each ſingle Particle in B is but $\frac{1}{100}$ of the whole Force, and conſequently is but equal to the Force acting on the ſingle Particle A; ſince then each ſingle Particle

AGAIN: Since the Attraction is always directly as the Quantity of Matter, and inverſely as the Square of the Diſtance, it follows, that were the internal Parts of the Earth a perfect Void, or hollow Concavity, a Body placed any where therein would be abſolutely light, or void of Gravity; but ſuppoſing the Earth a ſolid Body throughout, the Gravity from the Surface to the Centre will decreaſe with the Diſtance, or it will be directly proportional to the Diſtance from the Centre (XIX).

in B is acted upon with the ſame Force as the Particle A, the Velocities in each muſt be equal; that is, each Particle in the Body B (and therefore the whole Body) muſt deſcend with an equal Velocity with that of A. This would be eaſy to conceive, if the Particles in the Body B were to fall ſeparate; and I think 'tis as eaſy to underſtand that their Connection or Coheſion can no ways contribute to the Acceleration of their Motion, which is entirely the Effect of the Attracting Force of the Earth.

Plate II. Fig. 4.

(XIX) 1. For Illuſtration, ſuppoſe ABC be the outward Shell or Cruſt, and all the internal Parts of the Earth an hollow Space or Concavity, in which ſuppoſe any Body as R, to be placed in any Part whatſoever; through the Centre of the ſaid Body let there be drawn the Lines *ad* and *bc*, which by their Revolution would deſcribe Cones, whoſe Baſes *ab*, and *cd*, in the ſpherical Superficies, are in proportion to each other as the Squares of the Diſtances from R reſpectively. Now the Particle R is attracted by the Particles in thoſe two Baſes with an equal Force; for the Force is as the Square of the Diſtance of the Baſes inverſely, and as the Number of Particles in each Baſe directly, which Number of Particles is as the Square of the Diſtance from the Particle directly. But this *direct* and *inverſe Ratio* of the Forces makes a Ratio of Equality; that is, the Forces on each Side are equal, and being in contrary Directions they deſtroy each other; and this being the Caſe in any other Part of the concave Superficies, 'tis evident the Body R cannot be impell'd towards any Part, but will retain its Poſition as if not affected by Gravity at all.

2. The Forces with which a Body will be attracted on the Surface of two Spheres of the ſame Denſity, but unequal

HAVING premiſed the neceſſary *Præcognita* to the Science of MECHANICS, which entirely depends on the Principle of Gravitation, we come immediately to conſider the Object thereof, *viz.* the *Nature*, *Kinds*, and various *Affections* of *Motion*, and moving Bodies; and the Structure and Mechaniſm of all Kinds of Machines, commonly call'd *Mechanical Powers*, whether Simple or Compound.

MOTION is the continual and ſucceſſive Change of Space, and is either *Abſolute* or *Rela-*

Bulks, *will be directly as the Diameters of the Spheres.* For the Force (f) of the leſſer Sphere ABC will be to the Force (F) of the greater DEF inverſely as the Squares of the Diameter (d^2) of the leſſer to the Square of the Diameter (D^2) of the greater; that is, $f : F :: \frac{1}{dd} : \frac{1}{DD}$. Plate II. Fig. 5, 6.

Note, *This is ſo becauſe the Forces are inverſely as the Squares of the Diſtances from the Centres, which are equal to half the Diameters.*

3. Again: The Forces are as the Number of Attracting Particles, which are as the Bulks or Magnitudes of the Spheres, which are as the Cubes of the Diameter; therefore alſo $f : F :: d^3 : D^3$; whence both theſe Ratio's compounded give $f : F :: \frac{d^3}{d^2} : \frac{D^3}{D^2} :: d : D$; or, *the Forces of Attraction are in proportion to the Diameters of the Spheres directly.*

4. Hence it follows, that were there a Perforation made, and a Body to deſcend therein from the Surface of the Earth to the Center, its Gravity would always decreaſe with the Diſtance from the Center; becauſe in every Place it might be conſider'd as on that Surface of a Sphere whoſe Diameter is equal to twice the Diſtance from the Center, and which is proportional to the Sphere's Attraction; the Parts of the Earth above the Body being only Part of a Spherical Shell, which has no Effect on the Body, as was ſhewn above.

tive. *Abſolute Motion* is the Change of *Abſolute Space* or Place of Bodies, as the Flight of a Bird, the Motion of a Projectile, *&c.* But *Relative Motion* is the Change of *Relative Space*, or that which has Reference to ſome other Bodies: As of two Ships under Sail, the Difference of their Velocities is the *Relative Motion* of the Ship ſailing faſteſt; and is that alone which is diſcernible by us. The ſame is to be underſtood of *Abſolute and Relative Reſt* (XX).

(XX.) 1. To make this Matter yet plainer, we are to conſider, that SPACE is nothing elſe *but an abſolute and infinite Void*; and that the PLACE of a Body is that Part of the immenſe Void which it takes up or poſſeſſes; and that Place may be conſider'd either abſolutely, or in itſelf alone, and then it is call'd the *Abſolute Place* of the Body; or elſe with Regard to the Place of ſome other Body, and then it is call'd the *Relative or Apparent Place* of Bodies.

2. Now as Motion is only the Change of Place in Bodies, 'tis evident that will come under the ſame Diſtinction of *Abſolute*, and *Relative* or *Apparent*. All Motion is in itſelf *Abſolute*, or the Change of Abſolute Space; but when the Motions of Bodies are conſider'd and compared with each other, then are they *Relative and Apparent* only; they are *Relative*, as they are compared with each other; and they are *Apparent* only, inaſmuch as not their *true or abſolute Motion*, but the Sum or *Difference of the Motions* only is perceiveable by us.

3. In comparing the Motions of Bodies, we may conſider them as moving both the ſame way, or towards contrary Parts; in the firſt Caſe the *Difference of the Motions* is only perceived by us, in the latter the *Sum of the Motions*. Thus for Example; ſuppoſe two Ships, A and B, ſet ſail from the ſame Port upon the ſame Rhumb, and that A ſails at the Rate of five Miles *per* Hour, and B at the Rate of three; here the Difference of the Velocities (*viz.* two Miles *per* Hour) is that by which the Ship A will appear to go from the Ship B forwards, or the Ship B will appear at A to go with the ſame Velocity backwards, to a Spectator in either reſpectively.

4. If the two Ships A and B move with the ſame Degree of Velocity, then will the Difference be nothing, and ſo nei-

AGAIN: Motion is either *Equable* or *Accelerated*. *Equable Motion* is that by which a Body paſſes over equal Spaces in equal Times. *Accelerated Motion* is that which is continually augmented or increaſed; as *Retarded Motion* is that which continually decreaſes: And if the Increaſe or Decreaſe of Motion be equal in equal Times, the Motion is then ſaid to be *equably accelerated or retarded* (XXI).

ther Ship will appear to the other to move at all. Hence it is, that tho' the Earth is continually revolving about its Axis, yet as all Objects on its Surface partake of the ſame common Motion, they appear not to move at all, but are relatively at Reſt.

5. If the two Ships A and B, with the Degrees of Velocities as above, meet each other; the one will appear to the other to move with the Sum of both Velocities, *viz.* at the Rate of eight Miles *per* Hour; ſo that in this Caſe the *Apparent Motion* exceeds *the True*, as in the other it fell ſhort of it. Hence the Reaſon why a Perſon riding *againſt* the Wind finds the Force of it much greater than it really is, whereas if he rides *with it*, he finds it leſs.

SCHOLIUM.

6. The Reaſon of all theſe different Phænomena of Motion will be evident if we only conſider, that we muſt be abſolutely at Reſt, if we would diſcern the true or real Motion of Bodies about us. Thus a Perſon on the Strand will obſerve the Ships ſailing with their real Velocity; a Perſon ſtanding ſtill will experience the true Strength and Velocity of the Wind; and a Perſon placed in the Regions between the Planets will view all their true Motions; which he cannot otherwiſe do, becauſe in all other Caſes the Spectator's own Motion muſt be added to, or ſubducted from, that of the moving Body; and the Sum or Difference is therefore the *Apparent or Relative Motion*, and not the *True*.

(XXI.) 1. *Equable Motion* is generated by a ſingle *Impetus* or Stroke; thus the Motion of a Ball from a Cannon is produced by the ſingle Action of the Powder in the firſt Moment, and therefore the Velocity it firſt ſets out with would always continue the ſame were it void of Gravity, and to move in an unreſiſting Medium, which therefore would be always equable, or ſuch as would carry it through the ſame Length of Space in every equal Part of Time.

THE *Celerity* or *Velocity* of Motion is that Affection by which a Body paſſes over a given Space in a given Time, or what we commonly call the *Swiftneſs* or *Slowneſs* of Motion (XXII).

2. *Accelerated Motion* is produced by a conſtant Impulſe, or Power which keeps continually acting upon the Body, as that of Gravity, which produces the Motion of Falling Bodies, which ſort of Motion is conſtantly accelerated, becauſe Gravity every Moment adds a new Impulſe, which generates a new Degree of Velocity; and the Velocity thus increaſing the Motion muſt be quicken'd each Moment, or fall faſter and faſter the longer it falls.

3. In like Manner, a Body thrown perpendicularly upward, as a Ball from a Cannon, will have its Motion *continually retarded*, becauſe Gravity acts conſtantly upon it in a Direction contrary to that given it by the Powder, ſo that its Velocity upwards muſt be continually diminiſhed, and ſo its Motion as continually retarded, till at laſt it be all deſtroy'd. The Body has then attain'd its utmoſt Height, and is for a Moment motionleſs, after which it begins to deſcend with a Velocity in the ſame Manner accelerated, till it comes to the Earth's Surface.

4. And becauſe at a ſmall Diſtance from the Earth's Surface, the Power of Gravity is every where the ſame, it follows, that in equal Moments of Time the Impulſes on the Body will be equal, which will therefore generate an equal Increaſe of Velocity each Moment, and of Conſequence the Motion reſulting from thence will be equably quicken'd if downwards, or retarded if upwards, in Mediums without Reſiſtance.

5. In this Caſe we may obſerve in Bodies that are projected upwards, (1.) That the Time of the Aſcent is equal to that of the Deſcent. (2.) That at equal Heights above the Earth, the Velocity in the Aſcent and Deſcent is equal, or the ſame; and therefore, (3.) That the Velocity acquired (by falling) at the Earth's Surface, is equal to that which is generated by the Powder, or other *Impetus* (which threw it up) in the firſt Moment of its Motion. What Difference will ariſe from a reſiſting Medium (as to Projectiles thrown up in the Air, &c.) will be hereafter more fully conſider'd and ſpecified.

(XXII) 1. From the above Definitions we may determine the Theorems, for the Expreſſions of the Time (T), the Velocity (V), and the Space (S) paſſed over in Equable or Uniform Motion, very eaſily thus:

2. If the Time be given, or the ſame; the *Space paſs'd*

THE MOMENTUM or *Quantity of Motion* is all that Power or Force which a moving Body has to affect or ſtrike any Obſtacle or Impediment which oppoſes its Motion, and is equal to that impreſſed Force by which the Body is compell'd to change its Place.

SINCE the Force of a Body is greater or leſſer as the Quantity of Matter is ſo, when the Velocity of its Motion is the ſame; alſo ſince the ſaid Force in the ſame Body is proportional to its Velocity: Therefore it follows, that the *Momentum* or Force of Bodies in Motion is in the compound Ratio of their Quantities of Matter and their Velocities conjointly. Thus if one Body A ſtrike an Obſtacle X, with 3 Parts of Matter and 9 Degrees of Celerity; and another Body B ſtrike it Plate III. Fig. 3.

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over will be as the Velocity, viz. $S : V$; that is, with twice the Velocity, twice the Space; with three times the Velocity, three times the Space, will be paſs'd over in the ſame Time; and ſo on.

3. If the Velocity be given, or remain the ſame; *then the Space paſſed over will be as the Time* (viz. $S : T$) that is, it will be greater or leſſer as the Time is ſo.

4. But if neither the Time nor Velocity be given, or known; *then will the Space be in the compound Ratio of both*, viz. $S : TV$.

5. Hence, in general, ſince $S : TV$, we have $V : \frac{S}{T}$; that is, *the Velocity is always directly as the Space, and inverſely as the Time.*

6. And alſo $T : \frac{S}{V}$; that is, *the Time is as the Space directly, and as the Velocity inverſely*; or, in other Words, it *increaſes with the Space*, and *decreaſes with the Velocity.*

7. If therefore in any Rectangle ABCD one Side AB repreſent the Time, and the other Side BC the Velocity, 'tis evident the Area of the ſaid Rectangle will repreſent the Space paſs'd over by an uniform Motion in the Time AB and with the Velocity BC. Plate III. Fig. 5.

with 5 Parts of Matter, and 7 Degrees of Celerity; the *Momenta* of the reſpective Strokes will be as 27 to 35 (XXIII).

(XXIII). Since the *Momentum* (M) of a Body is compounded of the Quantity of Matter (Q), and the Velocity (V), we have this general Expreſſion $\mathrm{M}=\mathrm{QV}$, for the Force of any Body A; and ſuppoſe the Force of another Body B be repreſented by the ſame Letters in *Italicks*, viz. $M=QV$.

2. Let the two Bodies A and B in Motion impinge on each other directly; if they tend both the ſame Way, the Sum of their Motions towards the ſame Part will be $\mathrm{QV}+QV$. But if they tend towards contrary Parts, or meet, then the Sum of their Motions towards the ſame Part will be $\mathrm{QV}-QV$; for ſince the Motion of one of the Bodies is contrary to what it was before, it muſt be connected by a contrary Sign. Or thus; becauſe when the Motion of B conſpires with that of A it is *added to it*, ſo when it is contrary, it is *ſubducted from it*, and the *Sum or Difference* of the *abſolute Motions* is the whole *relative Motion*, or that which is made towards the ſame Part.

3. Again, this total Motion towards the ſame Parts is the ſame both before and after the Stroke, in caſe the two Bodies A and B impinge on each other; becauſe whatever Change of Motion is made in one of thoſe Bodies by the Stroke, the ſame is produced in the other Body towards the ſame Part; that is, as much as the Motion of B is increaſed or decreaſed towards the ſame Part by the Action of A, juſt ſo much is the Motion of A diminiſhed or augmented towards the ſame Part by the equal Re-action of B, by the *Third Law* of Motion.

4. In Bodies *not elaſtic*, let x be the Velocity of the Bodies after the Stroke (for ſince we ſuppoſe them not elaſtic, there can be nothing to ſeparate them after Colliſion, they muſt therefore both go on together, or with the ſame Celerity). Then the Sum of the Motions after Colliſion will be $\mathrm{Q}x+Qx$; whence if the Bodies tend the ſame Way, we have $\mathrm{QV}+QV=\mathrm{Q}x+Qx$, or if they meet, $\mathrm{QV}-QV=\mathrm{Q}x+Qx$; and accordingly $\frac{\mathrm{QV}+QV}{\mathrm{Q}+Q}=x$, or $\frac{\mathrm{QV}-QV}{\mathrm{Q}+Q}=x$.

Plate IV. Fig. 1.

5. If the Bodies (B) be at reſt, then $V=0$, and the Velocities of the Bodies after the Stroke will be $\frac{\mathrm{QV}}{\mathrm{Q}+Q}=x$.

THE general Laws of Motion, which all Bodies obſerve, are the three following, *viz.*

Thus if the Bodies be equal (*viz.* $\mathrm{Q} = \mathcal{Q}$) and A with 10 Degrees of Velocity impinge on B at reſt; then $\frac{\mathrm{QV}}{\mathrm{Q}+\mathcal{Q}} = \frac{10}{2} = 5 = x$. If $\mathrm{Q} = \mathcal{Q}$, and $\mathrm{V} : V :: 10 : 6$; we have $\frac{\mathrm{QV} - \mathcal{Q}V}{\mathrm{Q}+\mathcal{Q}} = \frac{16}{2} = 8 = x$, the Velocity after the Stroke. Plate IV. Fig. 2.

6. If the Bodies are both in Motion, and tend the ſame Way, then when $\mathrm{Q} = \mathcal{Q}$, and $\mathrm{V} = V$, 'tis plain $\frac{\mathrm{QV} - \mathcal{Q}V}{\mathrm{Q}+\mathcal{Q}} = 0 = x$; that is, the Bodies which meet with equal Bulks and Velocities will deſtroy each other's Motion after the Stroke, and remain at reſt. If $\mathrm{Q} = \mathcal{Q}$, but $\mathrm{V} : V :: 6 : 14$, then $\frac{\mathrm{QV} - \mathcal{Q}V}{\mathrm{Q}+\mathcal{Q}} = \frac{8}{2} = 4 = x$; which ſhews that equal Bodies meeting with unequal Velocities, they will after the Stroke both go on the ſame Way which the moſt prevalent Body moved before. Fig. 3. Fig. 4.

7. If the Velocity $\frac{\mathrm{QV} \pm \mathcal{Q}V}{\mathrm{Q}+\mathcal{Q}}$ be multiplied by the Quantities of Matter Q and $\mathcal{Q}$, we ſhall have $\frac{\mathrm{Q}^2\mathrm{V} \pm \mathrm{Q}\mathcal{Q}V}{\mathrm{Q}+\mathcal{Q}}$ = the *Momentum* of A after the Stroke; and $\frac{\mathrm{QV}\mathcal{Q} \pm \mathcal{Q}^2 V}{\mathrm{Q}+\mathcal{Q}}$ = the *Momentum* of B. Therefore $\mathrm{QV} - \frac{\mathrm{Q}^2\mathrm{V} \pm \mathrm{Q}\mathcal{Q}V}{\mathrm{Q}+\mathcal{Q}} = \frac{\mathrm{Q}\mathcal{Q}\mathrm{V} \pm \mathcal{Q}\mathrm{Q}V}{\mathrm{Q}+\mathcal{Q}} = \frac{\mathrm{Q}\mathcal{Q}}{\mathrm{Q}+\mathcal{Q}} \times \overline{\mathrm{V} \pm V}$ = the Quantity of Motion loſt in A after the Stroke, and conſequently is equal to what is gain'd in B, as may be ſhewn in the ſame Manner.

8. But ſince a Part of this Expreſſion (*viz.* $\frac{\mathrm{Q}\mathcal{Q}}{\mathrm{Q}+\mathcal{Q}}$) is conſtant, the Loſs of Motion will be ever proportional to the other Part $\mathrm{V} \pm V$. But this Loſs or Change of Motion in either Body is the whole Effect, and ſo meaſures the Magnitude or Energy of the Stroke. Wherefore any two Bodies, not elaſtic, ſtrike each other with a Stroke always proportional to the Sum of their Velocities ($\mathrm{V} + V$) if they meet, or to the Difference of their Velocities ($\mathrm{V} - V$) if they tend the ſame Way.

9. Hence if one Body (B) be at reſt before the Stroke,

LAW I. *Every Body will continue in its State of Reſt, or moving uniformly in a Right Line, ex-*

then $V = o$; and the Magnitude of the Stroke will be as V, that is, as the Velocity of the moving Body A; and not as the *Square of its Velocity*, as many Philoſophers (*viz.* the *Dutch* and *Italians*) maintain.

10. In Bodies perfectly Elaſtic, the reſtituent Power or Spring, by which the Parts diſplaced by the Stroke reſtore themſelves to their firſt Situation, is equal to the Force impreſs'd, becauſe it produces an equal Effect; therefore in this Sort of Bodies, there is a Power of Action twice as great as in the former Non-elaſtic Bodies, for theſe Bodies not only ſtrike each other by Impulſe, but likewiſe by Repulſe, they always repelling each other after the Stroke.

11. But we have ſhewn that the Force with which Non-elaſtic Bodies ſtrike each other is as $\mathrm{V} \pm V$; therefore the Re-action of Elaſtic Bodies is the ſame; that is, the Velocity with which Elaſtic Bodies recede from each other after the Stroke is equal to the Velocity with which they approach'd each other before the Stroke. Whence if x and y be the Velocities of two Bodies A and B, tending the ſame Way, after the Stroke, ſince $\mathrm{V} - V = y - x$, we have $x + \overline{\mathrm{V} - V} = y$; whence the Motion of A after the Stroke will be $\mathrm{Q}x$, and that of B will be $\mathcal{Q}x + \mathcal{Q}\mathrm{V} - \mathcal{Q}V$; and the Sum of theſe Motions will be equal to the Sum of the Motions before the Stroke, *viz.* $\mathrm{Q}x + \mathcal{Q}x + \mathcal{Q}\mathrm{V} - \mathcal{Q}V = \mathrm{QV} + \mathcal{Q}V$. Whence by reducing the Equation, it will be $\mathrm{Q}x + \mathcal{Q}x = \mathrm{QV} - \mathcal{Q}\mathrm{V} + 2\mathcal{Q}V$; and $x = \frac{\mathrm{QV} - \mathcal{Q}\mathrm{V} + 2\mathcal{Q}V}{\mathrm{Q} + \mathcal{Q}} =$ the Velocity of the Body A.

12. Again, the Velocity of B is $x + \mathrm{V} - V = \frac{\mathrm{QV} - \mathcal{Q}\mathrm{V} + 2\mathcal{Q}V}{\mathrm{Q} + \mathcal{Q}} + \mathrm{V} - V = \frac{2\mathrm{QV} - \mathrm{Q}V + \mathcal{Q}V}{\mathrm{Q} + \mathcal{Q}}$. Here we have ſuppoſed the Bodies tend the ſame Way before the Stroke; and 'tis evident from the Equation above, that ſo long as $\mathrm{QV} + 2\mathcal{Q}V$ is greater than $\mathcal{Q}\mathrm{V}$, the Velocity (x) of A after the Stroke will be affirmative, or the Body A will move the ſame Way after the Stroke as before; but when $\mathcal{Q}\mathrm{V}$ is greater than $\mathrm{QV} + 2V\mathcal{Q}$, the Velocity ($x$) will be negative, or the Body A will be reflected back.

13. If the Body B be at reſt, then $V = o$; and $x = \frac{\mathrm{QV} - \mathcal{Q}\mathrm{V}}{\mathrm{Q} + \mathcal{Q}}$; which ſhews the Body A will go forwards or

cept ſo far as it is compell'd to change that State by Forces impreſs'd.

backwards, as QV is greater or leſſer than $\mathcal{Q}\mathrm{V}$, or A greater or leſſer than B.

14. If $\mathrm{Q}=3$, $\mathcal{Q}=2$, $\mathrm{V}=10$, and $\mathit{V}=0$; then after Plate IV. the Stroke the Velocity of A will be $\frac{\mathrm{QV}-\mathcal{Q}\mathrm{V}}{\mathrm{Q}+\mathcal{Q}}=\frac{30-20}{5}$ Fig. 5. $=\frac{10}{5}=2$, and the Velocity of B will be $y=\frac{2\,\mathrm{QV}}{\mathrm{Q}+\mathcal{Q}}=\frac{60}{5}$ $=12$. If the Bodies are both in Motion, and $\mathit{V}=5$, the reſt the ſame as before; then $\frac{\mathrm{QV}-\mathcal{Q}\mathrm{V}+2\mathcal{Q}\mathit{V}}{\mathrm{Q}+\mathcal{Q}}=6=$ Velocity of A after the Stroke, and $\frac{2\mathrm{QV}-\mathrm{Q}\mathit{V}+\mathcal{Q}\mathit{V}}{\mathrm{Q}+\mathcal{Q}}=$ Fig. 6. $11=$ Velocity of B after the Stroke.

15. If the Bodies A and B move towards contrary Parts, or meet each other, then will the Relative Velocity, to which the Force of the Stroke is proportional, be $\mathrm{V}+\mathit{V}$, and ſo the Velocities of A and B after the Stroke will be x and $x+\mathrm{V}+\mathit{V}$; and ſo the Motion of A will be $\mathrm{Q}x$ and $\mathcal{Q}x+\mathcal{Q}\mathrm{V}+\mathcal{Q}\mathit{V}$; the Sum of theſe Motions is $\mathrm{Q}x=\mathcal{Q}x=\mathcal{Q}\mathrm{V}=\mathcal{Q}\mathit{V}=\mathrm{QV}-\mathcal{Q}\mathit{V}=$ the Motion towards the ſame Part before the Stroke. Whence we have $x=\frac{\mathrm{QV}-\mathcal{Q}\mathrm{V}-2\mathcal{Q}\mathit{V}}{\mathrm{Q}+\mathcal{Q}}$; and the Velocity of B will be $\frac{\mathrm{QV}-\mathcal{Q}\mathrm{V}-2\mathcal{Q}\mathit{V}}{\mathrm{Q}+\mathcal{Q}}+\mathrm{V}+\mathit{V}=\frac{2\mathrm{QV}+\mathrm{Q}\mathit{V}-\mathcal{Q}\mathit{V}}{\mathrm{Q}+\mathcal{Q}}$.

16. If $\mathrm{QV}+2\mathcal{Q}\mathit{V}$ be greater than QV, the Motion of the Body A will be backwards; otherwiſe it will go on forwards as before. If $\mathrm{Q}=3$, $\mathrm{Q}=2$, $\mathrm{V}=10$, and $\mathit{V}=5$; then will the Velocity of A be $\frac{\mathrm{QV}-\mathcal{Q}\mathrm{V}-2\mathcal{Q}\mathit{V}}{\mathrm{Q}+\mathcal{Q}}=\frac{-10}{5}$ Fig. 7. $=2$, and ſo the Body A will go back with two Degrees of Velocity. The Velocity of B, after the Stroke, will be $\frac{2\mathrm{QV}+\mathrm{Q}\mathit{V}-\mathcal{Q}\mathit{V}}{\mathrm{Q}+\mathcal{Q}}=13$.

17. If the Bodies are equal, that is, if $\mathrm{Q}=\mathcal{Q}$, then $x=\frac{-2\,\mathcal{Q}\mathit{V}}{2\,\mathcal{Q}}=-\mathit{V}$; which ſhews, that when equal Bodies meet each other, they are reflected back with interchanged Velocities; Fig. 8. for in that Caſe alſo the Velocity of B becomes $\frac{2\mathrm{QV}}{2\mathrm{Q}}=\mathrm{V}$. An Example of this you have in *Fig.* 8. of *Plate* IV.

LAW II. *The Change of Motion is always proportional to the moving Force impreſs'd, and is al-*

Plate IV. Fig. 9. 18. If the Bodies are equal, and one of them at reſt, as B; then ſince $Q = \mathcal{Q}$, and $V = 0$ we have the Velocity of A after the Stroke $x = 0$; or the Body A will abide at Reſt, and the Velocity of B will be $= V$, the Velocity of A before the Impulſe, as appears by the Example of *Fig.* 9. *Plate* IV.

Fig. 10. 19. If ſeveral Bodies B, C, D, E, F, are contiguous in a Right Line, if another equal Body A ſtrike B with any given Velocity, it ſhall loſe all its Motion, or be quieſcent after the Stroke; the Body B which receives it will communicate it to C, and C to D, and D to E, and E to F; and becauſe Action and Re-action between the Bodies B, C, D, E, are equal, as they were quieſcent before, they muſt continue ſo; but the Body F having no other Body to re-act upon it, has nothing to obſtruct its Motion, it will therefore move on with the ſame Velocity which A had at firſt, becauſe it has all the Motion of A, and the ſame Quantity of Matter by Hypotheſis.

Fig. 11. 20. Let there be three Bodies A, B, C; and let A ſtrike B at reſt; the Velocity generated in B by the Stroke will be $y = \frac{2QV}{Q+\mathcal{Q}}$, and ſo the *Momentum* of B will be $\frac{2QV\mathcal{Q}}{Q+\mathcal{Q}} = \mathcal{Q}y$. With this *Momentum* B will ſtrike C at Reſt and contiguous to it; the Velocity generated in C will be $\frac{2\mathcal{Q}y}{\mathcal{Q}+C}$; and its *Momentum* will be $\frac{2\mathcal{Q}yC}{\mathcal{Q}+C} = \frac{2QC}{\mathcal{Q}+C} \times \frac{2QV}{Q+\mathcal{Q}} = \frac{4QV\mathcal{Q}C}{Q\mathcal{Q}+QC+\mathcal{Q}^2+\mathcal{Q}C}$.

21. If now we ſuppoſe B a variable Quantity, while A and C remain the ſame, we ſhall find what Proportion it muſt have to each of them in order that the *Momentum* of C may be a *Maximum*, or the greateſt poſſible, by putting the Fluxion thereof equal to Nothing; that is, $\frac{4Q^2C^2V\dot{\mathcal{Q}} - 4QC\mathcal{Q}^2\dot{\mathcal{Q}}}{\overline{QC+Q\mathcal{Q}+\mathcal{Q}C+\mathcal{Q}^2}|^2} = 0$; whence we get $QC - \mathcal{Q}\mathcal{Q} = 0$, and ſo $QC = \mathcal{Q}\mathcal{Q}$; Conſequently $Q : \mathcal{Q} :: \mathcal{Q} : C$, or $A : B :: B : C$; that is, *the Body* B *is a Geometrical Mean between* A *and* C.

22. Hence if there be any Number (n) of Bodies in a Geometrical Ratio (r) to each other; and the firſt be A, the ſecond will be rA, the third r^2A, and ſo on to the laſt, which will be $r^{n-1}A$.

23. Alſo, the Velocity of the firſt being V, that of the ſe-

ways made according to the Right Line in which that Force is impreſs'd.

LAW III. *Re-action is always equal and contrary to Action; or the Actions of two Bodies upon each other are always equal, and in contrary Directions: That is, by Action and Re-action equal Changes of Motion are produced in Bodies acting upon each other; and theſe Changes are impreſs'd towards contrary Parts.*

THE firſt of theſe Laws is founded on the *Vis Inertiæ* of Matter, whereby it is indifferently dif-

cond will be $\frac{2V}{1+r}$, (for $\frac{2QV}{Q+\mathfrak{Q}}$ is here $=\frac{2AV}{A+rA}=\frac{2V}{1+r}$) that of the third $\frac{4V}{\overline{1+r}^2}$, that of the fourth $\frac{8V}{\overline{1+r}^3}$, and ſo on to the laſt, which will be $\overline{\frac{2}{1+r}}\Big|^{n-1}$ V.

24. The *Momentum* of the firſt will be AV, that of the ſecond $\frac{2rAV}{1+r}$, that of the third $\frac{4r^2AV}{\overline{1+r}^2}$, that of the fourth $\frac{8r^3AV}{\overline{1+r}^3}$, and ſo on to the laſt, which will be $\overline{\frac{2r}{1+r}}\Big|^{n-1}$ AV.

25. To give an Example; if $n=100$, and $r=2$; then will the firſt Body A be to the laſt $r^{n-1}A$, as 1 to 633825300000000000000000000000, nearly; and its Velocity to that of the laſt nearly as 271022000000000000 to 1: Laſtly, the *Momentum* of the firſt to the laſt will be nearly as 1 to 2338480000000.

26. If the Number (n) of Bodies be required, and the Ratio of the *Momenta* of the firſt and laſt be given as 1 to M, and the Ratio of the Series r given alſo; then putting $\frac{2r}{1+r}=R$, we have the *Momentum* of the laſt Body expreſs'd by $\overline{\frac{2r}{1+r}}\Big|^{n-1}=M=R^{n-1}$; therefore the Logarithm of M ($l.M$) is equal to the Logarithm of R ($l.R$) multiplied by the Power $n-1$; that is, $l.M=\overline{n-1}\times l.R$; conſequently $\frac{l.M}{l.R}+1=n$, the Number of Bodies required.

poſed to perſevere in its State of Motion or Reſt. 'Tis not more evident that Matter at Reſt requires an extrinſic Power to give it Motion, than that, when in Motion, the Force of ſome other Body reſiſting it is neceſſary to bring it to a State of Reſt. For want of ſuch Reſiſtance we ſee the Planets and Comets long conſerve their Motions undiminiſh'd; while moving Bowls, and Wheels, are gradually reduced to a State of Reſt, by the Friction or Rubbing of the Parts on which they move, againſt contiguous reſiſting Bodies; as is evident by the Experiment of the *Axis in Peritrochio*, moving firſt on fix'd Boxes, and afterwards on *Friction-Wheels.*

FROM this Law, and what will be farther demonſtrated hereafter, it follows, that no *perpetual Motion* can be effected, at leaſt by any human Power, with Bodies in a reſiſting Medium.

BY the *ſecond Law* we are inſtructed how to eſtimate the Sum of the Motions of Bodies moving the ſame or contrary Ways, when they directly ſtrike, or impinge on each other. Alſo we hence learn the *Compoſition and Reſolution of Motion* ariſing from Forces impreſs'd in oblique Directions; a Doctrine of the utmoſt Uſe in Philoſophy, and the Foundation of all *Mechanics.* To illuſtrate this: Let the Body B be impell'd by the Body A in the Direction *b c*, with a Force that would, in a given Time, cauſe it to move from *b* to *c*; at the ſame Inſtant, let another Body C ſtrike it in the Direction *b d*, with a Force that will carry it from *b* to *d* in the ſame Time; then

Plate III. Fig. 4.

compleat the Parallelogram *bced*, and draw the Diagonal *be*, that will repreſent the Direction and Diſtance through which the Body will move in the ſame Time by both the Forces conjointly (XXIV).

(XXIV.) This is evident if we conſider that the Force impreſs'd by the Body C does no ways diminiſh the Velocity of a Body approaching to the Line *ce*, at the End of the given Time, and therefore it will then be found ſomewhere in the ſaid Line *ce*. For the ſame Reaſon it will at the End of the ſaid Time be carried to a Diſtance from *bc* equal to *bd*, and therefore it muſt alſo at the ſame Moment be found ſomewhere in the Line *de*; but it cannot be in the Line *ce* and *de* at the ſame Time, unleſs in that Point *e*, where they interſect each other, as the Propoſition aſſerts.

2. We may now conceive the Body B moving by the ſingle Impulſe of ſome Power in the Direction *be*, ſuch as will carry it thro' the Space *be* in a given Time; then this may be reſolved into any other two Forces acting in the Directions *be* or *de*, and *bd* or *ce*, which Lines will alſo repreſent the Efficacy of the ſaid Forces in the ſame Time.

3. This Doctrine of the *Compoſition and Reſolution of Forces* will be found of very frequent Uſe; and will be farther illuſtrated in an Application to the Doctrine of *Oblique Percuſſion*, as follows. Let the Body A impinge on B at reſt in the Direction A C, which becauſe it does not go thro' the Center of the Body B, the Stroke will not be *Direct*, but *Oblique*; and let the Force of the Stroke be repreſented by the ſaid Line A C.

4. This Force A C may be reſolved into the two Forces A I, and I C; the former of which being parallel to the horizontal Line G F, cannot affect the Body B at all; for the Motion in that Line would only paſs by the Superficies of B, and touch the uppermoſt Point without any Stroke or Force to remove it.

5. But that Part of the Stroke repreſented by I C, paſſes thro' the Center of B, and therefore expreſſes the whole Force with which the Body A ſtrikes it in the Direction A C. *Therefore the Force of an Oblique Stroke is to the Force of a Direct Stroke, as* I C *to* A C, *paſſing thro' the Center of the Body* B.

6. Now ſince I C = A G, we have this Analogy, *The Direct Stroke is to the Oblique Stroke, as* A C *to* A G, *that is, as Radius to the Sine of the Angle of Inclination* A C G. Whence, 'tis evident, the leſs this Angle is, the more oblique the Stroke, and the leſs its Force, becauſe the leſs muſt A G (= I C) be in reſpect of A C.

THE *third general Law* is founded on Reaſon and Experience: We know from the Nature of Attraction or Gravity, that if a Stone fall towards the Earth, the Quantity of Motion both in the Earth and Stone is the ſame. That the Iron attracts the Loadſtone with an equal Power of Magnetiſm, is evident by Experiment.

7. If the Bodies A, B, are *Non-elaſtic*, in the Line G F, take C F = C G = A I, and this will be the horizontal Velocity of A after the Stroke, which will be the ſame as before, and ſince the whole Force I C of the Striking Body A is known, we can find (by *Annot*. XXIII. 7.) what Part thereof remains after the Stroke, which let be expreſs'd by F D, (taken from F to the Right) then drawing C D it ſhall repreſent the Motion and Direction of A after the Stroke, while B will go on towards E with the Motion B E generated by the Stroke, (which alſo may be found as before) and will be equal to F D, if the Bodies A and B are equal.

8. If the Bodies are *Elaſtic*, and B leſs than A, the Direction of A after the Stroke, (*viz.* C D) will make an *Acute Angle* with B E the Direction of B, or lie on the Right of C F. If B be greater than A, it will be reflected ſo that C D will make an Obtuſe Angle with C E, or lie on the Left of it. But if both the Bodies are equal, the whole Motion of A in the Direction C E will be deſtroy'd (by *Annot*. XXIII. 19.) and it will proceed with only the horizontal Velocity in the Direction C F, making a Right Angle with C E, the Direction of B.

Plate IV. Fig. 13.

9. If two Bodies A, B, in Motion, impinge on each other in the Point C in the oblique Directions A C, B C, theſe are each reſolvable into two, *viz.* A E and E C, and B D and D C. Now ſince E C and D C are parallel, they cannot obſtruct each other, ſo will be the ſame after the Stroke as before it. But A E and B D, being oppoſite, will expreſs the Forces with which the Bodies ſtrike each other directly, and may be found by the Rules above deliver'd.

10. If then we make C*E* = C E, and C*D* = C D: Alſo C*G* = Motion of A after the Stroke, and C*F* = Motion of B, and compleat the Parallelograms *EG*, and *DF*, their Diagonals C*A* and C*B* will be the Directions, and expreſs the Velocities A and B after the Stroke. This Conſtruction is general, and may be accommodated to particular Caſes, varying with the Magnitudes and Velocities, &c. of the impinging Bodies.

That *Action* and *Re-action* are equal between impinging Bodies, or that the ſame Quantity of Motion that is generated in one Body is deſtroy'd in the other by the Stroke, whether the Bodies be *elaſtic* or *non-elaſtic*, will alſo be made apparent to the Senſes by Experiment. Whence alſo it will appear, that the Action or Effect of *elaſtic Bodies* is twice as great, as that of Bodies void of Elaſticity (XXV).

(XXV.) Let $Q =$ Quantity of Matter in the Earth, and $V =$ the Velocity with which it moves by Attraction; and let q and v denote the ſame Things in the falling Stone; then ſince the Earth and Stone act mutually on each other by Attraction, the Velocity of each, and conſequently the Spaces they deſcribe in the ſame Time, will be as the Power acting on each Body, which therefore will be inverſely as the Quantities of Matter in each: Conſequently $Q : q :: v : V$; and ſo $QV = qv$ or the *Momentum* of the Earth is equal to that of the falling Body.

2. In like Manner, when a Horſe draws a Stone, the Cord being equally ſtretch'd between both, acts equally upon both, and that which has leaſt Reſiſtance yields, and is drawn along. This is uſually the Caſe of the Stone; but if its Weight be increaſed, and therewith its Reſiſtance, till it be equal to or greater than that of the Horſe, then neither Horſe nor Stone will move; unleſs the Stone be laid on a Deſcent, and then it will move the contrary Way; and draw the Horſe after it.

3. In the ſame Manner we may underſtand how Rowing, Swimming, Flying, &c. is performed; for the Boat, the Fiſh, and the Bird, are Bodies eaſily moveable with the leaſt *Impetus*, but the Water and Air in which they move, tho' fluid Bodies, yet give great Reſiſtance to others which ſtrike them, or re-act with an equal Force, in a contrary Direction; and by this means impel the Boat, the Fiſh, and Bird in a Direction nearly contrary to that in which they ſtrike it, and with an equal Force.

4. *Merſennus* tells us a Cannon 12 Feet in Length, weighing 6400 *lb.* gives a Ball of 24 *lb.* an uniform Velocity at the Rate of 640 Feet *per* Second. Put $W = 6400$, $w = 24$, $v = 640$, $V =$ Velocity, with which the Cannon recoils.

FROM this Law we have a Solution of divers *Phænomena* otherwiſe not to be accounted for: As,

Now ſince the *Momentum* of the Cannon and Ball are equal, we have $wv = wv$, and ſo $v = \frac{wv}{w} = \frac{640 \times 24}{6400} = 2{,}4$, the Velocity of the Cannon; that is, it would recoil at the Rate of $2\frac{4}{10}$ Feet *per* Second, if free to move.

5. But if the Cannon be fix'd, it will receive a Shock or Preſſure from the expanſive Force of the Powder, equal to the Preſſure of a certain Weight, which Weight may thus be found. As the Powder conſtantly acts on the Ball while in the Cannon, it will drive it along with an accelerated Velocity, which accelerated Velocity will produce an uniform Velocity of 24 Feet in the ſame Time (as we ſhall ſhew hereafter) when free from the Cannon. And ſince the Ball goes at the Rate of 640 Feet *per* Second, it will paſs over 24 Feet in $\frac{1}{26\frac{2}{3}}$ of a Second, which therefore is the Time of its paſſing thro' the Cannon.

6. Now ſince in accelerated Velocities the Spaces paſs'd over are as the Squares of the Times, therefore as $\overline{\frac{1}{26\frac{2}{3}}}\Big|^2 : \overline{1''}\Big|^2$, or as $1''$ is to $\overline{26\frac{2}{3}''}\Big|^2$ $(= 711\frac{1}{4})$ ſo is 12 Feet to $8533\frac{1}{3}$ Feet, which the Ball would be carried through in one Second by the accelerating Force of the Powder.

7. But ſince the accelerated Force of Gravity gives the Ball a Weight of 24 Pounds, the greater accelerated Force of the Powder will give it the Force of a Weight $= 12800$ Pounds: For the Weights of Bodies are as the accelerated Forces, and theſe as the Spaces paſs'd through in the ſame Time; therefore 16 F. : 24 *lb.* :: $8533\frac{1}{3}$ F. : 12800 *lb.* as required.

8. But if the Cannon be free to recoil, Part of this Force will be ſpent in giving it a Velocity of 2,4 Feet in a Second, and to find what Part of the whole Weight 12800 Pounds; that is, we are to conſider that as the whole Force produces a Velocity of 640 Feet in a Second one Way, ſo when the Piece recoils, this Velocity is divided into two Parts in contrary Directions, *viz.* into $640 - 2{,}4$ and $2{,}4$; the whole Force therefore of 12800 Pound muſt be divided into Parts of the ſame Proportion; thus, as $640 - 2{,}4 : 2{,}4 :: 637{,}6 : 2{,}4 :: 797 : 3 :: 12800 - x : x :: 797 + 3\ (= 800) : 3 :: 12800 : x = 48$ Pounds. Wherefore of the 12800 Pounds

why ſoft and unelaſtic Bodies *retain the Impreſſions* of others, while hard and elaſtic Bodies *rebound with equal Force*, and under *equal Angles*. Hence, the Reaſon of *Rowing with Oars*, and the *Swimming of Fiſhes* in Water; alſo, the *Flying of Birds* in Air; the *Recoiling of Guns*; the retro-vertiginous Motion of the *Æolipile*; the *Hydroſtatic Paradox*; and many other Things hereafter occaſionally mention'd, are accountable for on this and no other Principle.

WE proceed next to conſider the *Nature of Motion belonging to Bodies which deſcend freely by the Force of Gravity* in *Vacuo*, or an unreſiſting *Medium:* And this Kind of Motion we ſhall find affected with the following Properties, *viz.* (1.) That it is equably accelerated. (2.) That its Velocity is always proportional to the Time of the Fall. (3.) That the Spaces paſs'd through are as the Squares of the Times or Velocities. And therefore, (4.) That the Velocity, and conſequently the *Momentum*, which is proportional thereto, is as the Square Root of the Space deſcended through. (5.) That the Space paſs'd through the firſt Second is very nearly $16\frac{13}{100}$ Feet. (6.) That a Body will paſs over twice the Space in an horizontal Direction, with the laſt acquired Velocity of the deſcending Body, in the ſame Time (XXVI).

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only 48 are ſpent for giving Velocity to the Cannon, the other 12772 Pounds ſpend themſelves in Preſſure on the Gun and Ball, giving it a Velocity of 637,6 Feet *per* Second.

(XXVI.) 1. It has been ſhewn (*Annot.* XXI.) that the Motion of a Body falling freely by its Gravity is *equably accele-*

HENCE it follows, that if one Leg AB of a right-angled Triangle repreſent the *Time* of the Fall, and the other Leg BC the *Velocity* acquired at the End of the Fall; then will the Area ABC

rated; and that *its Velocity is at all Diſtances proportional to the Time of the Fall*, is evident from hence, that in every equal Particle of Time, the Body receives an equal Impulſe from Gravity, which generates an equal Increment of Velocity; which, therefore, as it increaſes with, muſt alſo be proportional to the Times.

Plate III. Fig. 5.

2. *That the Spaces paſs'd thro' are as the Squares of the Times or Velocities* is hence evident, that if the Time of the Fall thro' a certain Space be repreſented by A 1, and the Velocity acquired in the End of that Time be 1 *a*; then drawing A *a*, the Triangle A 1 *a* will repreſent the Space paſs'd thro' in that Time. For if we conceive the Line A 1 to be divided into an infinite Number of equal Parts, and thro' each of theſe Diviſions, Lines drawn parallel to 1 *a*, theſe will repreſent the Velocities in the ſeveral Particles of Time. Now the Space deſcribed in each Moment will be as the Velocity (becauſe for a Moment the Velocity may be conſider'd as uniform) conſequently, the Sum of all the Spaces deſcribed in the Moments in the Time A 1 will be as the Sum of all the Velocities, or Lines parallel to 1 *a*, which repreſent them: But the Sum of all theſe Lines make up the Area of the Triangle A 1 *a*, therefore the whole Space paſs'd thro' in the Time A 1 will be as the Area of the Triangle A 1 *a*.

3. Therefore the Triangle A 2 *b* repreſents the Space paſſed thro' in the Time A 2, and the Triangle A 3 *c* the Space paſs'd thro' in the Time A 3, and ſo on. But the Triangles A 1 *a*, A 2 *b*, &c. are ſimilar, and therefore are to each other as the Squares of their Sides A 1, A 2, or 1 *a*, 2 *b*, &c. That is, *the Spaces are as the Squares of the Times or Velocities*.

4. Conſequently the *Velocities and Times of the Fall are as the Square Roots of the Spaces paſs'd thro'*; and ſince the *Momentum* of a Body is always $M = QV$, and in this Caſe Q being a given Quantity, we have $M : V$, or the *Momentum of the Body as the Velocity, or as the Square Root of the Height thro' which the Body falls*.

5. It has been found by very accurate Experiments made by letting Bodies of various Sorts fall from the Height of the Dome of St. *Paul's*, to the Pavement, that Gravity accelerates Bodies in the beginning of their Fall, at the Rate of 16,13

of the Triangle repreſent the Space paſs'd through. And hence the Spaces deſcended through at the End of every Second will be as the Square Numbers 1. 4. 9. 16. 25. 36. &c. and therefore the

Feet in the firſt Second of Time. This may alſo be otherways ſhewn *à priori*, becauſe we demonſtrate that the Time (T) of the Vibration of a Pendulum (in the Arch of a Cycloid) is to the Time (*t*) of a Body's falling thro' half the Length of the Pendulum ($\frac{1}{2}$ L) as the Circumference of a Circle (P) to the Diameter (D). (*Annotat.* XXVIII. 9.)

6. Again, it is found that the Length of a Pendulum vibrating Seconds is 39,2 Inches. Now ſince $T : t :: P : D : 3{,}14159 : 1$, we have $T^2 : t^2 :: \overline{3{,}14159}^2 : 1^2 :: x : \frac{1}{2}L$; whence we have $x = \frac{1}{2}L \times \overline{3{,}14159}^2 = 16$ Feet = the Space deſcended in the Time of one Vibration, or 1 Second.

7. Since the Space paſs'd thro' by an uniform Motion in the Time AB, with the Velocity BC is AB × BC = ABCD = 2 ACB; it is evident that *a Body moving uniformly with the laſt acquired Velocity* BC *of a deſcending Body, will move thro' twice the Space in the Time* AB *of the Fall.*

Plate III. Fig. 5.

8. It is farther evident, that ſince the Spaces deſcended thro' in each Second are as the odd Numbers, 1, 3, 5, 9, 11, 13, 15, &c. and theſe Numbers conſtantly approach nearer and nearer to an Equality, ſo the accelerated Motion of the Deſcent does by Degrees approach nearer and nearer to an uniform Motion; thus the Spaces 101, 103, 105, differ but little; and the Spaces 1001, 1003, 1005, differ leſs; but when we come to the Spaces 1000000001, 1000000003, 1000000005, theſe are ſo very near equal, that the Motion may now be eſteem'd uniform.

9. If the Deſcent be in a reſiſting Medium, the Motion will actually become uniform at a certain Diſtance, and the ſooner as the Medium is denſer; thus a Body falling in Air will be longer in acquiring an uniform Motion than in Water, and in Quickſilver it will ſoon obtain it: Becauſe as the Denſity increaſes, ſo does the Reſiſtance, and conſequently the Increments of Velocity are annihilated in the ſame Ratio, and the Motion reduced to Uniformity.

10. Bodies of the ſame Matter and Figures will ſooner come to an uniform Motion as the Magnitudes are leſſer; for 'tis ſhewn (in Geometry) that the Quantities of Matter decreaſe in Bodies with the Cubes of the Diameters, but the Surfaces de-

Spaces paſs'd through in each Second ſeparately will be as the odd Numbers 1. 3. 5. 7. 9. 11. 13. 15. *&c.* as in the *Figure.*

THE next Sort of Motion is that of Bodies deſcending on *inclined Planes*, and *curved Surfaces*, which we find diſtinguiſh'd with the following Properties. (1.) The Motion on the inclined Planes is equably accelerated, as ariſing from Gra-

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creaſe only with the Squares of the Diameters. And ſince the Reſiſtance is proportional to the Surfaces of Bodies, moving in the ſame Medium with the ſame Velocity, it will follow that ſmaller Bodies will be more reſiſted than larger ones, and ſo come ſooner to an uniform Motion. Hence it is, that a Body reduced to Powder deſcends very ſlowly and with nearly an uniform Motion in Water, tho' in the Solid much heavier than it. Alſo a Bullet ſhot from a Gun will go much farther and with a greater Velocity than a Charge of ſmall Shot of the ſame Weight, or a Ball of Cork of the ſame Size, which *in Vacuo* would go much farther than the Bullet, as admitting a greater Velocity with a leſs Quantity of Matter, from the ſame *Momentum* of the Powder.

11. If Bodies (equal and homogeneous) move in the ſame Medium with different Velocities, thoſe which move faſteſt will ſooneſt acquire an uniform Motion, ſince (as will be ſhewn) the Reſiſtance in ſuch a Caſe increaſes with the Squares of the Velocity. If the Velocity with which a Body is projected downwards be equal to the uniform Motion of a free Deſcent, the Motion will be every where uniform, becauſe the Reſiſtance of the Medium is equal to Gravity. But if a Body (as a Ball from a Cannon) be projected with a Velocity greater than that, the Motion will be retarded, inaſmuch as the Reſiſtance is now greater than Gravity, and ſo will diminiſh the uniform Motion, ariſing from the Equilibrium juſt mention'd.

12. Hence light Bodies (as an Arrow) thrown directly upwards, ſpend leſs time in riſing than falling, becauſe the Motion upwards is altogether retarded, whereas that of the Deſcent is in ſome Part uniform. Alſo the greater the Velocity with which a Body (as a Bullet) enters a denſe Medium (as Water,) the leſs the Effect at a given Diſtance therein; and with the greater Force they are reflected or thrown out again, when projected obliquely, as we ſee in a Stone thus thrown into the Water, *&c.*

vity. (2.) The Force of Gravity compelling a Body, as A, to deſcend on an inclined Plane BD, is to the abſolute Force of Gravity as the Height of the Plane BC to its Length BD. (3.) The Spaces deſcended are as the Squares of the Times. (4.) The Times in which different Planes, BD, BH, of the ſame Altitude BC are paſs'd over, are as their Lengths reſpectively. (5.) The Velocities acquired by deſcending through ſuch Planes at the loweſt Points, D, H, are all equal. (6.) Therefore if a Body deſcends from the ſame Height through ſeveral contiguous Planes, ever ſo different in Number or Situation, it will always acquire the ſame Velocity in the loweſt Point. (7.) The Times or Velocities of Bodies deſcending through Planes ſimilarly ſituated, or alike inclined to the Horizon, as DB, EC, will be as the Square Roots of their Lengths, DB, EC (XXVII.)

Plate V. Fig. 1.

Fig. 2.

(XXVII.) Let BD be an inclined Plane, A a Body deſcending thereon; from the Center C draw CE perpendicular to the Plane, and CG perpendicular to the Horizon or Baſe CD, and meeting the Plane in F. Now ſince the Body gravitates in the Direction CF, let CF repreſent the Weight of the Body or abſolute Force of Gravity; this may be reſolv'd into two Forces, acting in the Direction CE and EF, of which the firſt is perpendicular to the Plane, and is that by which it preſſes or acts upon the Plane; but ſince the Plane equally acts upon the Body in a contrary Direction, that Force CE is wholly deſtroy'd. The Remainder therefore, EF, is that which carries the Body down the Plane, as acting in a Direction parallel thereto; and this is call'd the Relative Gravity.

Plate VI. Fig. 4.

2. Now becauſe of the ſimilar Triangles CFE, DFG, DBC, we have CF : FE :: DF : FG : BD : BC; that is, *the Abſolute Gravity* CF *is to the Reſidual or Relative Gravity* FE, *as the Length of the Plane* BD *to the Height thereof* BC.

3. Alſo ſince the Triangles BDC and BCI are ſimilar (CI

FROM theſe Properties of Bodies deſcending on Inclined Planes, we deduce the following Corollaries, *viz.* (1.) That the Times in which a Body deſcends through the Diameter B C, or any

being drawn perpendicular to B D); and ſince the Spaces deſcribed in the ſame Time in the Perpendicular and on the Plane will be as the Powers of Gravity, or accelerating Forces in thoſe Directions, that is, as BD to BC, or as BC to BI (for $BD : BC :: BC : BI$). Therefore BI will be the Diſtance deſcended on the Plane in the ſame Time as a Body would deſcend freely in the Perpendicular from B to C.

4. Since the Motion on the Plane is accelerated, the Time of deſcribing BI will be to the Time of deſcribing BD, as $\sqrt{BI} : \sqrt{BD} :: t : T$; and ſo $t^2 : T^2 :: BI : BD$; but, (becauſe $BI : BC :: BC : BD$) we have $BI : BD :: \overline{BI}^2 : \overline{BC}^2$; therefore $t^2 : T^2 :: \overline{BI}^2 : \overline{BC}^2$; and ſo $t : T :: BI : BC :: BC : BD$; *conſequently the Time of deſcribing* BI *or* BC *will be to the Time of deſcending through the Plane* BD, *as the Height of the Plane* BC *to its Length* BD.

5. Hence the Time ($\mathcal{T}$) of deſcending thro' any other Plane BH of the ſame Height (*Fig.* 6.) is to to the Time (t) of the Deſcent thro' the Perpendicular BC, as BH to BC; that is, $\mathcal{T} : t :: BH : BC$; but it is alſo $T : t :: BD : BC$; therefore *ex æquo*, we have $T : \mathcal{T} :: BD : BH$; *or the Times of deſcending thro' ſeveral Planes* BD, BH, *of the ſame Height, are as the Length of the Planes.*

6. The Velocity acquired in falling to C, is to the Velocity at I as BC to BI, (for the Velocities generated in the ſame Time, muſt be as the Powers which produce them) Alſo the Velocity at D is to that at I as $\sqrt{BD}$ to the $\sqrt{BI}$; or $V : \mathcal{V} :: \sqrt{BD} : \sqrt{BI}$; and ſo $V^2 : \mathcal{V}^2 :: BD : BI :: \overline{BC}^2 : \overline{BI}^2$, therefore $V : \mathcal{V} :: BC : BI$; ſince then the Velocity at C and D have both the ſame Ratio to the Velocity at I, they muſt be equal to each other.

7. In the ſame Manner it is ſhewn that the Velocity at H is equal to the Velocity at C; and conſequently the Velocities acquired in deſcending thro' Inclined Planes of the ſame Height are always equal.

Plate VI. Fig. 2. 8. Hence a Body deſcending thro' ſeveral Inclined Planes AF, FE, ED, contiguous to each other, will have the ſame Velocity at the Point D, as it would have acquired in falling freely thro' the ſame perpendicular Height BC, for

Chord BE of a Circle, are equal. Hence, (2.) All the Chords of a Circle are deſcribed in equal Times. (3.) The Velocities acquired in deſcending through any Arch of a Circle EC, in the

the Velocity at F is the ſame it would have had by deſcending thro' GF, which has the ſame Altitude; and the Velocity at E is the ſame it would have gain'd by deſcending thro' BE; conſequently the Velocity of D is the ſame it would acquire by deſcending thro' BD, and therefore the ſame it would have by falling freely thro' the Perpendicular BC.

9. If now we ſuppoſe the Number of thoſe contiguous Planes infinite, and their Lengths infinitely ſmall, they will then conſtitute a *Curve Line*; whence it follows, That a Body deſcending thro' the Arch of any Curve BH will acquire the ſame Velocity in the loweſt Point H, as it would have at C, by falling thro' the ſame perpendicular Height BC.

10. If there are two Planes BD, and CE, ſimilarly ſituated, or alike inclined to the Horizon AE; the Time of the Deſcent on DB, will be to the Time of the Deſcent on CE, as Square Roots of their Lengths; for the Time on BD is to the Time thro' BA, as DB to BA (by *Art.* 4.) and the Time on CE is to the Time thro' CA, as CE to CA; but the Times thro' BA and CA are as the Square Roots of BA and CA; and ſince by ſimilar Triangles we have $\sqrt{BA} : \sqrt{CA} :: \sqrt{BD} : \sqrt{CE}$, it follows, *That the Times of Deſcent on two ſimilar Planes* DB *and* CE *are as the Square Roots of their Lengths*, or as $\sqrt{BD}$ to $\sqrt{CE}$. Plate VI. Fig. 3.

11. From what has been demonſtrated in the preceding Articles, it follows (1.) That the Times of deſcending from B to I, and to E, are equal, being each equal to the Time of Deſcent thro' the Perpendicular BC. (2.) For the ſame Reaſon the Deſcent thro' IC and EC, and conſequently thro' all the Chords of a Circle, are equal. (3.) The Velocities at C acquired by the ſaid Deſcents are as the Lengths of the Chords IC and EC. For ſince $BC : EC :: EC : CF$ we have $BC = \frac{EC^2}{FC}$; and for the ſame Reaſon $BC = \frac{IC^2}{GC}$, whence $\frac{EC^2}{FC} = \frac{IC^2}{GC}$; and therefore $IC : EC :: \sqrt{GC} : \sqrt{FC}$; that is, as the Velocities in the Deſcents through IC and EC. (4.) Laſtly, ſince Times of Deſcent through the Planes DB and EC are as $\sqrt{DB}$ to $\sqrt{EC}$, they will alſo be as $\sqrt{BA}$ to $\sqrt{CA}$; for $DB : EC :: BA : CA$. Plate V. Fig. 1.

loweſt Point C, is equal to that which would be acquired in falling through the ſame perpendicular Height FC. (4.) The Velocities acquired in deſcending through the Chords IC, EC, of a Circle are, at the loweſt Point C, as the Lengths of thoſe Chords. (5.) The Times of Deſcent through Chords of ſimilar Arches, DB, EC, are as the Square Roots of the Semidiameters AB, AC, of the reſpective Circles.

Plate V. Fig. 3. FROM theſe Properties, and their Corollaries, the Doctrine of PENDULUMS is derived. A PENDULUM is any Body B, ſuſpended upon, and moveable about a Point A, as a Centre. The Nature of a Pendulum conſiſts in the following Particulars. (1.) The Times of the Vibrations of a Pendulum in very ſmall Arches are all equal. (2.) The Velocity of the Bob in the loweſt Point will be nearly as the Length of the Chord of the Arch which it deſcribes in the Deſcent. (3.) The Times of Vibration in different Pendulums, AB, AC, are as the Square Roots of their Lengths. (4.) Hence the Lengths of Pendulums AB, AC, are as the Squares of the Times of their Vibrations. (5.) The Time of one Vibration is to the Time of Deſcent through half the Length of the Pendulum, as the Circumference of a Circle to its Diameter. (6.) Whence the Length of a Pendulum vibrating Seconds will be found 39 Inches nearly; and of an *Half-Second* Pendulum 9,8 Inches. (7.) An Fig. 4. uniform homogeneous Body BG, as a *Rod*, *Staff*, &c. which is *one third Part* longer than a Pendulum AD, will vibrate in the ſame Time with it (XXVIII). (8.) This *Centre of Oſcillation* E

(XXVIII.) 1. As the Doctrine of Pendulums and Time-keeping Inſtruments depends in a great Meaſure upon the *Cycloid*, I think it neceſſary here to ſhew the Nature and Uſe of that Curve, with regard thereto. If a Circle ABC, inſiſting on a right Line AL, begin to revolve in the Manner of a Wheel, from A towards L, the Point A will by its twofold Motion deſcribe the Curve ACDIL, while the Circle makes one Revolution from A to L. Plate VI. Fig. 4.

2. This Curve is call'd the CYCLOID, and from the Definition 'tis evident (1.) That the *Baſe* of the Cycloid AL is equal to the Periphery of the *generating Circle* ABC, (2.) The Axis of the Cycloid FD is equal to the Diameter of the ſaid Circle. (3.) That the Part of the Baſe KL is equal to the Arch of the Circle IK. (4.) Therefore KF (= ME = IG) is equal to the remaining Arch IH, or GD. (5.) That the Chord of the Circle KI is perpendicular to the Cycloid in the Point I; and (6.) Therefore the Chord HI (being at right Angles with IK,) is a Tangent to the Curve in the Point I. (7.) The ſaid Tangent HI is parallel to the Chord DG.

3. Parallel to EI draw *ei* indefinitely near, and I*n* perpendicular thereto; then will the Triangles DGE, DGF, I*ni*, be ſimilar, and ſo we have DE : DG :: DG : DF :: I*n* : I*i*; that is, (putting DF = a, and DE = x DI = z,) $x : \sqrt{ax} :: \sqrt{ax} : a :: \dot{x} : \dot{z} = \mathrm{I}i = \frac{a\dot{x}}{\sqrt{ax}}$ = Fluxion of the Arch DI, whoſe Fluent is $2\sqrt{ax}$ = 2 DG = the Arch DI; and conſequently, the Semicycloid DIL = 2 DF, the Diameter of the generating Circle.

4. Let AID be a Semicycloid inverted; and ſuppoſe a flexible String faſten'd at one End in A, and ſtretch'd along the ſaid Curve AID, ſo that the other End of the String ſhould be made to coincide with the Point D; whence the Length of the String will be equal to that of the Curve: If now the End of the String be taken from D, and with a tight Hand be drawn from the Cycloid, it will, in its Evolution, deſcribe the Curve DBC, the Nature of which we are now to inveſtigate.

5. Let the String, in evolving, be in any Situation AIHB, then is IB (call'd the *Radius of Evolution*) equal to the Part of the Cycloid ID. Put FD = AK = a, DE = x,

CO $= y$, CB $= z$, then is $ni = \dot{x}$, Ba = O$p = \dot{y}$, B$b = \dot{z}$; and (by *Art.* 3.) DI = (IB =) $2\sqrt{ax}$, whoſe Fluxion is $\dot{x}\sqrt{\frac{a}{x}} =$ Ii; therefore $\overline{\dot{x} \times \sqrt{\frac{a}{x}}}^2 - \dot{x}^2 =$ $(\overline{\text{I}i}^2 - \overline{in}^2 =) \frac{\dot{x}^2 a - x\dot{x}^2}{x} = \overline{\text{I}n}^2$, whence I$n =$ $\dot{x}\sqrt{\frac{a-x}{x}}$; but I$n$: Ii :: ca : cB :: aB : B$b = \dot{z} =$ $\frac{\sqrt{a} \times \dot{y}}{\sqrt{a-x}}$; and I$i$: ni :: IB : MB $= 2x$; therefore BM — BN = MN = IQ = ED $= x$. Conſequently BH = HI; and alſo CK = KA, becauſe all the Radii of Evolution (of which AC is the laſt) are biſected by the Baſe LD; therefore CK — OK $= a - x =$ OC $= y$; whence $\sqrt{y} = \sqrt{a-x}$, and ſo $\dot{z} = \frac{\sqrt{a} \times \dot{y}}{\sqrt{y}}$; therefore $z = 2\sqrt{ay}$; conſequently the Curve CBD, having the ſame Property with AID, is every way equal and ſimilar thereto.

6. Whence it appears that if AC be the Length of a Pendulum ſo diſpoſed, as to vibrate between the two Semicycloids AD and AL, the Bob will deſcribe in its Motion the Cycloid DCL. The Properties of which Motion will now eaſily appear: For with reſpect to the Velocity acquired by deſcending thro' any Arch RC or BC, it is always as $\sqrt{\text{OC}}$ which is as the Chord SC $= \frac{1}{2}$ RC; conſequently *the Velocity at* C *is every where as the Space paſs'd through, or as the Arch of the Cycloid deſcribed in the Deſcent.*

7. But ſince it is always S = TV, in all kind of Motions; if in any Caſe S be as V, tis evident T muſt be a given Quantity, or always the ſame. That is, When the Spaces (S, s,) are as the Velocities (V, v,) the Times (T, t,) will be always equal; and therefore *all the Arches of a Cycloid, great or ſmall, are deſcribed in equal Times.*

8. If we put CK $= a$, KO $= x$, then will 2 SC = RC = $2\sqrt{aa - ax}$; if the Deſcent be from L to R, the Velocity at R will be $\sqrt{x} = \sqrt{\text{OK}}$. Now CO : CS (:: CS : CK) :: Rq : R$r = \frac{a\dot{x}}{\sqrt{aa - ax}} = \dot{z}$, the Fluxion of the Arch LR, which divided by the Velocity $\sqrt{x}$ gives the Fluxion

of the Time $= \frac{a\dot{x}}{\sqrt{aax - axx}} = \frac{a\dot{x}}{\sqrt{a} \times \sqrt{ax - xx}}$; but $\frac{a\dot{x}}{\sqrt{ax - xx}}$ = is the Fluxion of the circular Arch K S; therefore the Fluent of the Time of Deſcent thro' L R is Arch $\frac{KS}{\sqrt{a}}$ wherefore when S coincides with C, L R will become L C; and ſo the Time of Deſcent thro' the Semi-cycloid will be $\frac{KSC}{\sqrt{a}}$.

9. Therefore the Time of Vibration thro' the whole Cycloid L C D is $\frac{2KSC}{\sqrt{a}}$; and the Time of Deſcent thro' the Perpendicular K C = *a* being $\sqrt{a}$, we have $\frac{2KSC}{\sqrt{a}} : \sqrt{a} :: 2KSC : a$. *The Time of a Vibration in the Cycloid is to the Time of Deſcent thro' half its Length, as the Circumference of a Circle to its Diameter,* or as 3,14159 to 1.

10. If A C be the Length of a Pendulum vibrating Seconds, it may be eaſily found, ſince it is known by moſt accurate Experiments, that a Body deſcends freely by Gravity thro' 193½ Inches in the firſt Second of Time; therefore by making this Analogy, as $3,14159^2 : 1^2 :: 193\frac{1}{2} : 19,6 = AK$, and ſo A C = 39,2 Inches, *the Length of a ſecond Pendulum, as requir'd.* And ſince the Lengths are as the Squares of the Times of Vibrations, we have 4 : 1 :: 39,2 : 9,8 Inches, the Length of an half-ſecond Pendulum.

11. Since the Length of a Pendulum A C, for vibrating Seconds, is ſo conſiderable, the Bob, without the Cycloidal Checks A L and A D, will indeed deſcribe the Arch of a Circle, as *f*C*e*, when it vibrates, but then this Circular Arch will, for ſome Diſtance on each Side the loweſt Point C, coincide with the Cycloid very nearly, as for Example, to *g* and *h*; and hence it follows that a common Pendulum, vibrating thro' very ſmall Arches *gh*, will perform all its Vibrations in equal Times. And hence the above-mentioned Cycloidal Checks came into Diſuſe.

12. Hence the Time of Deſcent thro' the Chord C*g* is to the Time of Deſcent thro' the Arch of the Circle or Cycloid belonging to it as $\sqrt{4a}$ (ſuppoſing A C = 2*a*) to $\frac{\frac{1}{2}P}{\sqrt{}}$, or as 2*a* to ½P, that is, as *a* to ¼P $= \frac{3,14159}{4} = 0,7854$, or

in

THE greateſt Inconvenience attending this moſt uſeful Inſtrument is, that it is conſtantly liable to an Alteration of its Length from the Effects of *Heat* and *Cold*, which very ſenſibly expand and contract all Metalline Bodies, as will be very evident by the *Pyrometer* (XXXI).

the Gravity under the Equator is to the Gravity with us, as 391 to 392; as being proportional to the Lengths of the Pendulums (*Annotat.* XXIII. *Art.*13.) Whence it is evident the Surface of the Earth is farther diſtant from the Centre under the Equator than in our Latitude, and therefore is not Spherical. What the true Figure is, and whence it proceeds, will be ſhewn hereafter.

2. To the Uſes of a Pendulum above-mention'd may be added another moſt conſiderable, viz. *That a Pendulum is an univerſal Standard of Meaſure,* becauſe the Length of any Pendulum is known by the Time of its Vibration; and therefore may be compared with the Length of a Second Pendulum as a Standard. Thus ſuppoſe a *Japoneſe* was enquiring the Length of the *Engliſh* Foot, and was told it made 100 Vibrations in the Time a Second Pendulum made 55, he would immediately know the Time of a ſingle Vibration in each muſt be inverſely as thoſe Numbers, *viz.* as 55 to 100; and then, that the Lengths of thoſe Pendulums were as the Squares of the Times, *viz.* as $\overline{100}^2 : \overline{55}^2 :: 39,2 : 12 =$ the Length of one Foot.

3. Hence, if the Length of a Second Pendulum were made to conſiſt of 1000 equal Parts, the Lengths of the Meaſure made uſe of by all Nations might be compared therewith, and expreſs'd in thoſe Parts; thus the *Engliſh* Foot would be found equal to 306,9 nearly; for as 39,2 : 12 :: 1000 : 306,9 *fere*. And thus for any other. If this Method of aſcertaining the Meaſures were uſed by all Nations, it would entirely preclude all Doubts and Obſcurity in this Affair to Poſterity.

(XXXI.) 1. The PYROMETER which I have contrived to ſhew the Extenſion of heated Metals, &c. is perhaps the moſt ſimple, exact, and eaſy, that the Nature of ſuch a Machine will admit; and the Manner of computing its Effect is as follows.

2. AB is the Diameter of the Axis or Spindle on which the heated metalline Bar is laid, and is moved by its Exten-

WHEN Pendulums were firſt applied to Clocks, they were made very ſhort; and the Arches of the Circle deſcribed being large, the Times of Vibration through different Arches could not, in

ſion; on the other End is fix'd a ſmall Wheel CEDF, which is connected by a ſilken Thread to a ſmall Pinion Wheel GH, which on its Extremity carries a fine Index over the Diviſions of a large graduated Circle IKL.

3. Now admitting the metalline Rod extends $\frac{1}{10000}$ Part of an Inch, 'tis plain the Circumference of the Axis AB, on which it lies, muſt be moved through the ſame Space; and ſuppoſing the Wheel EF to be in Diameter 10 Times as large as the ſaid Axle AB, the Motion at D will be 10 Times as great, or the Point D will move through $\frac{1}{1000}$ Part of an Inch. But juſt ſo much as D moves, will the Periphery of the Pinion GH move, they both moving with the ſame Thread; if therefore the Circle IKL be in Diameter 10 Times as large as that of the Pinion GH, the Motion of the Index (which it carries) on its Circumference will be 10 Times as great, *viz.* $\frac{1}{100}$ Part of an Inch, which will be viſible to the Eye: Conſequently every Inch on the graduated Circle will ſhew the Extenſion of $(100 \times \frac{1}{10000} =) \frac{1}{100}$ Part of an Inch in the Metal. And ſo if the whole Circumference of the Circle IKL be 10 Inches, one Revolution of the Index will ſhew the Expanſion of $\frac{1}{10}$ of an Inch in the Metal.

4. Hence it is eaſy to obſerve, that if the Wheel EF, and Circle IKL, be enlarged in any Proportion to the Axles AB and GH greater than what I have aſſign'd, the Power or Effect of the Machine will be increaſed in the ſame Degree; and hence this Inſtrument may be conſtructed to ſhew very great Expanſions of the Metals with the utmoſt Degree of Accuracy.

5. The Reader may be curious to know in what Proportion the ſeveral Sorts of Metals are expanſible, which I ſhall here ſhew in a Table made from the Experiments of Profeſſor *Muſchenbroek*, with 1, 2, 3, 4, 5, Flames of Spirits of Wine.

	Iron.	*Steel.*	*Copper.*	*Braſs.*	*Tin.*	*Lead.*
Flame 1	80	85	89	110	153	155
2	117	123	115	220	*	274
3	142	168	193	275	*	*
4	211	270	270	361	*	*
5	230	310	310	377	*	*

that Caſe, be equal; to effect which, the Pendulum was contrived to vibrate in the Arch of a *Cycloid*, the Property of which Curve is, *that a Body will deſcend from any Part thereof to the low-*

6. Here we obſerve the Expanſion of *Iron* to be the leaſt of all Metals, and that of *Tin* and *Lead* the greateſt, and nearly equal with one Flame. Hence the Rods of Pendulums ought to be made of Iron, and alſo all Meaſures of Length, as Yards, *&c.* and whatever elſe we would not have alter'd by Heat and Cold.

7. The Method of remedying the Inconvenience ariſing from the Extenſion and Contraction of the Metalline Rod of Pendulums, is by applying the Bob with a Screw, by which means the Pendulum is at any time made longer or ſhorter, as the Bob is ſcrew'd downwards or upwards, and ſo the Time of its Vibration is continually the ſame.

8. If a Glaſs or Metalline Tube, uniform throughout, fill'd with Quickſilver, and the Length of 58,8 Inches, were applied to a Clock, it would *vibrate Seconds* (for $39.2 = \frac{2}{3}$ of 58,8) and ſuch a Pendulum admits of a twofold Expanſion and Contraction, *viz.* one of the Metal and the other of the Mercury, and theſe will be at the ſame Time contrary, and therefore will correct each other.

9. For by what we have ſhewn, the Metal will extend in Length with Heat, and ſo the Pendulum will vibrate ſlower on that Account. The Mercury alſo will expand with Heat, and ſince by this Expanſion it muſt extend the Length of the Column upward, and conſequently raiſe the *Centre of Oſcillation*, ſo that by this means its Diſtance from the Point of Suſpenſion will be ſhorten'd, and therefore the Pendulum on this Account will vibrate quicker: Wherefore if the Circumſtances of the Tube and Mercury are ſkilfully adjuſted, the Time of the Clock might by this means, for a long Courſe of Time, continue the ſame, without any ſenſible Gain or Loſs.

10. This is the Invention of Mr. *Graham* in the Year 1721, who made a Clock of this Sort, and compared it with one of the beſt of the common Sort for three Years together, and found the Errors of the former but about $\frac{1}{8}$ Part of the latter; of which the Reader may ſee a farther Account in *Phil. Tranſ.* N°. 392.

eſt Point in the ſame Time, and ſooner than by any other Way (XXXII).

(XXXII.) 1. How far the Cycloid is concern'd in the Pendulum, has been ſhewn; I ſhall now ſhew that this is the *Curve of quickeſt Deſcent*; for let AHC be the Curve of quickeſt Deſcent, then will a Body deſcending from A paſs from H to D ſooner in an Arch of this Curve than in any other, between H and D; let the Point G be taken ſuch, that, drawing HF, GE parallel to the Axis AB, they may cut off IG = ED, in the Perpendicular KG and LD. Since the Point D is given, LD is conſtant, as alſo IG = DE, becauſe the Point H is invariable, and therefore alſo KG is conſtant; but the variable Quantities are HI, HG, and GE, GD.

Plate VI. Fig. 7.

2. Now put KG = b, LD = c, and IG = ED = d; alſo HI = v and GE = z; then will the Velocity of the Body at G be as $\sqrt{KG}$, and at D as $\sqrt{LD}$. And ſince the Times are as the Spaces paſs'd over directly, and the Velocities inverſely, the Time of paſſing thro' HG will be as $\frac{HG}{\sqrt{KG}} = \sqrt{\frac{d^2+v^2}{b}}$; and the Time of deſcribing GD, as $\frac{GD}{\sqrt{LD}} - \sqrt{\frac{d^2+z^2}{c}}$.

3. But the Sum of thoſe Times is the leaſt poſſible, *viz.* $\sqrt{\frac{d^2+v^2}{b}} + \sqrt{\frac{d^2+z^2}{c}}$, a *Minimum*. Conſequently its Fluxion $\frac{v\dot{v}}{\frac{1}{2}\sqrt{d^2+v^2}} + \frac{z\dot{z}}{c\frac{1}{2}\sqrt{d^2+z^2}} = o$, or $\frac{v\dot{v}}{b\frac{1}{2}\sqrt{d^2+v^2}} = \frac{-\dot{z}z}{c^{\frac{1}{2}}\sqrt{d^2+z^2}}$; but ſince HF = $v + z$ = a conſtant Quantity, we have $\dot{v} + \dot{z} = o$, and ſo $\dot{v} = -\dot{z}$; and therefore $\frac{v}{b^{\frac{1}{2}}\sqrt{d^2+v^2}} = \frac{z}{c^{\frac{1}{2}}\sqrt{d^2+z^2}}$; therefore $v c^{\frac{1}{2}} \sqrt{d^2+z^2} = z b^{\frac{1}{2}} \sqrt{d^2+v^2}$, whence $\sqrt{d^2+v^2} : \sqrt{d^2+z^2}$ $v c^{\frac{1}{2}} : z b^{\frac{1}{2}}$; that is, GD : GH :: HI × $\overline{LD}^{\frac{1}{2}}$: GE × $\overline{KG}^{\frac{1}{2}}$.

4. But GD, HG are the Fluxions of the Curve; HI, GE, the Fluxions of the *Abſciſſæ* AM, AK; and KG, LD are the Ordinates to the Points G and D. *Therefore the Fluxions of the Curve of ſwifteſt Deſcent are every where as the*

THE Motion of PROJECTILES comes next to be conſider'd. A PROJECTILE is any Body A, thrown or projected in an *upright*, *obliqne*, or *horizontal* Direction; as a *Stone* from the Hand, an *Arrow* from the Bow, or a *Ball* or *Bomb* from a Cannon or Piece of Ordnance. The Force with which the Body is projected is call'd the IMPETUS, and the Diſtance to which it is thrown is call'd the HORIZONTAL RANDOM or AMPLITUDE of the Projection.

EVERY Projectile is acted upon by two Forces or Powers, *viz.* the *Impetus* of the *projectile Force*, and that of *Gravity*. By the firſt, the Body paſſes over *equal Spaces*, AB, BC, CD, *&c. in equal Times*; and by the ſecond, it deſcends through Spaces AG, AH, AI, *&c.* which are as the *Squares of the Times*; and therefore by theſe two Forces compounded, the Body will deſcribe, not a *Right Line*, but a *Curve* AQ; and of that

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Fluxions of the Abſciſſæ *directly, and the Square Roots of the Ordinates inverſely.*

5. But this is the Property of the Cycloid; for putting $AK = x$, $KG = BN = y$, and $BC = a$; the Triangles ONC and GED are ſimilar by the Nature of the Curve; and therefore ON : OC :: GE : GD; that is, $\overline{ay - yy}^{\frac{1}{2}} : \overline{aa - yy}^{\frac{1}{2}} :: \dot{x} : \frac{\dot{x} \times \overline{a^2 - y^2}^{\frac{1}{2}}}{\overline{ay - yy}^{\frac{1}{2}}} = \frac{\dot{x} \times a^{\frac{1}{2}} \times \overline{a - y}^{\frac{1}{2}}}{y^{\frac{1}{2}} \times \overline{a - y}^{\frac{1}{2}}} = \frac{\dot{x} \times a^{\frac{1}{2}}}{y^{\frac{1}{2}}} = GD$; but $a^{\frac{1}{2}}$ is a given Quantity, therefore GD is every where as $\frac{\dot{x}}{y^{\frac{1}{2}}}$, or directly as the Fluxion of the Abſciſs AK, and inverſely as the Square Root of the Ordinate RG, and conſequently *the Cycloid is the Curve of ſwifteſt Deſcent*. See *Annotat*. XXVIII. 3.

Sort which we call a *Parabola*; and this will be the Caſe in all Directions but that in the Perpendicular, when the Path of the Projectile will be (to Appearance) a Right Line. The greater the Angle of Elevation KAM of the Cannon is, the greater will be the Height DB to which the projected Body will riſe. Alſo, the greater will be the Diſtance or Amplitude of the Projection, till the ſaid Angle becomes equal to 45 Degrees KAO; upon which Elevation the Random AC will be the greateſt poſſible, and equal to twice the Altitude AG of the perpendicular Projection. On any Elevation AM or A*M*, equally above or below 45 Degrees, as on 40 and 50, 30 and 60, 20 and 70 Degrees, the Random AC will be the ſame; which Caſe an Engineer frequently finds of very great Uſe.

Plate V. Fig. 7.

IF the Object be ſituated above the Horizon, then, in order to ſtrike it with the leaſt *Impetus*, let a Piece of Looking-glaſs be fix'd to the Cannon perpendicular to its Axis; and holding a Plumb-Line over the Glaſs directly under the Eye, the Cannon is to be elevated till the Object appears exactly under the Plummet, and there fix'd; if then it be diſcharged, it will ſtrike the Object as required.

FROM what has been ſaid, we may eaſily underſtand how a Body projected upright from the Earth's Surface does really deſcribe a Parabola, though to Appearance it aſcends and deſcends in a Right Line. For it is urged by two Forces, *viz.* the Projectile upwards, and the Force ariſing from the Motion of the Earth about its Axis from *Weſt* to *Eaſt*; in which Caſe it muſt neceſſarily deſcribe a Parabola (XXXIII).

Plate V. Fig. 5, 6. (XXXIII.) 1. In the *Parabola*, it is demonſtrated that the ſeveral *Abſciſſæ* AG, AH, AI, &c. are as the Squares of the Ordinates GM, HN, IO, &c. and ſince this is the Property of the Curve which the Projectile deſcribes, 'tis evident the Projectile deſcribes a *Parabola*.

Fig. 7. 2. Let AM repreſent the Force with which a Ball is projected from the Cannon in the Direction AM; this may be reſolved into two others AL and LM, of which the firſt is perpendicular, and the latter parallel to the Horizon. The perpendicular Force or Velocity AL is that by which the Ball riſes, and the Heights to which it will riſe with the Velocities AL and AM are as the Squares of the Velocities, *viz.* as $\overline{AL}^2$ to $\overline{AM}^2$; but (becauſe AL : AM :: AM : AG) $\overline{AL}^2 : \overline{AM}^2 :: AL : AG$; therefore ſince AG repreſents the Height to which the Ball will riſe with the Velocity AM, AL will be the Height to which it will riſe with the Velocity AL, or in the Projection on the Elevation AM; thus BD = AL.

Fig. 6. 3. The Velocity of the Projection LM in the horizontal Direction is every where uniform, or carries the Ball through equal Spaces AR, RS, ST, TV, VW, in equal Times. For by the ſingle *Impetus* of the Powder, the Ball deſcribes equal Spaces AB, BC, CD, &c. in equal Times in the Direction AF, as has been ſhewn; therefore by the Similarity of Triangles AR, RS, ST, &c. will be as AB, BC, CD, &c. that is, *the horizontal Spaces paſs'd over in the Projection, will, in equal Times, be equal.*

Fig. 7. 4. Now ſince the perpendicular Velocity is to the horizontal Velocity as AL to LM, and AL is the Space paſs'd through by the former in half the Time of the Projection, LM would be the Space paſs'd through in the horizontal Direction in the ſame Time, if the Velocity LM were of the ſame Sort with AL, viz. *uniformly retarded and accelerated*; but ſince LM is not retarded, but uniform, it will carry the Body through twice the Space LM in the ſame Time (*Annot.* XXVI. 7.) that is, through AD = LB = 2 LM.

5. And ſince the Time of the Aſcent from A to B is equal to the Time of Deſcent, the Ball will paſs over DC =

I SHALL in the laſt Place conſider the Nature of CIRCULAR MOTION and CENTRAL FORCES. If a Body A be ſuſpended at the End of a String AC, moveable about a Point or Pin C as a Centre,

Plate V. Fig. 8.

AD in the Time of the Deſcent from B to C; therefore the horizontal Space AC, which the Ball paſſes over during the whole Time of the Projection, is equal to 4 times LM, wherever the Point M be taken in the Circumference AOG.

6. Hence when AO is the Direction of the Cannon, (O being the middle Point of the Semicircle) the Line NO, which is the horizontal Velocity, will be the greateſt poſſible (from the Nature of the Circle) and conſequently $4NO = AQ$, *the greateſt poſſible horizontal Random;* which therefore is made on an Angle $OAK = 45$ Degrees. For $OAK = AON = OAN$, becauſe $NO = AN$; therefore ſince $ANO = 90°$, $AON = OAN = OAK = 45°$.

7. Hence $AN = FE$ $(= NO = \frac{1}{4} AQ)$ will be the Height of the Projection on the Angle $OAK = 45°$. *Whence* AQ *will be the Parameter, or* Latus Rectum, *of the Parabola* AFQ.

8. If $MO = OR$, then will $LM = TR$; and conſequently the horizontal Random on the Elevation AM, and AR, will be equal, *viz.* AC; but the Height of the Projection on AR will be $AT = SD$. *Whence the Height* SD *is to the Height* DBD *(as* RK *to* MK) *as the Tangent of the Angle of Elevation* RAK *to the Tangent of the Angle* MAK.

9. From what has been demonſtrated 'tis eaſy to underſtand, that an Object ſituated on the Horizon may be hit by a Ball diſcharg'd on an Elevation of 45° with a leſs Charge of Powder, than on any other Angle of Elevation.

10. The Time of the Projection ABC is equal to the Time of the perpendicular Aſcent and Deſcent through AL or DB, (becauſe the horizontal Velocity does no way affect that in the Perpendicular.) But the Time of Aſcent and Deſcent in AL is to that in AG, as $\sqrt{AL}$ to $\sqrt{AG}$; that is, (ſince $AL : AG :: \overline{AM}^2 : \overline{AG}^2$) as AM to AG; that is, becauſe $AG : AM :: AM : AL = MK$) as MK to AM, or *as the Sine of the Angle of Elevation* MAK *to the Radius.*

11. Therefore the Times of two different Projections ABC, ASC, will be as the Chords AM and AR, or as the Sines of the Angles of Elevation to the Radius AG,

and in that Poſition it receive an Impulſe or Blow in an horizontal Direction, it will be thereby compell'd to deſcribe a Circle about the central Pin: While the circular Motion continues, the Body

12. Suppoſe it required to know from what Height a Body ought to fall to acquire ſuch a Velocity, that, being reflected in that Moment in the Direction A F, it ſhould deſcribe the given Parabola A O Q. In order to this we are to conſider that (the Equation being $xp = yy$) the Velocity of a Ball ſhot from the Cannon commences in the Direction of the Axis A L (or Abſciſs x) in the Point A; but that in the Direction A F (or of the Ordinate y) is from firſt to laſt the ſame; conſequently there muſt be a certain Point, to which the Ball having deſcended, acquires a Velocity equal to that in the Direction A F, which is uniform; and this muſt be when $\dot{x} = \dot{y}$. To determine which, putting the Equation $xp = yy$ in Fluxions, we have $\dot{x}p = 2y\dot{y}$; but ſince in this Caſe we have $\dot{x} = \dot{y}$, therefore $p = 2y$, and $\frac{p}{2} = y = \sqrt{xp}$, (*per* Equation;) therefore $\frac{1}{4}p^2 = xp$, that is, $\frac{1}{4}p = x$.

13. Conſequently, if an heavy Body fall from the Height of $\frac{1}{4}$ of the Parameter of the given Point A, and in that Point it be reflected (with the Velocity there acquired) in the Direction A F, it will deſcribe the given Parabola A O Q.

Plate VI. Fig. 9. 14. If it be required to ſtrike the Point B, whoſe Diſtance A D and Height B D are given, with the leaſt *Impetus* of all that will hit it, then in order to find the Elevation of the Cannon, draw A B, and E A perpendicular to A C; on the Point A deſcribe the Arch abc, which biſect in b, then a Line A F drawn through that Point will be the Elevation of the Cannon as required; and the leaſt *Impetus* will be equal to $\frac{AB + BD}{2}$. See the Demonſtration of this by the Inventor Dr. *Halley* in the *Phil. Tranſactions*, and by Dr. *Keill*, *Introd. Pag.* 280, 281. From a bare View of the Diagram the Reaſon of the Uſe of the *Looking-Glaſs* and *Plummet* is evident.

Fig. 10. 15. If a Ball be projected perpendicularly upright in the Direction A F with a Velocity that will carry it to the Height of one Mile, or 5280 Feet; it will be 18 Seconds (nearly) in the Aſcent, and as much in the Deſcent, (for, as 16,2 Feet will have a continual Endeavour to recede or fly off from the Centre, which is call'd its *Centrifugal Force*, and ariſes from the *Horizontal Impetus*; with this Force it acts upon the fix'd Centre Pin, and that, by its Renitency or Immobility, re-acts with an equal Force on the Body by means of the String, and ſolicits it towards the Centre of Motion, whence it is call'd the *Centripetal Force*; and when we ſpeak of either, or both of them indefinitely, they are call'd the *Central Forces* of the revolving Body.

IT is of the laſt Importance to underſtand the Nature of this Kind of Motion, ſince by it all the Machinery of the Planetary Syſtem is perform'd, as will be eaſy to underſtand, if, for the revolving Body, we ſubſtitute a Planet; for the Centre, we place the Sun; for the *Centripetal*

: 5280 Feet :: $1^2 : x^2$; whence $x = 18''$, *fere* the Time of Aſcent or Deſcent.) Now in the Parallel of *London*, the Motion of the Earth's Surface, and of all Bodies upon it, is at the Rate of 10,8 Miles *per* Minute, or 950 Feet *per* Second, from Weſt to Eaſt: Therefore in 36 Seconds, the Time of the Aſcent and Deſcent, the Ball will be carried over 34214 Feet, or more than ſix Miles, in the horizontal Direction Eaſtwards from the Point A to H. Now ſince the Ball is carried at the ſame Time with a retarded Force upwards, and an uniform Force forwards, it will deſcribe the Parabola ADH. And becauſe the horizontal Motion in the *Spectator* and in the *Project* is the ſame, they will be carried through equal Spaces AN, OB; AM, PC; AL, FD, &c. in the ſame Time; and conſequently the projected Ball will always appear perpendicularly over the Spectator in every Point of the Curve; which is the Occaſion of the Deception above-mention'd.

16. Hence the Objection uſually urged againſt the Motion of the Earth, from the apparent Aſcent and Deſcent of Projects in a Right Line, is the Reſult of Ignorance; as indeed are all other Arguments of this Kind.

Force, or String, its Power of Attraction; and for the *Projectile Force*, the Almighty Power of God in the firſt Creation of Things.

Plate V. Fig. 8. THE Theory of this Species of Motion is compriſed in the following Propoſitions, *viz.* (1.) The projectile Force A H is infinitely greater than the centrifugal Force H E. (2.) The central Force is proportional to the Quantity of Matter in the revolving Body A, all other Things being equal. (3.) If two equal Bodies A, B, deſcribe unequal Circles A M, A N, in equal Times, the central Forces will be as the Diſtances A C, B C, from the Centre C. (4.) If equal Bodies deſcribe unequal Circles with equal Celerities, the central Forces will be inverſely as the Diſtances. (5.) If equal Bodies deſcribe equal Circles, the central Forces will be as the Squares of the Celerities. (6.) If equal Bodies deſcribing unequal Circles have their central Forces equal, their Periodical Times will be as the Square Roots of the Diſtances. (7.) If equal Bodies deſcribe unequal Circles with equal Celerities, the Periodical Times will be as the Diſtances directly. (8.) Therefore the Squares of the Periodical Times are proportional to the Cubes of the Diſtances, when neither the Periodical Times nor the Celerities are given. And in that Caſe, (9.) The central Forces are as Squares of the Diſtances inverſely.

THESE are the Theorems of circular Motions, the two laſt of which are found by Aſtronomers to be ſtrictly obſerved by every Body of the Planetary and Cometary Syſtem. For Example:

The Periodical Time of *Venus* is 225 Days, and that of the *Earth* 365; the Squares of which Numbers are 50625 and 133225: Again, the Diſtance of *Venus* from the *Sun* is to that of the *Earth* as 72 to 100; the Cubes of which Numbers are 373248 and 1000000; but 50625 : 133225 :: 373248 : 1000000; that is, the Squares of the Periodical Times are as the Cubes of their Diſtances, very nearly. From hence alſo it will eaſily appear, that Bodies under the Equator have the greateſt centrifugal Force, which there acts in direct Oppoſition to Gravity, and diminiſhes towards the Poles, with the Squares of the Diſtances from the Earth's Axis. *Hence alſo it is evident, that if ever the Earth was in a fluid State, and at the ſame Time moving about its Axis, it muſt neceſſarily put on the Figure, not of a perfect Sphere or Globe, but of an oblate Spheroid, flatted towards both Poles; as is manifeſtly ſhewn by Experiment* (XXXIV.)

(XXXIV.) 1. Let AD = *a*, the Diameter of the Circle AMD; AH the Space which a Body A moves through in a conſtant Particle of Time by an uniform *Projectile Force*; HE the Diſtance it is drawn from the Tangent AH by the *Centripetal Force* (which is juſt ſufficient to retain the Body moving in a circular Orbit,) in the ſame Time. Plate V. Fig. 8.

2. Then ſince AH is indefinitely ſmall, the Arch AE, and its Chord, will nearly coincide and be equal to AH, and will therefore alſo repreſent the Projectile Force. But AD : AE :: AE : A*a* = HE = $\frac{\overline{AE}^2}{AD}$ = the Central Force. Now AE is infinitely ſmall in reſpect of AD, therefore HE or A*a* is infinitely ſmall with reſpect to AE.

3. If the Projectile Force or Velocity AE = V, and AD = *a*, the general Expreſſion of the Central Force will be $\frac{V^2}{a}$;

which therefore in different Circles will be as *the Squares of the Velocities directly, and inverſely as the Diameters or Semi-diameters of the Circles.*

4. But in the ſame Circle it will *be directly as the Squares of the Velocity*; becauſe in that Caſe a is given. But if the Velocity be given, it *will be as the Diſtance from the Centre inverſely*, (for then $\frac{V^2}{a}$ becomes $\frac{1}{a}$.)

5. Hence if the Central Forces in two Circles be equal; the *Diameters of thoſe Circles, or Diſtances from the Centre of Force, will be as the Squares of the Velocities directly.*

6. As AE : 1 :: 3,1416 a : P = the *Periodical Time* of deſcribing the whole Cirele (for in equable Motions the Spaces are as the Times.) Therefore $P = \frac{3{,}1416\,a}{V}$; whence $V^2 = \frac{\overline{3{,}1416}^2 \times a^2}{P^2}$, which ſubſtituted in the Expreſſion for the Central Force, we have $\frac{\overline{3{,}1416^2 \times a}}{P^2} = \frac{a}{P^2}$ (becauſe $\overline{3{,}1416}^2$ is given) for the Central Force in this Caſe; which therefore *is always as the Diameter or Radius of the Circle directly, and the Square of the Periodical Time inverſely.*

7. Hence if the Central Force be as any Power (n) of the Diſtance (a) from the Center C, we have $\frac{a}{P^2} = a^n$; whence $P = a^{\frac{1-n}{2}}$; and therefore when $n = 0$, we have $P = a^{\frac{1}{2}}$, or *the Periodical Time will be as the Square Root of the Diſtance.*

8. If $n = 1$, that is, if the Force be directly as the Diſtance, then will $P = a^{\frac{0}{2}} = a^0 = 1$; that is, *the Periodical Time will be given, or the ſame in every Circle.*

9. If $n = 2$, that is, if the Force be directly as the Square of the Diſtance, then $P = a^{-\frac{1}{2}} = \frac{1}{a^{\frac{1}{2}}}$; or *the Periodical Time will be as the Square Root of the Diſtance inverſely.*

10. If $n = -1$, that is, if the Force be inverſely as the Diſtance, then $P = a^{\frac{2}{2}} = a$, or *the Periodical Time will be directly as the Diſtance from the Centre.*

11. If $n = -2$, or the Force be inverſely as the Square of the Diſtance, then $P = a^{\frac{3}{2}}$, or (ſquaring both Sides) $P^2 = a^3$, that is, *The Square of the Periodical Time will be as the Cube of the Diſtance from the Center.*

12. When the Central Force is given, $\frac{a}{P^2} = 1$, and $P = a^{\frac{1}{2}}$, that is, *the Periodical Time will be as the Square Root of the Diſtances.*

13. Since the Central Force is as $\frac{V^2}{a}$, and as $\frac{a}{P^2}$, we have $PV = a$; *whence the Diſtance from the Centre is always in the Compound Ratio of the Periodical Time and Velocity.*

14. Hence if the Diſtance be given, or $a = 1$, then the *Periodical Time is inverſely as the Velocity, and the Velocity inverſely as the Periodical Time.*

15. If the Velocity be given, or $V = 1$, the *Periodical Time will be directly as the Diſtance from the Centre.*

16. If the Periodical Time be given, the *Velocity will be directly as the Diſtance likewiſe.*

17. If we put $p = 3{,}1416$; then $pa : P :: AE = \frac{AE \times P}{p\,a}$ = Time of deſcribing AE. Again, as $1^2 : 16{,}2 :: \frac{\overline{AE}^2 \times P^2}{p^2 \times a^2} : \frac{16{,}2\,\overline{AE}^2 \times P^2}{p^2\,a^2}$ = the Diſtance deſcended by an heavy Body (in the Time of deſcribing AE) by the Force of Gravity. Laſtly, this Force of Gravity $\left(\frac{16{,}2\,\overline{AE}^2 \times P^2}{p^2\,a^2}\right)$ is to the Central Force $\left(\frac{\overline{AE}^2}{a}\right)$ as 1 to $\frac{ap^2}{16{,}2\,P^2}$, that is, as $16{,}2\,P^2$ to ap^2, or as P^2 to $0{,}615\,a$.

18. If now we ſuppoſe a Body revolving about the Centre of the Earth at the Diſtance of its Surface with a Centrifugal Force equal to that of Gravity; then $P^2 = 0{,}615\,a$, and ſo $P = \sqrt{0{,}615\,a}$; and putting $a = 42000000$ Feet in the Diameter of the Earth, we have $P = 5083$ Seconds, or 84′ 43″; which is the Time of Revolution to acquire a Centrifugal Force equal to that of Gravity at the Earth's Surface. Conſequently, were the Earth to revolve in 84′ 43″ inſtead of 24 Hours, the Bodies on its Surface would loſe all their Weight, and be as liable to fall off as to abide thereon.

19. From what has been demonſtrated we may diſcover the Law of Gravitation at the Moon; for ſince the Diſtance of the Moon from the Earth's Centre is to the Diſtance of the Earth's Surface as 60 to 1; and ſince the Periodical Time at the Earth's Surface is 5083″, therefore the Periodical Time at the Moon may be found (by *Art.* 11.) thus; $1^3 : 60^3 :: \overline{5083}^2 : P^2$; whence $P = 5083 \times 60^{\frac{3}{2}} = 2362000'' =$ 27,3 Days, the Periodical Time of the Moon, if moved by

a

a Centripetal Power, as *Gravity, decreaſing with the Squares of the Diſtances inverſely*: And therefore ſince the *Periodical Time, thus found, is equal very nearly to the real Periodical Time of the Moon*, it ſhews that Law takes Place between the Earth and Moon. This might alſo be ſhewn from the Time of the Moon's Revolution, but *one Proof* is enough. I ſhall alſo hereafter ſhew, that this is the general Law of the whole Planetary Syſtem.

20. Since we had $\frac{a}{p^2} =$ the Central Force, when *a* is given, the Force will be as $\frac{1}{p^2}$; that is, in equal Circles the Centrifugal Forces will be as the Squares of the Velocities inverſely. And therefore ſince in the Time of the Earth's Revolution, *viz.* 24 Hours, there are 68400″; if we ſay, as $\overline{5083}^2 : \overline{68400}^2 :: 1 : 289$ nearly; that is, *the Centrifugal Force under the Equator, ariſing from the Earth's Rotation, is to the Power of Gravity on its Surface as 1 to 289 nearly.*

Plate VI. Fig. 11.

21. Since the Time of Revolution of a Body under the Equator EQ, and in any Parallel of Latitude BG, is equal, the Centrifugal Forces are as the Diſtances EC, BA, from the Centre of Motion (by *Art.* 16.) or, as Radius CB to the Coſine AB of the Latitude.

22. But in any Latitude B, the Centrifugal Force is not (as under the Equator) oppoſite to the whole Gravity, but only a Part thereof, which is to the Whole as the Co-ſine of Latitude to the Radius. For continue out AB (the Direction of the Centrifugal Force) to D; and CB (in which Gravity acts) to F; from D let fall a Perpendicular to F, then will BD repreſent the whole Centrifugal Force, and BF that Part of it which acts directly againſt Gravity; but BF : BD :: AB : BC = EC. Therefore on both theſe Accounts, the Centrifugal Force decreaſes from the Equator towards the Poles N and S in the Proportion of the Squares of the Co-ſines of the Latitude.

Fig. 12.

23. We proceed now to demonſtrate *the true Figure of the Earth*, which we ſhall find to be *Spheroidical* and not *Spherical*, if ever its Parts were in a fluid or yielding State, from the Conſequence of a Centrifugal Force. In order to this let NS be the Axis of Rotation, BC a Column of fluid Particles gravitating towards the Centre C, which (becauſe we ſuppoſe the Parts of the Fluid every where quieſcent) will be of the ſame Weight with every other Column of Particles CN, or CE. Let CE $= a$, CN $= b$; CB $= x$, AB $= s =$ Sine of the Angle BCN; and ſuppoſe the gravitating Power to be every where as the Power n of the Diſtance from the Centre; that is, ſuppoſe the Power of Gravity at E

(g) to be to that at B as CE^n to CB^n, then will the Gravity at B be $= \frac{g \times CB^n}{CE^n} = \frac{g \times x^n}{a^n}$.

24. Alſo the Centrifugal Force at E (f) is to that at B as CE to AB (as we ſhew'd *Art.* 16.) But $AB : CB :: s : 1 =$ Radius; whence $AB = s \times CB = sx$; conſequently we have the Centrifugal Force at B equal to $\frac{fsx}{a}$.

25. But ſince this Force acts in the Direction BD, that Part which oppoſes Gravity is FB (as was ſhewn above) whence ſince $BD : FB :: 1 : s :: \frac{fsx}{a} : \frac{fssx}{a} =$ to Centrifugal Force at B, which oppoſes Gravity.

26. Hence the Power of Gravity on a Particle at B will be as $\frac{g \times x^n}{a^n} - \frac{fs^2x}{a}$ impelling it towards the Center C. Such a Particle will be repreſented by $\dot{x}$, and therefore its Weight will be $\frac{g \times x^n\dot{x}}{a_n} - \frac{fssx\dot{x}}{a}$; and as this is the Fluxion of the whole Weight of the Column CB, the Fluent thereof $\frac{g \times x^{n+1}}{\overline{n+1}\, a^n} - \frac{fssx^2}{2a}$ will be the Weight of the ſaid Column of Particles CB.

27. If now we conſider the Column of Particles CE, which is of equal Weight, ſince in this Caſe the Angle NCE is a Right one, we ſhall have $x = a$, and $s = 1$; wherefore the Weight of the Column CE will be $\frac{g \times a^{n+1}}{\overline{n+1}\, a^n} - \frac{faa}{2a}$; therefore the Weight of a Column every where is equal to $\frac{2g - nf - f}{2\,\overline{n+1}} \times a \left(= \frac{g \times x^{n+1}}{\overline{n+1}\, a^n} - \frac{fssx^2}{2a}\right)$ therefore we have (by proper Reduction) $2gx^{n+1} - \overline{n+1} \times fssa^{n-1}x^2 = \overline{2g - nf - f} \times a^{n+1}$ for a general Equation.

28. Whence, putting $s = 0$, we have $2gx^{n+1} = \overline{2g - nf - f} \times a^{n+1}$; and therefore $a : x :: 2g^{\frac{1}{\overline{n+1}}} : \overline{2g - nf - f}^{\frac{1}{\overline{n+1}}} ::$ CE : CN, *ſo is the Diameter of the Equator to the Axis of the Earth.*

29. Hence, upon the Hypotheſis of an uniform Gravity, we have $n = 0$; in that Caſe $2g : 2g - f :$ CE : CN. But the Gravity (g) is to the Centrifugal Force (f) under the Equator, as 289 to 1 (*per Art.* 20.) Wherefore in this Caſe CE : CN :: 578 : 577.

30. If the Earth were to revolve in 84′ 43″, the Centrifugal Force would be equal to Gravity (*Art.* 18.) or $g = f$; and then CE : CN :: 2 : 1. If the Earth were to revolve quicker than that, the Particles would all fly off, and the Earth be reduced to a ſingle Atom.

31. If the Gravity be ſuppoſed proportional to the Diſtance from the Centre; then $n = 1$, and we have CE : CN :: $g^{\frac{1}{2}} : \overline{g - f}^{\frac{1}{2}}$ and if in this Hypotheſis the Earth were to revolve in 84′ 43″, or $g = f$, then would CE be infinitely greater then CN; that is, CN would be nothing, or the Spheroid would then become a circular Plane.

32. If Gravity be ſuppoſed inverſely as the Square of the Diſtance, we have $n = -2$; and then we ſhall have $2g^{\frac{1}{-n+1}} : \overline{2g + nf - f}^{\frac{1}{-n+1}}$ :: CE : CN :: $2g^{\frac{1}{-1}} : \overline{2g + f}^{\frac{1}{-1}} :: 2g + f : 2g :: 579 : 578$. And ſince of all the above Hypotheſes, this laſt is found to be the only true one (ſee *Art.* 19.) it follows *that the Earth in its Chaotic or Fluid State, revolving about its Axis, muſt neceſſarily put on a Spheroidical Figure,* having the Equatorial Diameter E Q, to the Axis N S, as 579 : 578.

33. And this would be the Caſe were all the Circumſtances of the Hypotheſis the ſame in Nature as we have preſumed in the Theory; but ſince they are found to be otherwiſe, the *Mathematical Theory* (which gives only the true Sort of Figure, but not the true Figure itſelf) muſt be adapted to Nature, and the Figure which the Earth really has inveſtigated from other Principles. For Bodies on the Earth's Surface do gravitate in ſuch Directions as paſs not through the Centre C (as has been ſuppoſed) any where but in the Equator or under the Poles, *viz.* at E and N.

34. In order to find the true Proportion between E Q and N S, Sir *Iſaac Newton* makes a Suppoſition they are as 101 to 100, and then by a Method (too prolix to be here explain'd, he finds the Gravity at E and N to be as 500 to 501. Now ſuppoſing the Matter of the Earth uniform and at Reſt, the Weight of the Column of Particles EC will be to that of the Column NC in the Ratio of their Magnitudes 101 to 100, and of their ſpecific Gravities 500 to 501 conjointly, (See *Annot.* LVIII. 10.) that is, as 505 to 501.

35. Now 'tis plain if the Weight of each Particle in the Column CE were divided into 505 equal Parts, a Centrifugal Force, that ſhould take off 4 of thoſe equal Parts, would leave the Weight of the Column E C equal to that of the

Column N C, whence an Equilibrium muſt enſue between them.

36. Whence our great Author makes this Analogy: If a Centrifugal Force which is $\frac{4}{505}$ of Gravity cauſes an Exceſs of $\frac{1}{100}$ Part in the Altitude E C, a Centrifugal Force which is but $\frac{1}{289}$ Part of Gravity can cauſe an Exceſs only of $\frac{1}{229}$ Part of E C. Therefore the true Proportion between the Diameter of the Equator E Q, and the Earth's Axis N S, is that of 230 to 229. Whence, ſince the former is about 8000 Miles, it will exceed the latter by about 34 $\frac{3}{10}$ Miles *Engliſh* Meaſure.

37. Whence, ſince in an Equilibrium of Fluids communicating with each other the Altitudes are as the ſpecific Gravities inverſely, it follows that the Gravity at the Equator E is to that at the Pole N as 229 to 230; or a Body which under the Poles weighs 230 *lb.* will under the Equator weigh but 229 *lb.*

38. The Gravity increaſes from the Equator E towards the Pole N *with the Squares of the Sines of the Latitudes.* For continue out C F to G, and draw G D perpendicular to A D; then if B D be the whole Centrifugal Force at the Equator, and B G the whole Dimunition of Gravity occaſion'd there by it; and ſince in the Latitude B the Centrifugal Force which acts directly againſt Gravity is reduced to F B, the Line F B will alſo repreſent the Diminution of Gravity in the Latitude B, and therefore the Difference G F will be the Increaſe of Gravity at B. And ſo (becauſe B G : G D :: G D : G F) we have B G : G F :: ($\overline{GB}^2 : \overline{GD}^2$ ::) $\overline{BC}^2 : \overline{AC}^2$ :: *Radius Square* : *Square of the Sine of the Latitude* :: *Decreaſe at the Equator* : *the Increaſe at* B.

39. If we put the Diminution of Gravity at the Equator $=$ 10000, then will the Gravities at the Equator, *London*, and the Pole be as the Numbers 2290000, 2296124, 2300000; therefore the Lengths of Pendulums vibrating Seconds in each of thoſe Places will be in the ſame Proportion (by *Annot.* XXVIII. 13.) But 2296124 : 2290000 :: 39,2 : 39,1 $=$ Inches of a Second Pendulum under the Equator, the ſame nearly as found by Obſervation.

40. Since in the elliptic Meridian E N Q S, the Curvature at E, Q, is much greater than that at N, S; the Radius of Curvature (ſuppoſe E *b*) in the former will be ſhorter than that in the latter, (ſuppoſe S *d*;) and therefore if at the Equator the Angle E *b a*, or the Arch E *a*, be equal to *one Degree*, the Angle of one Degree S *d e* at the Pole S will contain a greater Arch *e* S, as is evident from the Diagram. Whence it appears that in this ſpheroidical Figure of the

Earth, the Degrees increaſe from the Equator to the Pole; ſo that if in the Equator a Degree conſiſts of 60 Miles, in the ſeveral Latitudes the Miles will be as below:

Lat.	0.	10°.	20°.	30°.	40°.	50°.	60°.	70°.	80°.	90°.
Miles in a Deg.	60.	59,5.	59,57.	59,67.	59,8.	59,93.	60,06.	60,16.	60,235.	60,26.

SCHOLIUM.

What has been ſaid hitherto, has been upon Suppoſition that the Earth is of an uniform Denſity throughout, but it is reaſonable to ſuppoſe (from the Earth's having been originally in a fluid State) that the heavieſt Matter ſubſided firſt, according to the Laws of Gravity; and therefore that the Earth is more denſe and compact the nearer we go to the Centre, whence it muſt follow, that as there is a greater Number of Particles, there will be, on the Whole, a greater Quantity of Centrifugal Force in the Column E C, and conſequently a greater Exceſs in its Altitude above that of N C, than was before ſtated; that is, E C will exceed N C more than in the Proportion of 230 to 229, which is alſo found to be ſtill more conſonant to Experiments.

LECTURE III.

Of the CENTRES *of* MAGNITUDE, MOTION, *and* GRAVITY. *The* METHOD *of finding the* CENTRE *of* GRAVITY *in all* BODIES. *The* PHÆNOMENON *of the* DOUBLE CONE *and* CYLINDER. *Of the* COMMON CENTRE *of* GRAVITY *between two or more* BODIES; *conſider'd between the* EARTH *and* MOON, *the* SUN *and* PLANETS. *The* FUNDAMENTAL PRINCIPLE *of* MECHANICS *demonſtrated. The Impoſſibility of a* PERPETUAL MOTION. *The* NATURE *and* KINDS *of* LEVERS, BALANCES, PULLIES, *the* AXIS *in* PERITROCHIO, INCLINED PLANE, WEDGE, *and the* SCREW. *Of* FRICTION *and* FRICTION-WHEELS. *Of* COMPOUND MACHINES. *The* THEORY *of* CLOCK-WORK. *a new* ORRERY, COMETARIUM, *&c. Of the* MOVEMENT *of* WIND-MILLS, WATER-MILLS, SHIPS, *&c. Of the* MAXIMA *and* MINIMA *in* MECHANICS. *The* THEORY *of* WHEEL-CARRIAGES *at large.*

HAVING conſidered the Nature of every Kind of Motion, with the Properties of each, we ſhould have come immediately to treat of the MECHANICAL POWERS or MACHINES, but that ſomething ſtill remains to be premiſed thereto, relating to the *Centres of Magnitude*, of *Motion*, and of *Gravity* in Bodies.

THE CENTRE *of* MAGNITUDE is that Point which is equally diſtant from all the external Parts of the Body: And in Bodies that are uniform and homogeneal, it is the ſame with

THE CENTRE *of* MOTION, which is that Point which remains at Reſt, while all the other Parts of the Body move about it: And this is again the ſame in uniform Bodies, of the ſame Matter throughout, as

THE CENTRE *of* GRAVITY, which is that Point about which all the Parts of a Body do in any Situation exactly balance each other.

THIS Centre of Gravity in Bodies is of the greateſt Conſequence to be well underſtood, as being the ſole *Principle of all Mechanical Motions.* The particular Properties hereof are as follow. (1.) If a Body be ſuſpended by this Point, as the Centre of Motion, it will remain at Reſt in any Poſition indifferently. (2.) If a Body be ſuſpended in any other Point, it can reſt only in two Poſitions, *viz.* when the ſaid Centre of Gravity is exactly above or below the Point of Suſpenſion. (3.) When the Centre of Gravity is ſupported, the whole Body is kept from falling. (4.) Becauſe this Point has a conſtant Endeavour to deſcend to the Centre of the Earth; therefore, (5.) When this Point is at Liberty to deſcend, the whole Body muſt alſo deſcend or fall, either by ſliding, rolling, or tumbling down. (6.) The Centre of Gravity in regular, uniform, and homogeneal Bodies, as *Squares*, *Circles*, *Spheres*, &c. is the middle Point in a Line connecting any two oppoſite

Points or Angles. (7.) In a Triangle it is in a Line drawn from any Angle biſecting the oppoſite Side, one Third of the Length diſtant from that Side or Baſe. (8.) It is alſo *one Third* of the Side diſtant from the Baſe of an hollow Cone. (9.) But in a ſolid Cone it is *one Fourth* of the Side diſtant from the Baſe. (10.) In the Human Body, the Centre of Gravity is ſituated in that Part which is call'd the *Pelvis*, or in the Middle between the Hips.

HENCE the Solution of ſeveral very curious *Phænomena* will be evident with the leaſt Attention; as why ſome Bodies ſtand more firmly on their Baſes than others; why ſome ſtand firmly in an inclined Poſition; why ſome Bodies fall in one manner, ſome in another; why ſome may ſeem to riſe, while the Centre of Gravity really deſcends, as the *rolling Cone*, and *Cylinder*. Hence the Form of that particular Bucket which deſcends empty with the Mouth downwards, but is drawn up full with the Mouth upwards. Hence alſo it appears that a Waggon loaded with heavy Matter, as Iron, Stone, *&c.* will go ſafely on the Side of a Hill or riſing Ground, where a Load of Hay or Corn would be overturn'd. Again, we hence ſee the Reaſon why no Man, ſtanding ſtill, can move or ſtir, without firſt moving the Centre of Gravity out of its Place; alſo, why we ſtand firmly, while the Centre of Gravity falls between, or on the Baſe Line of the Feet; and how neceſſarily we fall, when the Centre of Gravity falls on one Side or other of the ſame: With many other

Particulars, which naturally reſult from this Principle (XXXV).

Plate VI. Fig. 13. (XXXV.) 1. That the Centre of Magnitude, of Motion, and of Gravity is the ſame Point C in regular and homogeneous Bodies, as the circular Area, or Square A B D E is evident; becauſe any Right Line as A D drawn through the ſaid Point and terminated by the Peripheries of thoſe Bodies, has an equal Number of Particles on each Side; therefore the ſaid Point is Centre of Magnitude and Gravity, and conſequently of uniform Motion in every ſuch Line, and therefore of the whole Area, which is made up of them.

2. But if a Body be not homogeneous but conſiſts of different Matter in different Parts, the Centre of Magnitude C will not be the Centre of Gravity; thus ſuppoſe A B F a circular Piece of Wood with a round Piece of Lead *e f* in one Side of it; this Lead will remove the Centre of Gravity from C to ſome other Point D in the Diameter paſſing through the Centers C and *g*; and this Centre of Gravity D will of Courſe be alſo the Centre of Motion and Reſt, for on no other Point can a Body reſt but that on which it is equilibrated, ſince Reſt is only the Reſult of an Equilibrium of the Parts.

3. If the Centre of Gravity be ſupported, the Body will be ſupported likewiſe, for ſince the Power of every Particle to deſcend is there referr'd, and all unite in one; if that one Point be upheld, all the reſt muſt, as having a mutual Dependance thereon by a neceſſary Connection of Parts.

4. If a Body be ſuſpended on any other Point C, than the Centre of Gravity D, it will deſcend in every Poſition but two, *viz.* when the Centre of Gravity is in E above, or in D below the Point of Suſpenſion C, becauſe in two Caſes it gravitates on a fix'd Point, which ſuſpends the Body, and therefore upholds it; but in all other Caſes it has Liberty to deſcend, and therefore will carry the Body down, till it comes to reſt in D.

Plate VII. Fig. 1. 5. If the Centre of Gravity C in the Body A B D E gravitate on the Baſe A E in the Perpendicular C F, that Body will ſtand firm, and ſo much the more ſo, as the Point F falls nearer the middle Point between A and F; for ſince, were the Body to move, the Point or Angle E would be the Centre of Motion, it follows, that in that Caſe the Centre of Gravity C muſt riſe in the Arch of a Circle from C to G, which it cannot do without ſome extraneous Force; and therefore when left to itſelf, the Body muſt ſtand firm.

6. But in the Body of the next Figure, becauſe the Line

If two or more Bodies in Motion be connected together, as *Chain-Shot*, &c. or any how depend

Plate VII. Fig. 2.

CF, in which it gravitates, falls off from the Baſe AE, the Centre of Gravity C will be free to deſcend, as not being ſupported, and conſequently the Body muſt fall.

Fig. 3.

7. If a Body ABDE, laid on an inclined Plane XY, gravitates in the Direction CI, within the Baſe AE, that Body will move down the Plane (by *Annot.* XXVII. 1.) but it cannot move over the angular Point A (by 5th of this) it muſt therefore deſcend by *ſliding down* the Plane. If the Baſe AE, had been ſo ſmall that the Point F had fallen off it, it would have turn'd on the Point A in its Motion, and ſo have *tumbled down* the Plane. But if the Baſes AE be ſuppoſed infinitely ſmall, or a Point, the falling from one Point to an other, will be ſo momentaneous, that the Body (which then becomes a Circle or Sphere) will deſcend by *rolling down* the Plane.

Fig. 4.

8. To find the Centre of Gravity of a *Line*, has been ſhewn (*Annot.* XXIX. 2.) To find that of a Superficies as of a *Triangle*, *Parabola*, &c. this is the Method. Let AG biſect the Baſe AC of the Triangle ABC, it will alſo biſect every other Line DE drawn parallel to the Baſe; conſequently the Centre of Gravity of the Triangle will be found ſomewhere in the Line BG. The Area of the Triangle may be conſider'd, as conſiſting of an infinite Number of indefinitely ſmall Parallelograms DE$ba$D, each of which is to be conſider'd as a ſmall Weight, and alſo as the Fluxion of the Area of the Triangle, and ſo may be expreſs'd by $2y\dot{x}$, (putting BF $= x$, and FE $= y$;) if this fluxionary Weight be multiplied by its Velocity x, we ſhall have $2yx\dot{x}$ for its *Momentum*.

9. Now put BG $= a$, and AC $= b$; then BG (a) : AC (b) :: BF (x) : DE $= \frac{bx}{a} = 2y$; therefore the Fluxion of the Weights $2y\dot{x} = \frac{bx\dot{x}}{a}$; and the Fluxion of the *Momenta* $2yx\dot{x} = \frac{bxx\dot{x}}{a}$; whence the Fluent of the latter, *viz.* $\frac{bx^3}{3a}$, divided by the Fluent of the former, *viz.* $\frac{bx^2}{2a}$, will give $\frac{2}{3}x$ for the Diſtance of the Point from B in the Line

on each other, they will have a *Common Centre of Gravity*, which will be a Point in the Line join-

BF, which has a Velocity equal to the Mean Velocity of all the Particles in the Triangle DBE, and is therefore its Centre of Gravity. *Conſequently the Centre of Gravity of any Triangle* ABC, *is diſtant from the Vertex* B, $\frac{2}{3}$ BG *a right Line drawn from the Angle* B *biſecting the Baſe* AC.

10. And ſince the Section of a ſuperficial or hollow Cone is a Triangle, and Circles have the ſame Ratio as their Diameters, it follows *that the Circle whoſe Plane paſſes through the Centre of Gravity of the Cone is* $\frac{2}{3}$ *of the Length of the Side diſtant from the Vertex of the ſaid Cone.*

Plate VII. Fig. 5. 11. But in a *ſolid Cone*, which conſiſts of an infinite Number of circular Areas, which therefore may be conſider'd as ſo many Weights, the Centre of Gravity may be found as before, by putting $BE = x$, $BG = a$; the circular Area $DFE = y$, and $AGC = b$; and from the Nature of the Cone, $a^2 : x^2 :: b : y = \frac{bx^2}{a^2}$ but $\dot{x}y = \frac{bx^2\dot{x}}{a^2}$ = Fluxion of the Weights; and $yx\dot{x} = \frac{bx^3x}{a^2}$ = Fluxion of the *Momenta*, whence the Fluent of the latter, *viz.* $\frac{bx^4}{4a^2}$ divided by the Fluent of the former $\frac{bx^3}{3a^2}$, will give $\frac{3}{4}x$ of the Centre of Gravity of the Part DBEF; *conſequently the Centre of Gravity of the Cone* ABCG *is diſtant from the Vertex* B, $\frac{3}{4}$ *of the Side* BG, *in a Circle parallel to the Baſe.*

Fig. 6. 12. If ABCD be a regular Superficies, and ABEF another ſimilar thereto, their Difference will be FECD; and the Centres of Gravity of each will be in the Points H, G, I, as is evident from what has been ſhewn. Now (from *Euclid*'s Elements) we have FECD to ABEF as the Triangle FBD to the Triangle ACF; but FBD : ACF :: FD : AF :: GH : HI; wherefore if any two regular and ſimilar Superficies or Solids together with their Centres of Gravity G, H, are given, the Centre of Gravity I of their Difference will be given alſo by the above Analogy. This in many Mechanical Affairs will prove of very conſiderable Uſe.

Fig. 7. 13. Thus for Example, ſuppoſe ABCG and BDEF were two ſimilar Cones, and their Centres of Gravity when hollow ing the Centres, ſo ſituated, that its *Diſtance from the ſaid Centres will be reciprocally as the Quantity of Matter in each Body.*

or ſuperficial were *c* and *a*; but when ſolid, *d* and *b*; if then we ſuppoſe the Cone DBEF cut off from the Cone ABCG, there will remain the Difference ADECG, whoſe Centres of Gravity, when hollow and when ſolid, may be found as above; which let be in the Circles *eif* and *gkb*, at *i* and *k*. If now the truncated Part were made into a Bucket by fixing in a thin Bottom at AC, and ſuſpending it moveable in a Handle by Pins fix'd between the ſaid Circles *eif* and *gkb*, on each Side, 'tis evident ſuch a Bucket when empty muſt hang with its Mouth downwards; and when full of Water, with the Mouth upwards; thus may ſuch a Bucket be applicable to divers Uſes, as being capable of filling, and emptying, and moving itſelf in different Forms, as the Bottom is fix'd in the wide or narrow End.

14. The Centre of Gravity of a Parabola ABC, is found as in the Triangle and Cone; thus let BF $= x$, DE $= y$; then will $y\dot{x}$ be the Fluxionary Weight, and $yx\dot{x}$ the Fluxion of the *Momenta*; but from the Nature of the Curve we have $y = x^{\frac{1}{2}}$, whence $y\dot{x} = x^{\frac{1}{2}}\dot{x}$, and $yx\dot{x} = x^{\frac{1}{2}}\dot{x}x$, whoſe Fluent $\frac{2}{5}x^{\frac{1}{2}}$ divided by $\frac{2}{3}x^{\frac{3}{2}}$ (the Fluent of $x^{\frac{1}{2}}\dot{x}$) will give $\frac{3}{5}x = \frac{3}{5}$, BF for the Diſtance of the Centre of Gravity from the Vertex B in the Part DBE; and ſo $\frac{2}{3}$ of BG is the Point in the Axis of the whole Parabola ABC, from the Vertex B. Plate VII. Fig. 8.

15. The Phænomenon of the double Cone is eaſily accounted for, from what we have ſaid of the Centre of Gravity. For let ABD be the common Baſe of the two Cones, its Centre C will be the Centre of Gravity of the Whole; therefore if DF be the Leg of a Ruler elevated to an Angle FDG, whoſe Sine FG is leſs then the Semidiameter of the Cone CD, 'tis plain the Centre of Gravity C at the Poſition of the Cone in D is more diſtant from the Centre of the Earth than in its Poſition between the Legs of the Ruler at F; and therefore it will deſcend (as on an inclined Plane CFE) from C to F, where it will ſtop, as being ſupported on the Ends of the Ruler. Fig. 9.

16. Let ABED repreſent a Section of a Cylinder of Wood biaſs'd on one Side with a cylindric Piece of Lead as B, this will bring the Centre of Gravity out of the Centre of Magnitude C to ſome Point G between C and B; let FH Fig. 10.

WHENCE, ſince the Earth and Moon are to each other as about 40 to 1, and the Diſtance of

be an inclined Plane, whoſe Baſe is F L. It is evident the Cylinder laid upon the Plane will no where reſt but there, where a Perpendicular to the Horizon F L paſſes through the Centre of Gravity G, and that Point of the Plane E in which the Cylinder touches it; and this in all Angles of Inclination of the Plane leſs than that whoſe Sine is equal to C G (the Radius being C D) will be in two Situations A B E D and *a b e d*, becauſe when the Cylinder moves, the Centre of Gravity deſcribing a Circle round the Centre of Magnitude C, this Circle will meet the Perpendicular in two Points G and *g*, in each of which the Centre of Gravity being ſupported, the Cylinder will reſt. Therefore the Cylinder moves from E to *e* by the Deſcent of the Centre of Gravity from G to *g*, in the Arch of the Cycloid G*bg*.

Plate VII. Fig. 11. 17. If the Cylinder A B E D inſiſting on the horizontal Line E L in the Point E, has the Centre of Gravity G in the horizontal Diameter D B, it will gravitate in the Perpendicular G*e*; if therefore a Plane F H touch the Cylinder in the Point *e*, it is plain the Cylinder cannot either aſcend or deſcend on ſuch a Plane; becauſe G in any Situation between *e* and H, or *e* and F, it will gravitate to the Left or Right from the Point in which the Cylinder touches the Plane, and ſo will in either Caſe bring it back to the Point *e*.

18. In this Caſe the Angle of the Plane's Inclination H F L is equal to the Angle E C*e* or C*e*G, becauſe E L and F H are Tangents to the Cylinder in the Points E and *e*, and conſequently the Angles $ECe + EFe =$ two Right Angles $= EFe + HFL$; therefore $ECe = HFL$. Hence it follows, that the Cylinder cannot aſcend on a Plane whoſe Inclination is greater then the Angle E C*e*, but on a Plane of leſs Inclination it may; and there is a certain Inclination of the Plane on which the Aſcent will be a *Maximum*, but 'tis not worth while to inſiſt farther on ſo uſeleſs a Subject.

Fig. 12. 19. If D L M E and F P Q O repreſent two Waggons loaded, the former with heavy Matter, as *Iron*, *Stone*, &c. which lies low, and the former with a light Subſtance that riſes high, both on the Side of an Hill A B; 'tis evident the Centre of Gravity of the Load L M, acting in the Direction B I, which falls between the Wheels D E will keep the Waggon firm on the Side of the Hill, (by *Art.* 5.) But in the other Load N P Q O, where the Centre of Gravity K riſes ſo high above the Bottom of the Waggon, it muſt act in a the Moon 60 Semidiameters of the Earth, the Diſtance of the common Centre of Gravity of the Earth and Moon will be found about 1854 Miles from the Earth's Surface; and it is this common Centre of Gravity that deſcribes the *Annual Orbit* about the Sun, and not the Earth itſelf, as is commonly ſaid and thought.

IN like manner there is a common Centre of Gravity of the Sun and all the Planets which circulate about him; and were they all placed in a Right Line on one Side the Sun, then would the common Centre of Gravity of the whole Syſtem be diſtant from the Sun's Surface *eight Tenths* of his Semidiameter: And it is about this common Centre of Gravity, and not about the Sun, that not only all the Planets, but even the Sun itſelf, do conſtantly move (XXXVI.)

Direction K H, which falling without the Baſe Line F G of the Wheels, muſt neceſſarily cauſe the Waggon to overturn.

20. For the ſame Reaſon a Man muſt fall when the Centre of Gravity acts in a Direction falling without the Baſe-Line of the Feet; and 'tis by the artful adjuſting of this Point over the Rope that People can walk or fly thereon; and that Tumblers and Equilibriſts perform ſuch Wonders. In fine, 'tis this ſingle Principle that regulates every Kind of Motion, both Animal and Mechanical, and to which we are naturally prone to have a much ſtricter Regard then we commonly think of; as would be evident were we nicely to conſider the Gate, Poſture and Configuration of the Body under the various Modes of Acting, *viz.* of *Walking*, *Sitting*, *Riding*, *Stooping*, *Carrying*, *Lifting*, *Drawing*, *Puſhing*, *Reaching*, *Striking*, *Throwing*, &c. and the particular Manner in which Artiſts fix a Wind-Mill, Crane, or any ſuch like Machine.

(XXXVI.) 1. If two Bodies A and B be connected together, by a Wire or Chain A B, they will each of them affect the other by their gravitating on the Line A B; and alſo every Particle of that Line will be affected with thoſe Gravities, Plate VII. Fig. 13.

THE common Centre of Gravity of any Number of Bodies being ſupported, none of thoſe Bo-

or be carried downwards, but with an unequal Force, except *one,* on which the Force of each Body is equal; and ſince we ſhew'd the Force of any Body A or B ariſes from its Quantity of Matter multiplied into its Velocity, 'tis evident, that one Particle muſt be ſo ſituated at C, that $B:A(::v:V)::AC:BC$. Becauſe then we have $\overline{B\times V=A\times v}=\overline{B\times AC=A\times BC}$. Or the Product of each Body into its Velocity is the ſame at the Point C, but no other.

2. The Point C is therefore call'd the *Common Centre of Gravity* of both the Bodies. If the Diſtance between the Centres of the Bodies be given, and the Magnitude of each, the Diſtance of the Point C from either may be thus found; as $B+A:B::AC+BC(=AB):AC$. Let A repreſent the Earth, and B the Moon; then will $A:B::40:1$, nearly; and ſince $AB=60$ Semidiameters of the Earth; we have $40=1:1::60:\frac{60}{41}=AC$; and ſince a Semidiameter contains about 4000 Miles, we have $\frac{60\times40000}{41}=5854=AC$, from whence taking one Semidiameter $AG=4000$, there will remain $GC=1854$ Miles, for the Diſtance of the Point C from the Earth's Surface.

3. Since the Earth and Moon act on each other by Attraction, 'tis evident they muſt both revolve about the common Centre of Gravity C; whence this Point C, and not the Centre of the Earth A is that which the Moon regards in her periodical Revolutions, and where there no other Bodies in the Heavens but the Earth and Moon, this common Centre of Gravity C would be at Reſt, or a fix'd Point.

4. But ſince the large Body of the Sun commands (by the ſame Power of Attraction) the Earth and Moon to revolve about itſelf, it will follow that the Point C is that which muſt deſcribe the Circle (or *Orbis Magnus*) about the Sun; becauſe no other Point between A and B can keep always at the ſame Diſtance from the Sun, on Account of the mutual Revolutions of thoſe Bodies about that Point at the ſame time they are carried about the Sun.

Plate VII. Fig. 14. 5. But to illuſtrate this Matter further, let S be the Sun, and CDEF a Part of the annual Orbit; A and B the Earth and Moon in her Conjunction at C, in her Quadrature at D, in her Oppoſition at E, and in her laſt Quarter at F; during

dies can fall; which is the Reaſon of many very ſurpriſing Appearances in Nature, *as that common Experiment of ſuſpending a Bucket of Water at the End of a Stick off the Table without falling; the Toy of the Man a ſawing*, &c.

all theſe Motions from C to F, 'tis evident the Centre of the Earth deſcribes an irregular Curve A D A F; but the Centre of the Moon, one much more ſo, as being at very unequal Diſtances from the Sun S, continually increaſing and decreaſing; and that the Point C is that alone which is at an equal Diſtance at all Times, and therefore deſcribes the circular Orbit about the Sun. Plate VII. Fig. 13.

6. The Point C is that in which the Force of each Body A, and B, is exerted; and therefore every Action of that Point is the ſame as it would be if both the Bodies were there united in one; thus if A and B were two Chain-Shots, the Point C in the Chain, and no other, would ſtrike an Obſtacle with the greateſt Force, or that of a Body equal to the Sum of A and B.

7. If therefore a Wire be inſerted in the Point C, and at the other End any Body D were affixed, the common Centre of Gravity of all the three Bodies A, B, and D may be found as before; for let the Body $C = A + B$; then will $C + D : D :: CD : CE$; or $A + B + D : D :: CD : CE$; and thus E will be the Point ſought. And in the ſame Manner the common Centre of Gravity in a Syſtem of any Number of Bodies may be found.

8. Hence the common Centre of Gravity of the Sun and Planets may be eaſily found by comparing their Quantities of Matter and Diſtances; thus let A B C repreſent the Sun's Body, and S its Centre. Now ſince the Bulks and Diſtances of the four firſt Planets, *Mercury*, *Venus*, the *Earth* and *Mars*, are very inconſiderable in regard of the Sun, they would not (if placed in the Right Line C G) remove the common Centre of Gravity between the Sun and themſelves far from the Centre of the Sun, ſuppoſe to L. But when we come to *Jupiter*, his Bulk and Diſtance give him a conſiderable *Momentum*, which will (if placed in the ſame Line C G) remove the Centre of Gravity from L to I, a Point without the Surface of the Sun's Body. Laſtly, if we conſider *Saturn* placed in the Line C G, with all the reſt; though his Quantity of Matter falls ſhort of, yet his Diſtance far ex- Fig. 15.

We have now premiſed every thing neceſſary for underſtanding the Nature of thoſe Inſtruments which are commonly call'd MECHANICAL POWERS, or MACHINES: They are Six in Number, as follow, *viz.* The *Lever*, the *Pulley*, the *Wheel and Axle*, the *Inclined Plane*, the *Wedge*, and the *Screw.* They are call'd *Mechanical Powers*, becauſe they increaſe our Power of moving or raiſing heavy Bodies, which are often unmanageable by any natural human Strength, not thus aſſiſted; and of two or more of theſe all other compound Inſtruments and Machines are contrived and compoſed.

I. A LEVER is any inflexible Line, Rod,

ceeds that of *Jupiter*, and therefore his *Momentum* will be conſiderable enough to bring the common Centre of Gravity from I to K, at ſuch a Diſtance CK from the Sun's Surface as is equal to $\frac{8}{10}$ of the Sun's Semidiameter SC; or SC : CK :: 10 : 8. Now it is this Point K which is the fix'd and immoveable Centre of the Syſtem, about which the Sun, as well as all the Planets continually move. But the Sun being always very near it, and its Diſtance therefrom varying with the different Poſitions of the Planets, the Motion of the Sun about this common Centre K will be very irregular and unequal; while that of the Planets, on account of their great Diſtance, may be eſteem'd nearly uniform and circular

9. If the two Bodies A and B move to or from each other in the ſame Right Line AB, with Velocities proper to their Quantities of Matter inverſely, the common Centre of Gravity C will remain at Reſt, becauſe their Diſtances from it will by that means be always in the ſame Ratio, *viz.* of their Maſſes inverſely, which is the only thing that makes that Point the Centre of Gravity between them. But if both Bodies were to move in two different Planes, either in the ſame or different Directions, the Centre of Gravity C would deſcribe a Right Line, and with a Velocity of the ſame Sort as that with which the Bodies themſelves do move.

or Beam, moveable about or upon a fix'd Point, (call'd the Prop or *Fulcrum*); upon one End of which is the Weight to be raiſed, at the other End is the Power applied to raiſe it, as the Hand, &c. Since (as we have before proved) the *Momentums of the Weight, and Power, are as the Quantities of Matter in each multiplied by their reſpective Celerities; and the Celerities are as the Diſtances from the Centre of Motion, and alſo as the Spaces paſs'd through in a perpendicular Direction in the ſame Time*; it muſt follow, *that there will be an Equilibrium between the Weight and Power, when they are to each other reciprocally as the Diſtances from the Centre, or as the Celerities of the Motions*, or *as the perpendicular Aſcent or Deſcent in the ſame Time*; and this univerſally in all Mechanical Powers whatſoever, which is therefore the *fundamental Principle* of all *Mechanics* (XXXVII.)

(XXXVII.) 1. The Nature of this Propoſition being not underſtood by Smatterers in Mechanics, gave them Occaſion to imagine the Poſſibility of a *Perpetual Motion* from one Part of it, which they did not ſee was utterly impoſſible from another Part of it.

2. That Part which ſeem'd to promiſe the Poſſibility thereof, was this, viz. *That the* Momenta *of equal Bodies were as their Diſtances from the Centre of Motion.* Hence, ſay the Perpetual-Motion-Men, if a Wheel were conſtructed of the Form of that in the Figure ABCD, with circular Cells going from the inner Part EFGH to the outer, containing equal Balls C, D, E, F, then upon turning the Wheel, they muſt move towards the Centre N on one Part, as the Ball E, and from it on the oppoſite Part, as the Ball C; and by this Means the Ball C will have a greater *Momentum* than the Ball F, and ſo will determine the Wheel to move round; and ſince this muſt be the Caſe of all the Balls E and C that come into the Situation E, C, the Wheel muſt neceſſarily move con- Pl. VIII. Fig. 1.

To

Plate V. Fig. 9. To illuſtrate this, let AB be the Lever ſuppoſed without Weight, and F its *Fulcrum* or Prop; let W be a Weight ſuſpended from the End A, and P the Power applied to the other End B. Then let the Lever be moved into the Situation CD, 'tis evident the Velocities of the Points A and B will be as the Arches AC and BD deſcribed in the ſame Time: Alſo the perpendicular Diſtances, through which the Weight W and Power P move in the ſame Time, are CE and GD, which are as the Arches AC and BD; and theſe are as the Radii CF and DF, which are equal to AF and BF. Therefore in order to produce an *Equilibrium*, it muſt be W ×

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tinually, becauſe it will continually bring two Balls into that Situation.

3. 'Tis true, were there but two Balls E and C, the Ball C would by this Contrivance move the Wheel one Quarter round, *viz.* while it deſcended from C to D; and by this Means would raiſe the Ball E to F; and there they will abide in the Situation DF; but, ſay the Gentlemen of this Perſuaſion, two other Balls ſucceeding to the Places E and C, will ſtill keep the Wheel moving.——Yes, ſo they would, if the Balls at D and E could be taken away the Moment they came into that Poſition, not elſe; for the Balls C and E, in order to move the Wheel, muſt move the Balls D and F, which have equal *Momenta*, (as being at the ſame Diſtance each from the Centre, as are the other two reſpectively) which is abſurd by the *general Propoſition*.

4. The Abſurdity of a Perpetual Motion will ſtill farther appear, if we conſider, that the *Momenta* of Bodies are always proportion'd to the perpendicular Deſcent or Aſcent to or from the Centre of the Earth. Since, therefore, in the Wheel, the Bodies are all equal by Suppoſition, and the perpendicular Spaces through which they *deſcend and aſcend* below and above the Horizontal Line or Diameter AC, are equal; it follows, that an *Equilibrium* muſt neceſſarily enſue. Thus ſo far is this Wheel from producing a Perpetual Motion, that it admits of none at all.

$AC = P \times BD$; or $W \times CE = P \times DG$, or $W \times AF = P \times BF$: Conſequently, $P : W :: AC : BD :: CE : DG :: AF : BF$. Note, *that in eſtimating the Effects of Machines, we regard only the Diſtances of the Power or Weight which are perpendicular to the Lines of Direction in which thoſe Powers act, as* F B, *or* F M, *which are perpendicular to the Directions* P B *and* L M.

THE *Lever* is of five Kinds. (1.) The common Sort, where the Prop is between the Weight and the Power, but neareſt the former. (2.) When the Prop is at one End, the Power applied at the other, and the Weight between both. (3.) When the Prop is at one End, the Weight at the other, and the Power applied between both. (4.) The bended Lever, which differs only in Form from the firſt Sort. (5.) When the Prop is placed at an equal Diſtance between the Weight and the Power, and this is commonly call'd

THE BALANCE, whoſe Uſe is, with a Pair of Scales, to bring one Body to an equal Weight with another that is a Standard. The *Proportional Balance* is without Scales, and is uſed for diſcovering or aſſigning any Proportion of Weight in Bodies. The *Falſe Balance* makes Bodies of unequal Quantities of Matter appear to have equal Weight. Laſtly, the *Statera*, or *Roman Balance*, commonly call'd the *Steelyard*, is a moſt uſeful Kind of univerſal Balance, the Structure and Uſe whereof will be eaſy to underſtand from the above Principles.

II. The PULLEY is an Inſtrument well known; if ſingle, it is reduced to the Lever of the fifth Sort, or Balance, and ſo affords no Advantage in raiſing Weights. If two or more be combined together in the common Way, *Then the Power is to the Weight as Unity to the Number of Ropes which goes to the Pulleys of the lower Box.* But there are different Ways of applying Pulleys, whoſe *Advantage or Power* will be obvious from a View of the Structure of the ſeveral Sorts of *Tackles*, andthe Experiments with them.

III. The WHEEL and AXLE (commonly call'd the *Axis in Peritrochio*) is the third Mechanical Power. We eaſily ſee by its Make it is reducible to a Lever of the firſt Sort; and that therefore *the Power is to the Weight, as the Diameter of the Axle to the Diameter of the Wheel,* in an *Equilibrium* in this Machine.

IV. The INCLINED PLANE is the fourth Mechanical Power, and from a due Conſideration of it, it will appear, that, for an *Equilibrium, the Power muſt be to the Weight, as the Height of the Plane to the Length.*

V. The WEDGE is only a double Inclined Plane, intended to ſeparate the Parts of Wood, *&c.* which ſtrongly cohere together; whence, in the common Form of it, *the Power will be to the Reſiſtance to be overcome, as half the Thickneſs of the Wedge to the Length thereof.*

VI. The SCREW is the laſt mention'd Mechanical Power, whoſe Uſe is both for *Preſſure* and *raiſing of Weights*, but chiefly the former. *The Power is to the Weight, as the Velocity of the Weight to the Velocity of the Power,* that is, *as the Diſtance between two Threads of the Screw to the Circumference of a Circle deſcribed by the Power* (XXXVIII.)

(XXXVIII.) 1. Notwithſtanding the Demonſtration of the Fundamental Principle of Mechanics, as deliver'd in the Lectures, is moſt natural and perſpicuous, and eaſily reſults from what has been ſaid of the *Momentum* of Bodies and their common Centre of Gravity: Yet as Sir *Iſaac Newton* has demonſtrated the ſame Thing in a different and moſt extenſive Manner, which alſo is the Conſequence of a different Principle, *viz.* the *Compoſition and Reſolution of Forces and Ratios*, it will be proper to exhibit and explain that alſo, which is as follows.

2. Suppoſe two Weights A, P, appended by the Strings M A, N P, at the Ends of unequal Radii O M, O N, iſſuing from the Center O of any Wheel, in a State of *Equilibrium*; thoſe Weights will be to each other reciprocally as O L to O K, that is, A : P :: O L : O K, or as the neareſt Diſtances of their Lines of Direction from the Centre reciprocally. Pl. VIII. Fig. 5.

3. For on the Centre O with the Radius O L deſcribe an Arch cutting the Thread M A in D, and draw O D which continue out to E, to which draw A E perpendicular and compleat the Parallelogram A E D C. Now ſince K L paſſes through the Centre O and is perpendicular, to each String in the Points K and L by Suppoſition; it matters not whether the Body A and P be ſuſpended from the Points M and N, or K and L, or D and L, ſince the Weight of the Bodies is the ſame in either of thoſe Points reſpectively.

4. Therefore let A D expound the whole Force of the Weight of the Body A, and let it be reſolved into the two Forces D E and A E, of which the former drawing directly from the Centre avails nothing in moving round the Wheel; but the other Part A E or C D, acting perpendicularly upon the End of the Radius O D, has the ſame Force or produces the ſame Effect, as if it had acted perpendicularly at the End of the equal Radius O L.

5. Therefore the Weight of P will be expreſs'd by D C (or A E) becauſe of the *Equilibrium*; and hence the Weight of A is to that of P as A D to A E; and becauſe of the ſi-

WE have here conſider'd the Action or Effect of each of theſe Machines, as they would

milar Triangles AED and DOK, we have $AD : AE :: DO (= LO) : OK$; therefore $A : P :: OL : OK$.

6. If the Weight p, (equal to P) ſuſpended by the String Np, does at the ſame time in part reſt upon the inclined Plane pG; let pQ be drawn perpendicular to pN, pH perpendicular to the Horizon, and HG through the Point N perpendicular to pG: Then will the Tenſion of the String PN be to that of the String pN, as pH to pN. For ſuppoſe the Body p wholly ſupported by the two Planes pG, pQ, it will preſs them with its whole Force, which may be expreſs'd by pH, and which is reſolvable into the two Forces HN and pN, of which the firſt preſſes the Plane pG, and the laſt the Plane pQ; if therefore the Plane pQ be removed, the ſame Force will ſtretch the String pN; but ſince $p = P$, the Force with which P ſtretches the String PN is as pH; therefore the Tenſion of the String PN is to that of the String pN as pH to pN.

7. Let the Force of p to move the Wheel (as ſupported on the Plane be call'd x. Then $P : x :: pH : pN$; and ſo $P = \frac{x \times pH}{pN} = \frac{A \times OK}{OL}$ (by 5th), whence we have this Equation, $x \times pH \times OL = A \times OK \times pN$; therefore $A : x :: pH \times OL : pN \times OK$. But here 'tis to be obſerved that as the String PN is chang'd into the Poſition pN, its neareſt Diſtance OL will become OR, ſo that the Analogy between A and x corrected will be $A : x :: pH \times OR : pN \times OK$.

8. From what is demonſtrated is follows (1.) That the Line KOL, is a LEVER, whoſe Power is always expreſs'd by the Ratio $\frac{OL}{OK}$. (2.) That the Lines MOL, MON, DOL, are alſo Levers of equal Force, becauſe the neareſt Diſtance of their Lines of Direction from the Centre O is the ſame in all. (3.) Whenever the Lever KL is moved, the Velocity of the Points K and L will be proportional to the Diſtances from the Centre KO and OL. (4.) Therefore the Velocities alſo, and perpendicular Spaces paſs'd through by the Bodies A and P, will be proportional to the ſaid Diſtances from the Centre.

9. Since $P : A :: KO : LO$; 'tis evident the Power P may be in any Proportion leſs then the Weight A, if its Diſtance from the Centre of Motion OL be in the ſame Proportion

anſwer to the Strictneſs of the Mathematical Theory, were there no ſuch thing as Friction or

greater than the Diſtance of the Weight KO, and yet its Force or *Momentum* ſhall be equal to that of the Weight A. Whence the Nature of a Lever for increaſing a Power ſufficiently appears.

10. If the Diſtances from the Centre OK = OL; then will the Lever become a *common Balance*, becauſe then, in order to an *Equilibrium*, the two Bodies A and P muſt have equal Weight.

11. If the Arms of the Balance KO and OL differ but a very little in Length, there will be the ſame Difference in the Weights A and P. Whence if OK : OL 31 : 32 :: P : A; and therefore if A is an *Averdupois* Pound, P will want $\frac{1}{2}$ an Ounce of it and yet be *in Equilibrio*. Hence the Nature of a *Falſe Balance* is evident; and in order to detect it we need only interchange the Weights, for then, in order to an *Equilibrium*, we muſt have KO : OL :: (31 : 32 ::) 32 : $33\frac{1}{31}$; but of this there wants $2\frac{1}{31}$ half Ounces, which will render the Fraud very notorious.

12. The Arms of a Balance being nicely equated in Weight and Length, and divided into an equal Number of equal Parts, becomes the *Proportional Balance* to be uſed without Scales. For any two Bodies hanging on the Arms of this Balance *in Equilibrio*, will have the Proportion of their Weights expreſs'd by the Numbers at the Diviſions (whence they hang) alternately.

13. If AC the ſhorter Arm of a Balance be made equal Pl. VIII,
in Weight to CE the longer Arm, by the Addition of the Fig. 3.
Ball A, ſo that the whole Beam AE may be nicely equipoiſed on the Centre of Motion C, it then becomes the *Statera* or STEEL-YARD in common Uſe. For if B be taken as a fix'd Point from whence to hang any Weight W, and P be any conſtant Weight, which by moving backwards and forwards on the longer Arm CE, comes to a Point D of the *Equilibrium*; then will P : W :: CB : CD; but becauſe P and CB are given Quantities, or always the ſame, therefore W will always be as CD. Conſequently if the Arm CE be divided into Parts each equal to CB, and number'd; the Number at which P hangs will always ſhew how many times P is contain'd in W, and thence its Weight will be known.

14. If ADBE be a Pulley, upon which hang the Weights P, W; then ſince the neareſt Diſtances of the Strings AW and BP, from the Centre of Motion C, are AC and BC, Fig. 4.

rubbing of Parts which move upon each other, by which means *one Third Part* of the Effect of

the Pulley will be reduced to the Lever or Balance AB with reſpect to its Power; and from thence it appears that ſince $AC = BC$, we ſhall always have $P = W$ for an *Equilibrium*; and therefore no Advantage in raiſing a Weight, *&c.* can be had from a *ſingle Pulley*.

Pl. VIII. Fig. 5. 15. In a Combination of two Pullies AB and DFEG, the Power is doubled; for the Pulley DFEG is reducible to the Lever ED, which muſt be conſider'd as fix'd in the Point E to the immoveable String HE; and the Power acting at D is equal to P, and the Weight W is ſuſtain'd from the Centre C of the Pulley, but $P : W :: CE : DE$; therefore ſince $DE = 2\,CE$, it is $W = 2P$, or $P = \frac{1}{2}W$.

16. The Force of the PULLIES may alſo be eaſily ſhewn by comparing the Velocities of the Power and Weight; for 'tis evident, if the Weight W be raiſed one Inch, each String HE, BD, will be ſhorten'd one Inch, and conſequently the String AP will be lengthen'd *two Inches*, and ſo P will paſs through twice the Space that W does, in the ſame Time, whence its Velocity will be twice as great, and therefore it will be equipollent to a Body W of twice its Weight.

Fig. 6, 7. 17. In the two other Forms of Pullies 'tis evident the *Power* P *is to the Weight* W *as Unity or* 1 *to the Number of Ropes going to the lower Pullies*, becauſe if the Weight be rais'd *one Inch*, each Rope belonging to the lower Pullies will be ſhorten'd one Inch, all which will go into the Rope to which the Power P is applied, which therefore muſt deſcend through ſo many Inches in the ſame Time. Conſequently the Tackle of Pullies in the Form of *Fig.* 6. will increaſe the Power five Times; and that of *Fig.* 7. will increaſe it ſix Times.

18. In the Diſpoſition of Pullies according to *Fig.* 8. 'tis plain, ſince each Pulley has a fix'd Rope, it muſt be conſider'd as a Lever of the ſecond Sort, and ſo will double the Power of the foregoing Pulley, and ſo four Pullies will increaſe the Power ſixteen Times.

19. Though the laſt mention'd Form be of the greateſt Force from the ſame Number of Pullies; yet if we conſider the Simplicity, Force, and Conveniency of a Tackle of Pullies all together, none is ſuperior to that in the Form of *Fig.* 9. where the uppermoſt Pulley is fix'd, and each has a Rope annex'd to the Weight; its Power is therefore thus eſtimated.

the Machine is, at a Medium, deſtroy'd, as is evident by an Experiment of the Inclined

When the Weight W is raiſed one Inch, the Rope A B will be lengthen'd as much, and ſo the Pulley C will deſcend one Inch, by which means the Rope C D will be lengthen'd two Inches, and one by the Riſing of the Weight W; wherefore the Pulley E will deſcend three Inches; and thus the Rope E P will be lengthen'd ſix Inches by that means (*viz.* three on each Side) alſo the Riſing of the Weight will cauſe it to lengthen one Inch more; ſo that the Power P goes through ſeven Inches while the Weight W riſes one; therefore P : W :: 1 : 7, and thus you proceed for any other Number.

20. The Wheel and Axle is a Mechanical Power upon the ſame Principle; for the Weight W hanging from the Axis A will be to the Power P, which keeps it in *Equilibrio*, inverſely as their Velocities; but the Velocities are as the Circumferences of the Wheel and Axle; which again are as their Diameters or Semidiameters, that is, as their Diſtances from the Centre of Motion. If the Spokes or Handles F, G, H, I, K, &c. be added, the Power of the Machine is ſtill farther augmented in Proportion to their Lengths. Pl. VIII. Fig. 10.

21. That the Inclined Plane is a Mechanical Power appears ſufficiently in its diminiſhing the Weight of a Body laid upon it in regard to the Power which holds it in *Equilibrio*. Let A be a Body ſuſtain'd on the Inclined Plane B D; from the Centre C, draw C F perpendicular to the Horizon or Baſe D C; and C E perpendicular to the Plane, then C F will repreſent the whole Weight or Force of Gravity of the Body A, which is reſolvable into the two Forces C E and E F; but the Force C E being perpendicular to the Plane acts wholly upon it, and is equally re-acted on or ſuſtain'd by the Plane; the other Force E F, being parallel to the Plane, is that by which the Body deſcends, or is kept from deſcending by an equal Power acting in a contrary Direction. Therefore the whole Weight of the Body is to the Power which keeps it in *Equilibrio* on the Plane as C F to F E, or (becauſe the Triangles C F E and B D C are ſimilar) as B D to B C, that is, as the Length of the Plane to its Height, as was more particularly ſhewn in *Annot.* XXVIII. 1, 2.

22. The Power of the Wedge A C B H is evident from its conſiſting of two equal Inclined Planes A H C and B H C; as it is chiefly of uſe to ſeparate the adhering Parts of Wood, Fig. 11.

Plane. And farther, concerning Friction we are to obſerve, that it is not proportional to the

and ſince the Power of Coheſion in Wood is every where variable and uncertain, 'tis evident there can be no regular Calculation of the actual Effect of the Wedge; but if we ſuppoſe the Power of Coheſion in the Wood A D E B to be uniform, or to make every where an equal Reſiſtance to the Wedge A B C, dividing its Parts A F and B G, then the Power of the Wedge would be to the Reſiſtance of the Wood, as their Velocities inverſely, that is, as the Spaces moved through in the ſame Time, that is, *as the Height of the Wedge* H C *to half its Width* A H.

Pl. VIII. Fig. 12. 23. In the SCREW A B, ſuppoſe the Diſtance of the Spiral Threads $\frac{1}{10}$ of an Inch, and the Length of the Lever $FG = 16\frac{1}{2}$ Inches, then will the Circles deſcribed by the Hand at F be $= 16{,}5 \times 6{,}3 = 103{,}9$, or 104 Inches nearly, that is, 1040 Tenths of an Inch; whence the Velocity of the Power is to that of the Weight, as 1040 to 1; therefore the Screw is a great Mechanical Power, either for raiſing Weights or Preſſure, even though we allow the greateſt Part for Friction.

SCHOLIUM.

24. Having treated of the *Simple Pendulum* in the preceding Lecture, and of the Nature of the Lever in this; I ſhall here let the Reader ſee how nearly theſe two Inſtruments are allied, or rather ſhew him that the *Lever* is in reality nothing but a *Compound Pendulum*; from whence many Particulars concerning the Properties of each, not hitherto mention'd, will occur worthy of Obſervation.

Plate. IX. Fig. 1. 25. Let C D be a fine Rod or Wire (whoſe Weight is inconſiderable) and moveable about the Point C. If A and B be two Bodies (of the ſame Kind) fix'd at any Diſtance Ca, Cb on the Rod; and then the Rod permitted to deſcend freely; the Bodies A and B will begin their Motions with Velocities proportional to Ca and Cb, and ſo their reſpective *Momenta* will be $A \times Ca$ and $B \times Cb$, in the Points a and b (by *Annot.* XXXVIII.)

26. Now if $A : B :: mb : ma$, then will $A \times am = B \times bm$; and conſequently the Point m will be the common Centre of Gravity between the Bodies A and B (by *Annot.* XXXV.) Wherefore ſince the Velocity of the Point m is as Cm, its *Momentum* will be as $Cm \times \overline{A + B}$; and from the Nature of the common Centre of Gravity, we have $A \times Ca + B \times$

$Cb = Cm \times \overline{A+B}$; and therefore $\frac{A \times Ca + B \times Cb}{A+B} = Cm$; or by putting $Ca = a$, $Cb = b$, and $Cm = d$; then $\frac{Aa+Bb}{A+B} = d$. Suppoſe the Moment the Rod CD began to deſcend, the Bodies A and B were to be diſengaged from it in ſuch a Manner as to deſcend upon it freely; then will they deſcend (not in the Curves *ap* and *bq*, but) in the Tangents *ac* and *bd*, deſcribing the Spaces *ac*, *bd*, which will be as the Inceptive Velocities C*a*, C*b*; and of courſe when the Rod comes into the Situation C*d*, the Bodies will be at *c* and *d*; it being in this Caſe $Ca : Cb :: ac : bd :: Cc : Cd$. And *mn* will be the Line deſcribed by the common Centre of Gravity.

27. At *c* and *d*, let us ſuppoſe the Bodies again fix'd to the Rod, they will then conſtitute a *Compound Pendulum*, which will now vibrate forwards and backwards through the Angle *d*C*r*, the Body A deſcribing the Arch *ces*, and B the Arch *dfr*, and the common Centre of Gravity *n* the Arch *nol*. As this Pendulum vibrates, the Velocities of the Bodies will be every where as their Diſtances, and greateſt of all in the Perpendicular Situation C*ef*: Where the Velocities of the Points *e*, *o*, *f*, may ſtill be repreſented by *a*, *d*, and *b*.

28. Now this *Compound Pendulum* is iſochronous to (*i.e.* vibrates in the ſame Time with) ſome ſimple Pendulum of a determinate Length PQ. If then in the *Compound Pendulum* C*d* there be taken $Cx = PQ$, the Point *x* is call'd the Centre of Oſcillation in this *Compound Pendulum*; for if the Bodies A and B were there fix'd, the Times of Vibration would be the ſame then as now: To determine this Point therefore, is to determine the Time of Vibration in any *Compound Pendulum* whatſoever.

Plate IX. Fig. 2.

29. In order to this; let us once more ſuppoſe the Moment the Bodies A and B come to the Points *e* and *f* in the Perpendicular, they were diſengaged from the Rod, and were to aſcend at the End of a String ſeparately, with the Velocities they had acquired by their Deſcent in the Points *e* and *f*. Then the Altitudes *ig* and *kh*, to which they will aſcend above tho Horizontal Line *ei* and *fh*, will be as the Squares of the Velocities at *e* and *f*; that is, as *aa* and *bb*; as is evident from what has been elſewhere ſhewn (*Annotat.* XXVI. 2, 3.)

30. The Bodies A and B will in this Caſe arrive to the Points *g* and *h* before their Motion will be deſtroy'd. And 'tis plain that *eg* is leſs than *ec* or *es*, becauſe the Body A is retarded in coming from C to *e* by the Body B. Alſo *fh* is greater than *df* or *fr*, becauſe B has its Motion quicken'd

by the Body A. And as when in paſſing from d to f, the perpendicular Spaces deſcended thro' are as the Velocities ſingly, and the *Momenta* as the Velocities and Maſſes of Matter conjointly; ſo on the other Side, in paſſing from f and e to b and g, the perpendicular Spaces aſcended through are as the Squares of the initial Velocities, the *Momenta* will be as the Maſſes of Matter, and thoſe perpendicular Spaces conjointly; that is, as Aaa and Bbb.

31. Now in all Caſes, the Sum of the *Momenta* divided by the Sum of the Maſſes gives a Quotient expreſſing the Velocity, or Aſcent or Deſcent of the Centre of Gravity. (See *Art*. 26.) Therefore $\frac{Aaa + Bbb}{A + B} = d =$ the Aſcent of the Centre of Gravity; for the *Momentum* of this Centre of Gravity is on both Sides the Perpendicular Cf the ſame at equal Diſtances, and therefore it will always aſcend or deſcend through the ſame perpendicular Heights.

32. Having thus obtain'd the Expreſſion of the Deſcent of the Centre of Gravity, we have that for the Centre x of Oſcillation of courſe, for we have As $Cn : Cx :: d : x :: \frac{Aaa + Bbb}{A + B} : \frac{Aaax + Bbbx}{Ad + Bd} =$ Deſcent of the Centre of Oſcillation. But the Deſcent is equal to the Aſcent; for the Point x is actuated and moved in the ſame Manner as the Body Q, which in one Oſcillation deſcends and aſcends through equal perpendicular Spaces, and thoſe Spaces are always as the Squares of the Velocities in the loweſt Point; and therefore for the Point x, it will be as $\overline{Cx}^2 = xx$. Whence we have $\frac{Aaax + Bbbx}{Ad + Bd} = xx$; and therefore $\frac{Aaa + Bbb}{Ad + Bd} = x = Cx$, the Diſtance of the Centre of Oſcillation from the Centre of Motion.

33. Becauſe (in *Art*. 26.) we had $\frac{Aa + Bb}{A\ B} = d$; if we ſubſtitute this Equivalent for d in the above Equation, we have $\frac{Aaa + Bbb}{Aa + Bb} = x$; and therefore $Aaa + Bbb = Aax + Bbx$. Hence $Aaa - Aax = Bbb - Bbx$; and ſo $Aa : Bb :: b - x : x - a$; that is, $A \times Cc : B \times Cd :: xd : xc$. From whence it appears, *that the* Momenta *of the Bodies, or their Power to move the Pendulum, are inverſely proportional to their Diſtance from this Centre*; which is the true Definition of the Centre of Oſcillation.

34. Therefore when $A : B :: b : a$, and ſo $Aa = Bb$; then alſo $b - x = x - a$, or $xd = xc$; or the Centre of Oſcillation would biſect the Diſtance between the Bodies A and B.

35. If $a = o$, or the Body A be removed to the Centre C, then $Aaa = o$, and $Aa = o$; whence $x = \frac{Bbb}{Bb} = b$, that is, the Inſtrument then becomes a ſimple Pendulum Cd.

36. If the Bodies A and B are not placed both on one Rod, but on two Rods Cc and Cd, making an Angle cCd, being united in the Angle C. Then theſe Rods ſuſpended on the Angle C will be the ſame *Compound Pendulum* as before, and the Centres of Gravity and Oſcillation n and x, when the Bodies are at reſt, will be both in the Perpendicular Cx, as in the other Caſe, and expreſs'd by the ſame Equations, *viz.* $Cn = d = \frac{Aa + Bb}{A + B}$; $Cx = x = \frac{Aaa + Bbb}{Aa + Bb}$. And it will oſcillate in the ſame Time with the ſimple Pendulum PQ, when $Cx = PQ$. Plate IX. Fig. 3.

37. If the Body A be ſuppoſed removed on the other Side the Line E D, its Diſtance, $Cr = a$, is now negative, or it will be $-a$ in the Equations for the Centres; thus $d = \frac{-Aa + Bb}{A + B}$; and $x = \frac{Aaa + Bbb}{-Aa + Bb}$, becauſe $-a \times -a = aa$. Hence in this Caſe 'tis very obvious that the *Compound Pendulum* is now become a *Lever*, whoſe *Fulcrum* is the Point C; and A may be conſider'd as a Weight to be moved by the Power B. And hence becauſe when Bb is greater than Aa, d will remain affirmative, therefore n the Centre of Gravity will be in the Part Cd, and ſo will cauſe B to preponderate.

38. On the contrary, if Aa be greater than Bb, then will d be negative, or the Centre of Gravity n will be in the Arm Cr, and cauſe the Weight A to deſcend. But when $Aa = Bb$, then $-Aa + Bb = o$, and $d = \frac{o}{A + B} = o$, that is, n or the Centre of Gravity will fall upon the Point C, and produce an *Equilibrium*; and $A : B :: b : a :: Cd : Cr$. As was before ſhewn to be the Property of the Lever. (See *Annot.* XXXVIII.)

39. Thus alſo, with reſpect to the Centre of Oſcillation, $x = \frac{Aaa + Bbb}{-Aa + Bb}$; 'tis plain, in caſe of an *Equilibrium* where $Aa = Bb$, we have $x = \frac{Aaa + Bbb}{o} =$ Infinite; for ſince the Lever in this Caſe has no Motion, it can only be iſochronous to a ſimple Pendulum of an infinite Length, whoſe Times of Vibration are infinitely great, and its Motions, of courſe, not ſenſible.

Plate IX. Fig. 4. 40. It is farther evident, that the Centres of Oſcillation and Gravity are both on the ſame Side of the Centre of Motion C; and that when Bb is greater than Aa, the Centre of Oſcillation will be ſomewhere on the Side of B in the Line Cd continued out; if Bb exceeds An but a little, it will be at a great Diſtance, as at q; if the Exceſs be greater, it will be nearer, as at p. If it be very great, the Centres will be very near as at o, and when Bb is infinite with reſpect to Aa, then the ſaid Centre will be in the Centre of the Body B.

41. Let the Lever cCd be *in Equilibrio* with the Weights A and B; and let it be required to raiſe the Weight A from c to m, by the Power B; then if Bb be ever ſo little greater than Aa it will deſcend, till the Lever comes to the Situation mn, as required. The whole Action of the Machine conſiſts in raiſing the given Weight A to a given Height H, in a given Time; and this Effect of the Machine will *(cæteris paribus)* be greater as the Time is leſs. But the Time is the Time of half the Vibration of the Lever; and the Times of Vibrations are leſs as the Intenſity of the Power B is increaſed, while its Diſtance continues the ſame, becauſe the Diſtance of the Centre of Oſcillation is diminiſhed.

Fig. 5.

42. Now putting A = Body to be raiſed, H = Height, and T = Time, then $\frac{AH}{T} = E$ = the Effect, or Intenſity of the Power B; whence $AH = ET$ = whole Action of the Machine; and therefore when ET is leaſt of all, the Machine will be in its greateſt Perfection; becauſe as E increaſes T will decreaſe, and though it be not in the ſame Proportion, yet it will cauſe that ET will become a *Minimum* at a certain Limit; as will be evident by the following Example.

43. Suppoſe A = 100 *lb*, and B = 10 *lb*; and if $Cc : Cd :: 10 : 100$; then there will be an *Equilibrium*, in which Caſe T will be infinite, and therefore alſo TE, and conſequently cannot be expreſs'd in Numbers.

44 If B = 11 *lb*, then $\frac{Aaa + Bbb}{-Aa + Bb} = \frac{100 + 1100}{-100 + 110} = \frac{1200}{10} = 120 = x$, the Diſtance of the Centre of Oſcillation, the Square Root of which 10,95 will be proportional to the Time (T) of Vibration, (ſee *Annotat.* XXIX.) This multiplied by E = 11, the Intenſity of the Power, gives 120,45 for the Expreſſion of the Action of the Lever in this particular Caſe.

45. If B = 15 *lb*, then, as before, we ſhall find $x = 32$, whoſe Square Root $5{,}66 \times 15$ ($= T \times E =$) $= 84{,}9$ the

Quantity of Surface, but *to the Weight* of the incumbent Part; as we ſhall alſo ſhew by Experiment (XXXIX.).

Lever, which is now leſs than before. Again, ſuppoſe B or $E = 20$ *lb*, then we ſhall have $x = 21$, and its Square Root 4,6; then $4{,}6 \times 20 = T \times E = 92$, which is again greater than the laſt; the Limit then is between 11 *lb*. and 20 *lb*. and is very near 15 *lb*, as will appear by the following Table; where the Numbers in the ſecond Column expreſs the whole Action of the Lever in Parts, of which the leaſt contains 100000, all which correſponds to the Intenſities of the Power increaſed from 10 to 20 *lb*.

Powers.	*Actions.*	*Powers.*	*Actions.*
10	Infinite.	15,16	100000
11	142360	16	100368
12	114036	17	101611
13	104677	18	103397
14	101053	19	105575
15	100016	20	108330

46. This Conſideration of the Power is but of little Uſe in the Lever; but as that is the moſt ſimple Machine, it was beſt adapted to explain and exemplify this Doctrine. In the Axis *in Peritrochio*, and Pulley, it is of greater Uſe, and ſhould not be neglected; but as it is there a Buſineſs of great Difficulty and Labour, I ſhall refer the Reader to *S'Graveſande*'s third Edition of his *Principia*, where he may ſee the Computations at large; the Reſult of which in general is, *That the Power which ſuſtains the Weight* in Equilibrio *ſhould be increaſed by one half in the Pulley and Axis* in Peritrochio, *that the Action of thoſe Machines may be a* Minimum.

(XXXIX.) The Doctrine of FRICTION is contain'd under the following Particulars, *viz.*

1. When one Body inſiſts on another upon an horizontal Plane, it preſſes it with all its Weight, which being equally re-acted on (and therefore the whole Effect of its Gravity deſtroy'd) by the Plane, it will be abſolutely free to move in any lateral or horizontal Direction by any the leaſt Power applied thereto, provided both the touching Surfaces be perfectly ſmooth or even.

2. But ſince we find no ſuch thing as perfect Politure or Evenneſs in the Surfaces of Bodies (at leaſt ſuch as are produced by Art,) but an evident Roughneſs or Unevenneſs of the Parts

As to *Compound Engines and Machines*, they are as *numerous*, as they are *various* in their Structure; concerning all which we may obſerve in

in the Surface, ariſing from the Poroſity and peculiar Texture of the Body, it is eaſy to underſtand that when two ſuch Surfaces come together the prominent Parts of the one will in ſome meaſure fall into the concave Parts of the other, and therefore, when an horizontal Motion is attempted in one, the fix'd prominent Parts of the other will give more or leſs Reſiſtance to the moving Surface by holding and detaining its Parts, which is what we call *Friction*.

3. Now ſince any Body will require a Force proportional to its Weight to draw it over a given Obſtacle, it follows that the Friction ariſing to the moving Body will always be in proportion to its Weight only, and not the Quantity of the Surface, by which it bears upon the reſiſting Plane or Surface. Thus if a Piece of Wood four Inches wide, and one Inch thick, be ground, and thereby made exactly fit to the Surface of another fix'd Piece of the ſame Wood, it will require the ſame Weight to draw it along on the ſame, whether it be laid on its broad or narrow Side.

4. For though on the broad Side there be four times the Number of touching Particles (*cæteris paribus*) yet each Particle is preſs'd without $\frac{1}{4}$ of the Weight that thoſe are on the narrow Side; and ſince four times the Number multiplied by $\frac{1}{4}$ of the Weight is equal to $\frac{1}{4}$ of the Number multiplied by four times the Weight, 'tis plain the Effect, that is, the Reſiſtance, is equal in both Caſes, and ſo requires the ſame Force to overcome it.

5. The Reaſon why the Friction is proportional to the Weight of the moving Body, is, becauſe the Power applied to move the Body muſt *raiſe* it in ſome Meaſure *upon and over* the prominent Parts of the Surface on which it is drawn; and this Motion of the Body, as it is not upright, ſo it will not require a Power equal to its whole Weight; but being in the Nature of the Motion on an *Inclined Plane*, (ſince the Body bears on the prominent Parts all the while) the Power which moves it will be proportional to but *a Part of its Weight only*; and this will vary with the various Degrees of Smoothneſs or Aſperity between the rubbing Surfaces, and the other concurring Circumſtances.

Plate X. Fig. 1. 6. I find by Experiment, that a Body ABCD (of Wood, Braſs, &c.) laid on the Surface EFGH, will be drawn a-general, that they conſiſt of one, two, or more of the Simple Powers combined together; that in moſt of them the *Axis in Peritrochio*, the

long by a Weight P, nearly equal to *one third* of its own Weight; if the Surfaces be hard and well poliſh'd, it will be leſs than a *third Part*; but if the Parts be ſoft or rugged, it will be much greater. Thus alſo the Cylinder of Wood AB, if very ſmooth, and laid on two well-poliſh'd Supporters C, D, (having been firſt oil'd or greaſed) and then charged with the Weight of two Pounds in the two equal Balls G, H, it will require an additional Weight *x* (equal to about a third Part of the two Pounds) to give Motion to, or overcome the Friction of the ſaid Cylinder. Plate X. Fig. 2.

7. Now this additional Weight, as it cauſes a greater Preſſure of the Cylinder, will likewiſe encreaſe the Friction, and therefore require the Addition of another Weight *y*, equal to the third Part of its own; for the ſame Reaſon the Weight *y* will require another *z*, a third Part leſs; and ſo on *ad infinitum*. Hence upon Suppoſition that the Friction is preciſely equal to a *third of the Weight*, the firſt Weight with all the additional ones, *viz.* 2, $\frac{2}{3}$, $\frac{2}{9}$, $\frac{2}{27}$, *&c.* will be a Series of Numbers in Geometrical Progreſſion decreaſing. Now the Sum of all thoſe Terms, except the firſt (*i. e.* the Sum of all the infinite Number of additional Weights $x+y+z$, *&c.*) is found (by a well known Theorem in Arithmetic) to be equal to *one Pound*. So that if the Weight of the Cylinder be inconſiderable, the Way to overcome the Friction would be to double the Power G, or H, at once.

8. But though we may at a Medium allow about a *third Part* of the Weight with which any ſimple Machine is charged for the Friction ariſing from thence, yet this is very precarious and ſeldom the Caſe; for if ABCD be a Piece of Braſs of ſix Ounces, and EFGH be alſo a Plate of Braſs, and both the Surfaces well ground and poliſh'd, the Weight P of near two Ounces will be required to draw along the Body AC alone; but if AC be loaded with 6, 8, or 10 *lb*, then a ſixth Part of the Weight will be ſufficient to draw it along the Plane. If the Plane be cover'd with a linen or woollen Cloth, then a third, or half Part, and ſometimes more, will be requiſite to draw it along on the Plane. Fig. 14.

9. Yet notwithſtanding the Difficulty and Uncertainty attending the Eſtimation of the Quantity of Friction, 'tis ſtill a moſt uſeful and neceſſary Enquiry how, and by what Means

Lever, and the *Screw*, are the conſtituent Parts; that in all, a certain Power is applied to produce an Effect of much greater Moment; and that,

the Friction of any Machine may be abated or diminiſhed. In order to this we muſt conſider it mechanically, that is, we Plate X. Fig. 3. muſt conſider Friction as a Force acting againſt a Power applied to overcome it. Thus ſuppoſe AB an upright Stem or Shaft turning freely in the Socket B fix'd in the Table or Plane IKLM; and AC, DE two Arms fix'd in the ſaid Shaft, the latter of which DE has three Pins going into a Socket in the Middle of a heavy Weight F, G, or H, in ſuch a Manner, that when a Power applied at C moves the Lever AC, it cauſes the Lever DE to protrude or thruſt along the Weight at F, G, or H, in a circular Manner upon the Table.

10. Now ſince we ſuppoſe the Weight all the while it is in Motion is freely and wholly ſupported by the Plane, it follows that all the Reſiſtance it can give to the Power applied at C, is only what ariſes from its Friction on the Plane. What this Friction is, will be found by applying the Weight at G, ſo that BG be equal to AC; for then the Power applied to C, acting in a Tangent to the Circle CRS, that ſhall juſt move the Weight G, will be equal to its Friction (by *Annotat*. XXXVIII. 10.) But if the Weight be applied at F, becauſe BF is greater than AC, the ſame Power at C, as before, will not move it, becauſe here its Force is increaſed by having a greater Velocity than the Power; as on the other Hand, if placed at H, a leſs Power at C ſhall move it, becauſe of its having there leſs Velocity than the Power. All which is plain from the Properties of the Lever, demonſtrated in *Annotat*. XXXVIII.

11. Hence we underſtand, that though the Weight of a Machine remains the ſame, yet the Friction may be diminiſh'd by contriving that the Parts on which it moves and rubs ſhall have leſs Velocity than the Power which moves it. Thus if the Cylinder AB were to move on the two ſmall Pins or Gudgeons E, F, the Friction would be abated in the ſame Proportion as the Diameter of thoſe Gudgeons is leſs than the Diameter of the Cylinder.

12. The Friction on theſe Gudgeons is ſtill farther diminiſh'd by cauſing them to move on the Circumference of a Wheel; thus let F be the Gudgeon of the Cylinder revol-

in the laſt Place, it is known, that the greateſt Effect or Perfection of the Machine is then, when it is ſet to work with *four Ninths* of that Charge

ving on the Wheel CDE; the Velocity of the Wheel's Circumference will be the ſame with that of the Gudgeon; but the Velocity of the Wheel's Axis AB (which is now to be conſider'd as the rubbing Part) is leſs than that of the Wheel in Proportion as its Diameter is leſs than that of the Wheel. For Example, if the Friction of the Cylinder moving on its Surface be $\frac{1}{3}$ Part of the Weight, and the Gudgeons be to the Cylinder as 1 to 10, they will reduce the Friction to $\frac{1}{30}$ Part; and if again the Axis of the Wheel be to the Wheel as 1 to 10, the Wheel will reduce it to $\frac{1}{300}$ Part; and if the Axis of this Wheel be laid on the Perimeter of another Wheel, the Friction will be reduced to a ſtill leſſer Part of the Weight, and ſo you may proceed to diminiſh the Friction *ad infinitum*. Hence Wheels applied in this Manner are call'd FRICTION-WHEELS. Plate X. Fig. 4.

13. Beſides what we have now premiſed, ſomewhat farther is neceſſary to be underſtood to diminiſh Friction by *Wheel-Carriages*. It was before obſerv'd that Friction aroſe chiefly by lifting the Body over the prominent Parts of the Plane on which it moved; now if we can contrive to move the Body along without lifting or ſuſtaining its Weight, we ſhall move it without much Friction, and this may be done by laying the Body on any moveable circular Subject, as Rollers, Wheels, &c. Thus let AB be the Section of an heavy Body laid on a Roller EF, upon the Plane CD, and drawn by the Power P; 'tis evident when AB moves, the Aſperity of its Surface will lay hold on that of the Roller, and move it likewiſe; and 'tis as plain, that when the Body AB is drawn againſt the prominent Parts of the Roller, they immediately give way, and make no Reſiſtance; thus the perpendicular Diameter *ab* yields into the Situation *ef*, and *cd* ſucceeds in its Place. By this circular Motion of the Roller, its prominent Parts below do only deſcend and move upon, or over, and are not drawn againſt the fix'd prominent Parts of the Plane, and ſo receive no Reſiſtance from them; hence the Body AB is convey'd along without being lifted up, in the Manner as a Wheel is moved by a Pinion, without any conſiderable Reſiſtance. And this is the true Foundation of the *Theory of Wheels*, or Doctrine of *Wheel-Carriages*. Fig. 5.

which is equivalent to the Power, or will but juſt keep the Machine *in Equilibrio* (XL.)

(XL.) 1. To demonſtrate this Propoſition, I ſhall chuſe a Water-Wheel A D E F, driven round by a Current of Water G A, ſtriking the lower Float-Boards A in a perpendicular Direction, in the Manner of an underſhot Mill. Now if the Wheel be not loaded or charged with any Weight, but moves freely on the Gudgeons of its Axis C, then the Water, coming on the Floats, will put the Wheel in Motion, and acting, upon it continually will ſoon accelerate its Motion ſo far, as to give it a Velocity equal to its own.

late X. Fig. 6.

2. But if the Axle of the Wheel C be charged with a Weight P, which it is obliged to raiſe, this will give Reſiſtance to the Wheel, and diminiſh its Velocity, or cauſe it to move ſlower than the Water; as the Weight P is increas'd, the Motion of the Wheel will be proportionably retarded; till the Weight P, coming to have an equal *Momentum* with the Water, the Wheel will loſe all its Motion, or be reduc'd to a State of *Equilibrium*.

3. Now let $F =$ Force of the Water, $V =$ its Velocity, $v =$ Velocity of the Wheel, $P =$ Weight that holds the Wheel *in Equilibrio*, $z =$ Weight raiſed by the Wheel in Motion. Then the Difference of thoſe Velocities, *viz.* $V - v$ will be that with which the Water ſtrikes the Wheel; and ſince the Force of Striking Fluids is always as the Square of the Velocity, (as will hereafter be ſhewn) and Cauſes are proportional to their Effects, we ſhall have $\overline{V-v}^2$ always proportional to z; and when $v = o$, we ſhall have $z = P$; and then V^2 will be as P, ſo that it will always be $V^2 : P :: \overline{V-v}^2 : z$; and ſo $\frac{V\sqrt{z}}{\sqrt{P}} = V - v$, and $v = \frac{V\sqrt{P} - V\sqrt{z}}{\sqrt{P}}$.

4. But from the Principles of Mechanics, we have the Radius of the Wheel A C ($= R$) to the Radius of the Axle C B ($= r$) as the Velocity of the Wheel v to the Velocity of the Weight z; that is, $R : r :: \frac{V\sqrt{P} - V\sqrt{z}}{\sqrt{P}} : \frac{rV}{R} \times \frac{\sqrt{P} - \sqrt{z}}{\sqrt{P}} =$ Velocity of z. Which Velocity of the

THE *Common* JACK is a compound Engine, where the Weight is the Power applied; the

Weight being multiplied by the Weight z, gives an Expreſſion $\frac{rV}{R} \times \frac{z\sqrt{P} - z\sqrt{z}}{\sqrt{P}}$ for the Effect of the Engine.

5. This Expreſſion is to be determined to a *Maximum* by making the Fluxion of $\frac{z\sqrt{P} - z\sqrt{z}}{\sqrt{P}}$ equal to Nothing, (the Part $\frac{rV}{R}$ being conſtant, we neglect.) Let $\frac{z\sqrt{P} - z\sqrt{z}}{\sqrt{P}}$ $= y$, then $z\sqrt{P} - z\sqrt{z} = z \times P^{\frac{1}{2}} - z^{\frac{3}{2}} = y \times P^{\frac{1}{2}} = y$, (becauſe $P^{\frac{1}{2}}$ is conſtant) therefore $\dot{z} P^{\frac{1}{2}} - \frac{3}{2} z^{\frac{1}{2}} \dot{z} = \dot{y} = o$; hence $\dot{z} P^{\frac{1}{2}} = \frac{3}{2} z^{\frac{1}{2}} \dot{z}$, therefore $P^{\frac{1}{2}} = \frac{3}{2} z^{\frac{1}{2}}$; whence $P = \frac{9}{4} z$, and $z = \frac{4}{9} P$; that is, the Weight z, when the Machine is in its greateſt Perfection, is equal to $\frac{4}{9}$ of the Weight P, that will keep it *in Equilibrio*. *Q. E. D.*

6. Hence, if in the Expreſſion $v = \frac{V\sqrt{P} - V\sqrt{z}}{\sqrt{P}}$, we ſubſtitute the Value of z, *viz.* $\frac{4}{9}$ P, we ſhall have $v = \frac{V\sqrt{P} - V\sqrt{\frac{4}{9}P}}{\sqrt{P}}$, or $v\sqrt{P} = V\sqrt{P} - V\sqrt{\frac{4}{9}P} = V \times \frac{1}{3}\sqrt{P}$, therefore $v = \frac{1}{3}$ V. That is, the Velocity of the Wheel (v) is equal to the *Third Part* of the Velocity of the Water, when the Machine is in the greateſt Perfection.

7. When the Wheel is kept *in Equilibrio* by the Weight P, we have $F : P :: CB (= r) : CA (= R)$ by the Principles of Mechanics; whence we have $\frac{FR}{r} = P$; whence $z =$ $(\frac{4}{9} P =) \frac{4}{9} \frac{FR}{r}$; which Values of z and P, ſubſtituted in the Expreſſion $\frac{rV}{R} \times \frac{z\sqrt{P} - z\sqrt{z}}{\sqrt{P}}$, will give $\frac{4}{27}$ V F for the Exponent of the greateſt Effect of the Engine.

8. To illuſtrate this by Example; ſuppoſe G A a Body of Water iſſuing from an Aperture in the Pen-Stock *one Foot Square*, and four Feet below the Surface o the Water-Head; in this Caſe the Force of the Water F = 250 *lb*, and the Ve-

Friction of the Parts, and the Weight with which the Spit is charged, is the Force to be overcome; and a ſteady, uniform Motion, by means of the Fly, is the End or Intention of the Machine (XLI.)

locity $V = 16$ Feet *per* Second (as will appear hereafter); whence $\frac{4}{27}VF = \frac{4 \times 250 \times 16}{27} = 592,6$, and this is the greateſt Effect of the Engine; now if $R : r :: 8 : 1$; then $P = \frac{FR}{r} = 2000$; and $z = \frac{4}{9}P = 888,9$ *lb.* And its Velocity will be $\frac{1}{8}$ of $\frac{1}{3}$ of 16, or $\frac{2}{3}$ of a Foot *per* Second. To prove the Truth of all that has been ſaid, one need only take z either greater or leſſer than 888,9 *lb*, and then the Expreſſion for the Effect of the Engine $\frac{rVz}{R} \times \frac{\sqrt{P} - \sqrt{z}}{\sqrt{P}}$, will give a Number in each Caſe leſs than 592,6; which therefore is the *Maximum*, as above found.

(XLI.) 1. The FLY is the only Part I ſhall here take notice of in the Chimney-Jack, and ſhall explain its Nature and Uſe both there and in other Mechanical Machines. For it may be applied to any Sort of Engines to very good Purpoſes, which have a quick and circular Motion, and where the Power or Reſiſtance acts unequally in the different Parts of a Revolution.

2. The Uſe of the Fly is to facilitate the Motion of Engines, by accumulating and retaining the Power communicated to it, and exerting it gradually and equally in each Revolution of the Machine; whence it comes to paſs that the Motion of the Machine is render'd very nearly uniform and of an equal Tenour in all Parts of the Revolution, and therefore more eaſy, pleaſant, and convenient to be acted by the impelling Power. For neither the Strength of Men, nor other Powers or Reſiſtances affecting the Engine, can or do act equally and uniformly in every Part of a Revolution. Thus Meat on a Spit gives always more Reſiſtance on one Part than another, by which means the Motion would be ſo irregular and jolting, that it would ſoon become ſhaken and looſed from the Spit, and ſo not be carried round, were not this Irregularity prevented by the Fly.

IN CLOCKS, WATCHES, *&c.* the Power is the Weight or Spring; the Force to be overcome is the Friction of the numerous Parts, which are chiefly a Combination of Wheels and Axes, whoſe Uſe is to divide a large Portion of Time,

3. The Fly does not add any new Power to an Engine, as ſome have imagin'd; as is evident for the following Reaſons. (1.) The Fly has no Motion but what it receives at firſt from the Machine. (2.) A Degree of Force is always neceſſary to maintain the Motion of the Fly, which muſt be ſupplied from the Machine. (3.) The Friction of the Pivot, Screw, *&c.* of the Fly is a Reſiſtance to the impreſs'd Force, and muſt abate it. (4.) The Air likewiſe makes Reſiſtance to the Weights at the End of the Fly. Upon all which Accounts 'tis eaſy to underſtand, that the Fly, inſtead of adding, does very much decreaſe or leſſen the Power impreſs'd on the Machine.

4. The beſt Form for a Fly is that of a heavy Wheel or Circle, of a fit Size; for this will meet with leſs Reſiſtance from the Air; and being continuous, and the Weight every where equally diſtributed through the Perimeter of the Wheel, the Motion will be more eaſy, equable, and regular. In this Form the Fly is moſt aptly applied to the perpendicular Drill, where it not only gives Weight and Regularity of Motion, but contributes to keep the Drill upright by its Centrifugal Power.

5. In this Form it is alſo beſt applied to a *Windlaſs* or common *Winch*, where the Motion is pretty quick; for when a Man turns the bended Handle of the Winch, his Strength is not, nor can be equally exerted in every Part of the Revolution; for in pulling upwards from the lower Quarter, he can exerciſe more Power than in thruſting forward in the upper Quarter, where, of courſe, part of his former Force would be loſt, were it not accumulated and conſerved in the equable Motion of the Fly. By this means a Man may work all Day in drawing up a Weight of 40*lb.* whereas 30*lb.* would create him more Labour in a Day without the Fly.

6. The Fly is ſometimes made uſe of to increaſe the Force of Mechanical Engines, as of the Lever and Screw, in that for *Stamping* of Money in the Mint. Here the Power exerted by the Man is accumulated by the large Weights at each End of the Lever; the Lever increaſes and communi-

as a *Day* or an *Hour*, into very minute equal Parts, as *Minutes*, *Seconds*, &c. and to point out thoſe Diviſions by an equable Motion of a proper Hand or Index round a graduated Circle (XLII.)

cates it to the Cylinder upon which it is fix'd; the Screw does again increaſe or condenſe it upon the Medal, by means of which the Impreſſion of the Image is made.

7. The great Power of this (or any other Machine of this Kind) may be thus computed. Suppoſe the Arms of the Fly 15 Inches each, or the Length of the Lever 30 Inches, and each Weight to be 50*lb.* and the Diameter of the Cylinder at the Screw be *one Inch.* If each Stroke be made in a Half-Circle, which will be four Feet, and in half a Second of Time, the Velocity (being equably accelerated by the continued Action of the Man) will at the Inſtant of the Stroke be at the Rate of eight Feet *per* Second; therefore 8 x 100*lb.* = 800*lb.* for the *Momentum* of the Lever or Fly. But ſince the Diameter of the Screw is but $\frac{1}{30}$ of the Length of the Lever, we ſhall have 30 x 800*lb.* = 24000*lb.* for the *Momentum* of the Engine, which is equal to that of 100*lb.* falling 120 Feet; which prodigious Force is ſtill farther increaſed for coining larger Pieces of Money, by increaſing the Weights and Arms of the Fly.

(XLII.) 1. I ſhall here ſhew the Principles of Watch and Clock-Work, and how to calculate the Numbers for the Movements, in as clear and compendious a Manner as I poſſibly can. In all *Automata*, or Machines of Clock-Work, there is a natural Agent, or Principle of Motion; which, by acting on one Part, gives Motion to that, and all the other Parts depending upon it, and thus becomes the *Primum Mobile*, or firſt Mover to the whole Machine.

2. In common *Clocks* and *Watches* this is of two Sorts, *viz.* A SPRING or a WEIGHT; either of which may be made to act with any determinate Force; the Spring by its *Elaſticity*, and the Weight by its *Gravity*. In theſe Machines this Force is requir'd to be ſuch as will overcome the *Vis Inertiæ* and Friction of all the Parts in Motion; which in Watches is very inconſiderable, but in Clocks is much greater, and that in Proportion as they are more compounded.

3. The Manner in which a Weight acts upon the Cylinder, about which the Line or Cord (to which it hangs) is wound, is eaſy to be underſtood by all; but the Action of the Spring coil'd up within the Cylindric Barrel or Box of a Clock or Watch,

is ſomewhat more nice and myſterious; and the Manner how it acts upon the Fuſee always with an equal Force by means of the Chain, and the proper Figure of the Fuſee for that Purpoſe, is next to be explain'd.

4. The Chain being fix'd at one End to the Fuſee, and at the other to the Barrel, when the Machine is winding up the Fuſee is turn'd round, and of courſe the Barrel; on the Inſide of the Barrel is fix'd one End of the Spring, the other End being fix'd to an immoveable Axis in the Centre. As the Barrel moves round, it coils the Spring ſeveral Times about the Axis, thereby increaſing its elaſtic Force to a proper Degree; all this while the Chain is drawn off the Barrel upon the Fuſee; and then when the Inſtrument is wound up, the Spring by its elaſtic Force, endeavouring conſtantly to unbend itſelf, acts upon the Barrel, by carrying it round; by which means the Chain is drawn off from the Fuſee, and thus turns the Fuſee, and conſequently the whole Machinery.

5. Now as the Spring unbends by degrees, its elaſtic Force, by which it affects the Fuſee, will gradually decreaſe; and therefore unleſs there were ſome Mechanical Contrivance in the Figure of the Superficies of the Fuſee to cauſe that as the Spring is weaker, the Chain ſhall be removed farther from the Centre of the Fuſee, ſo that what is loſt in the Spring's Elaſticity is gain'd in the Length of the Lever; I ſay, unleſs it were for this Contrivance, the Spring's Force would always be unequal upon the Fuſee, and ſo would produce an unequable Motion of the Parts of the Machine.

Plate XI. Fig. 1.

6. The Figure of the Curve, which ſhall form the Superficies of the Fuſee by a Revolution about its Axis, may be inveſtigated as follows. Let BCD be the Curve, AL the Axis of the Fuſee produced; let D be the Point where the End of the Chain is fix'd on the Fuſee when the Watch is down, or the Spring uncoil'd, and B the Point where it touches it when the Spring or Machine is wound up. From the Points B and D let fall the Perpendiculars to the Axis BA and DH; in which produced, let there be taken AE and HI proportional to the Force or Strength of the Spring, when the Chain is at B and D. Through E, I, draw the Right Line EIK interſecting the Axis ſomewhere in K; and from any Point C in the Curve, draw CF perpendicular to the Axis in G; then will FG be as the Strength of the Spring when the Chain is at G.

7. Now, ſince the Force acting on the Fuſee ought always to be uniformly the ſame; and this Force being always as the Strength of the Spring expreſs'd by FG, and the Diſtance at which the Chain acts from the Axis of the

Fuſee conjointly: Therefore the Force at any Point C will be as the Rectangle $FG \times GC$, and ſince this is a given Quantity it may be made $FG \times GC = ab$, and ſo we have $FG = \frac{ab}{GC}$.

8. Therefore to determine the Equation of the Curve BCD, let $KH = a$, $HI = b$, $HG = x$, and $GC = y$. Then becauſe of the ſimilar Triangle HKI and GKF, we have $HK : HI :: GK : FG = \frac{ab}{y}$; that is, $a : b :: a + x : \frac{ab}{y}$; whence we have $aa = ay + xy$, which is the Equation of the Curve, and ſhews it to be of the *Hyperbolic Kind.*

9. The Form or Figure of the Fuſee being thus determined by the given Force of the Spring in the Points B and D, and the Points G taken at Pleaſure ſucceſſively from H towards A; we next conſider that it is acted upon, or put into Motion, by an uniform Force; by which means alſo the *Great Wheel*, which is fix'd to it, is put into Motion, and that drives the Pinion of the *Centre Wheel*, which *Centre Wheel* drives the Pinion of the *Third Wheel*, and this drives the Pinion of the *Contrate Wheel*, and this the Pinion of the *Balance Wheel*, which plies the two Pallets on the Axis of the Balance, and keeps the Balance in Motion.

10. The Balance in a Watch is inſtead of the Pendulum in the Clock; both ſerving to govern the Motion of the whole Machinery. To this Balance is fix'd a ſmall Steel ſpiral Spring which regulates the Motion thereof, and makes it equable; whence it has its Name of *Regulator*. N. B. *The Reader will do well to have his Eye upon the internal Parts of the Watch, as he reads this Deſcription.*

11. When the Watch is wound up, the Chain from the Spring exerts a Force upon the Fuſee, which gives Motion to all the Parts of the Machine in the following Manner; which will be eaſy to underſtand when the Number of Teeth in each Wheel, and Leaves in the Pinions which they drive, are ſpecified. And theſe in Modern Thirty-Hour Watches are as follows.

	Teeth.	Leaves.
Great-Wheel	48	12
Centre-Wheel	54	6
Third-Wheel	48	6
Contrate-Wheel	48	6
Balance-Wheel	15	2 Pallets.

12. Hence 'tis eaſy to underſtand how often any one Wheel moves round in the Time of one Revolution of that which drives it. Thus the Great-Wheel on the Fuſee, having 48 Teeth, and driving the Centre-Wheel by a Pinion of 12, muſt cauſe the Centre-Wheel to move round 4 Times in one Turn of the Fuſee. And ſo for all the reſt, as below.

$12)48(4 =$ Turns of the Centre-Wheel.
$6)54(9 =$ Turns of the Third-Wheel.
$6)48(8 =$ Turns of the Contrate-Wheel.
$6)48(8 =$ Turns of the Balance-Wheel.

13. Whence it follows that the Turns of each of thoſe Wheels reſpectively in one Turn of the Fuſee, will be had by multiplying thoſe ſeveral Quotients together ſucceſſively, as follows:

1 Turn of the Fuſee or Great-Wheel.
$4\times1=4$ Turns of the Centre-Wheel.
$9\times4\times1=36$ Turns of the Third-Weel.
$8\times9\times4\times1=288$ Turns of the Contrate-Wheel.
$8\times8\times9\times4\times1=2304$ Turns of the Balance-Wheel.

14. The Balance-Wheel having 15 Teeth, and each ſtriking a Pallet twice in one Revolution, there will be 30 Strokes upon the Axis of the Balance, which we call the *Beats* of the Balance; and therefore there will be $2304\times30=69120$ Beats in one Turn of the Fuſee or Great-Wheel.

15. The Centre-Wheel is that which we muſt have the principal Regard to in the *Diviſion of Time*; the Wheels beyond this, towards the Balance, ſerving only to multiply the Strokes of the Balance, and cauſe it to move with an inſenſible Power, and be thereby ſubject to a more perfect Regulation. But the Centre-Wheel is that upon which both the *Hour and Minute Hand* is moved or carried round upon the Face of the Watch to indicate the Time, *viz.* the Hour of the Day, or Minute of the Hour.

16. Since the Time of the Watch's going is 30 Hours, and the Minute Hand, and conſequently the Centre-Wheel, goes round once in an Hour, the ſaid Centre-Wheel will have 30 Turns in the Time of the Watch's going; and becauſe it has four Turns in one of the Fuſee, therefore $4)30(=7\frac{1}{2}=$ the Number of Turns of the Fuſee in winding up the Watch. Whence $69120\times7{,}5=518400=$ the Number of Beats during the whole Time of the Watch's going.

17. If then we divide 518400 by 30, we ſhall have the Quotient $17280 =$ Number of Beats in an Hour, which is call'd the TRAIN of a Watch; and it is ſaid to be a *ſwifter or ſlower Train* as the Number of Beats in an Hour is greater or leſſer.

If we divide the Train 17280 by 3600, the Seconds in an Hour, the Quotient will be almoſt 5; that is, there will be near 5 Beats *per* Second in ſuch a Watch.

18. From this Analyſis of a Watch, it will be eaſy to form an Idea of the Manner of Calculation for the Numbers of the Teeth and Leaves for the ſeveral Wheels and Pinions throughout the Work; and I ſhall endeavour to facilitate and illuſtrate this by an Example of the Numbers for a Watch whoſe *Train* is 14400, and which therefore will beat Quarter-Seconds, becauſe ſuch an one will be uſeful for many Philoſophical Purpoſes, as well as the common Meaſure of Time.

19. The Time which this Watch ſhall go, may be 32 Hours; then $14400 \times 32 = 460800 =$ the Beats of the Balance in the whole Time. Suppoſe the Number of Turns in the Fuſee be 8; then 8)460800(= 57600 = the Beats in one Turn of the Fuſee. Again, let the Number of Teeth in the Balance-Wheel be 15, there will be 30 Beats in one Turn of this Wheel; then 30)57600(= 1920, which will be the Number ariſing from the continued Multiplication of all the Quotients of the Wheels, divided by the Pinions they drive from the Great-Wheel to the Balance-Wheel, as will be eaſy to underſtand by *Art.* 12, 13.

20. The Buſineſs is now to break this Number into 4 convenient ſmall Numbers, which multiplied together ſhall make the ſame Number 1920. This may ſoon be done by a few Trials; thus ſuppoſe I take the Number 4 for one of them; then 4)1920(480. This Number, 480, I plainly ſee can be divided by 6 without a Remainder; therefore 6)480(= 80; and as I plainly ſee that $80 = 8 \times 10$, therefore the four Numbers ſought are 4, 6, 8, and 10. For theſe, multiplied together, make 1920, *viz.* $4 \times 6 \times 8 \times 10 = 1920$.

21. Having thus got the Quotients, it will be very eaſy to find what large Numbers, divided by ſmall ones, will produce the ſaid Quotients; thus 12)48(= 4; wherefore, if I allow 48 Teeth to the Great-Wheel on the Fuſee, it muſt drive a Pinion of 12 on the Centre Wheel.

22. In the next Place, for the Quotient 6, I chuſe the Numbers 54 and 9; thus 9)54(= 6, which ſhews that the Teeth of the Centre-Wheel may be 54, and it muſt then drive a Pinion of 9 on the Third Wheel. Or, if inſtead of 54 and 9, I rather make choice of 48 and 8, it will anſwer the ſame End; for 8)48(= 6, as before.

23. Then for the Quotient 10, 'tis eaſy to ſee, that 50 and 5 will produce it, *viz.* 5)50(= 10. That is, the Third Wheel having 50 Teeth muſt drive a Pinion of 5 on the Contrate-Wheel. If the ſaid Wheel has 40, or 60 Teeth,

and drive a Pinion of 4 or 6, the ſame Number of Turns will be obtain'd.

24. Laſtly, for the Quotient 8, we have the Numbers 48 and 6; for 6)48(=8; or 7)56(=8; or 5)40(=8. Whence if the Contrate-Wheel be allow'd 40, or 48, or 56 Teeth, it muſt drive a Pinion of 5, 6, or 7 Leaves on the Balance-Wheel.

25. By this Means all the Wheels and Pinions are determined, and adjuſted in the Body of the Watch from the Fuſee to the Balance; but all that we have hitherto done ſhews only the *Minutes of an Hour*, and *Seconds*, or *Quarter-Seconds* of a Minute; but nothing has yet been mention'd relating to the Mechaniſm for ſhewing the *Hour of the Day*, of which we ſhall treat in the next Place.

26. This Part of the Work lies conceal'd from Sight between the upper Plate of the Watch-Frame and the Dial-Plate; and ſince few People have the Opportunity of viewing this Work, I have here repreſented it by a Figure; wherein ABC is the uppermoſt Side of the Frame-Plate, as it appears when detached from the Dial-Plate; the Middle of this Plate is perforated with a Hole receiving that End of the Arbor of the Centre-Wheel which carries the Minute-Hand; on this End of the Arbor, near the Plate, is fix'd a Pinion *a b* of 10 Teeth; this is call'd the *Pinion of Report*; it drives a Wheel *c d* of 40 Teeth; this Wheel *c d* carries a Pinion *e f* of 12 Teeth, and this drives a Wheel *g h* with 36 Teeth.

Plate XI. Fig. 2.

27. As in the Body of the Watch the Wheels every where drive the Pinions, and ſo quicken or increaſe the Motion; here, on the contrary, the Pinions drive the Wheels, and by that means decreaſe the Motion, which is here neceſſary; for the Hour-Hand, which is carried by a Socket fix'd on the Wheel *g h*, is required to move but once round while the Pinion *a b* moves 12 Times round. To this End the Motion of the Wheel *c d* is $\frac{1}{4}$ of the Pinion *a b*, becauſe 10)40(=4. Again, while the Wheel *c d* or the Pinion *e f*, goes once round, it turns the Wheel *g h* but $\frac{1}{3}$ Part round, for 12)36(=3. Conſequently the Motion of *g h* is but $\frac{1}{3}$ of $\frac{1}{4}$ of the Motion of *a b*; but $\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$, that is, the Hour-Wheel *g h* moves once round in the Time that the Pinion of Report on the Arbor of the Centre or Minute-Wheel makes twelve Revolutions as requir'd.

28. Having thus ſhewn the Nature and Manner of the Mechaniſm of a *Watch*, the Structure of that Part of a CLOCK, which is concern'd in ſhewing the Time, will eaſily be underſtood, eſpecially as it is repreſented in the Draught. The Mechaniſm of a Clock conſiſts of *two Parts*, one to *ſhew the*

Time, the other to *report it*, by ſtriking the Hour upon a Bell. How each of theſe is effected, I ſhall ſhew by the following Steps.

Plate XI. Fig. 3. 29. Each Part is actuated or moved by *Weights*, as in common Clocks, or *Springs* included in Boxes or Barrels, as A; this Cylinder moves the Fuſee B, and the great Wheel C, (to which it is fix'd) by the Line or Cord that goes round each, and anſwers to the Chain of the Watch. The Method of Calculation is here nearly the ſame as before. For ſuppoſe here the Great-Wheel C goes round once in twelve Hours; then if it be a *Royal Pendulum* Clock, ſwinging Seconds, we have $60 \times 60 \times 12 = 432000$ Seconds or Beats, in one Turn of the Great-Wheel.

30. But becauſe there are 60 Swings or Seconds in one Minute, and the Seconds are ſhewn by an Index on the End of the Arbor of the Swing-Wheel (which in thoſe Clocks is in an horizontal Poſition) therefore 'tis neceſſary that the Swing-Wheel ſhould have 30 Teeth. Whence 60)43200(= 720, the Number to be broken into Quotients for finding the Number of Teeth for the other Wheels and Pinions, as before, in *Art.* 20.

31. But ſince the Minute-Wheel D goes 12 Times round in one Turn of the Great-Wheel C, we know one of the Quotients muſt be 12; whence 12)720(=60; and 60 is compounded of 6×10, or, 8×7,5. So that either 6 and 10, or 8 and $7\frac{1}{2}$ are the two other Quotients; but the latter Numbers are preferable to the former, as affording a greater Uniformity of the Parts and Eaſe in the Execution. Therefore 8)60(=$7\frac{1}{2}$; hence if we give 60 Teeth to the Middle or Minute-Wheel D, and it drives a Pinion of 8 on the Contrate-Wheel E,

32. The other Quotient 8 will give us 8)64(=8, or 7)56(=8; that is, if we give 64 Teeth to the Contrate-Wheel E, it muſt drive a Pinion of 8 on the Arbor of the Swing-Wheel. But if only 56 be allow'd, then it muſt drive a Pinion of 7, and theſe laſt Numbers are now generally uſed. Thus much for the *Time Part*, as far as the Minutes and Seconds: As for the Pinions and Wheels to ſhew the Hours, they are the ſame as was before ſhewn in the Watch, *Art.* 26, 27.

33. In Spring Clocks the Diſpoſition of the Wheels in the Watch Part is ſuch as is here repreſented in the Figure, where the Swing-Wheel F is in an horizontal Poſition, the Seconds not being ſhewn there by an Index, as is done in the large Pendulum Clocks, according to the foregoing Calculation: Whence in theſe Clocks the Wheels are diſpoſed in a

different Manner, as repreſented in *Fig.* 4. where C is the Great-Wheel, D the Centre or Minute-Wheel, both as before; but the Contrate-Wheel E is placed on one Side, and F the Swing-Wheel is placed with its Centre in the ſame perpendicular Line G H with the Minute-Wheel, and with its Plane perpendicular to the Horizon, as are all the others. Thus the Minute and Hour Hands turn on the End of the Arbor of the Minute-Wheel at *a*, and the Second-Hand on the Arbor of the Swing-Wheel at *b*.

34. It remains now that we give an Account of the Machinery of the *Striking-Part* of a Clock. Here, as in the *Watch-Part*, the *Primum Mobile* is a large Spring, in the *Spring-Barrel* G; but in long Pendulums it is a Weight, as is well known. This by its Cord and Fuſee moves the *Great Wheel* H; that gives Motion to the *Pin-Wheel* I; that continues it to the *Detent* or *Hoop-Wheel* K; and that to the *Warning Wheel* L; which at laſt is ſpent on the *Flying-Pinion* Q, which carries the *Fly* or *Fan*; and by its great Velocity it meets with great Reſiſtance from the Air it ſtrikes, and by this means bridles the Rapidity of the Clock's Motion, and renders it equable.

35. All theſe Wheels are *quieſcent* or motionleſs, unleſs when at the Beginning of each Hour the Detent O is lifted up, by which means the Work is unlock'd, and the Whole put into Motion by Virtue of the Spring in the Box G. During this Motion the Pins *e*, *e*, *e*, *e*, of the Pin-Wheel I takes the Tail of the Hammer T, and carrying it upwards removes the Head of the Hammer *s* from the Bell R; then being let go by the Pin it is made by a ſtrong Spring to give a forcible Stroke upon the Bell, and this is repeated as often as the Hour requires, by means of a Contrivance in another Part.

36. This conſiſts of moveable Wheels, and ſeveral Levers, and other Parts, which cannot be underſtood by a bare Deſcription, or even by a Repreſentation in a Draught, ſo well as any Perſon may have an Idea of them by taking off the Face or Dial-Plate of a late-made Eight-Day Clock; for, within 8 or 10 Years paſt, great Improvements have been made in this Part of the Mechaniſm. Of which, perhaps, I may give a more ample Account in another Place, the Scope of my preſent Deſign not admitting it here.

37. To the Invention of Mr. *Maurice Wheeler* we owe the curious Contrivance of a *Clock deſcending on an Inclined Plane*; the Theory of which is worth any Gentleman's Knowledge, and may be ſeen in N° 161. of the *Philoſophical Tranſactions*. Alſo the Clock itſelf may be ſeen at *Don Saltero*'s Coffee-Houſe in *Chelſea*.

Plate X. Fig. 7.

38. But ſince many People may have a Curioſity to be acquainted with this odd Kind of Machinery, who have neither of the Opportunities abovemention'd, I will here preſent them with a ſhort Account of this Clock. D E is the inclined Plane on which the Clock A B C deſcends; it conſiſts externally of a Hoop, and two Sides or Plates ſtanding out beyond the Hoop about $\frac{1}{8}$ of an Inch all round, and indented that the Clock may not *ſlide*, but turn round as it moves down.

39. One of thoſe Plates is inſcribed with the 24 Hours, which paſs ſucceſſively under the Index, which is always in a Poſition perpendicular to the Horizon, and ſhews the Hour on the Top of the Machine: For this Reaſon the lower Part of the Index is heavieſt, that it may preponderate the other, and always keep it upright as the Movement goes on.

40. For the internal Part or Mechaniſm, let L E D Q be the external Circumference of the Hoop, and *f f* the Frame-Plate on which is placed the Train of Wheel-Work 1, 2, 3, 4. which is much the ſame as in other Clocks, and is govern'd by a Balance and Regulator as in them. But for a Spring and Fuſee, there needs none in this Clock, their Effects being otherwiſe anſwer'd, as we ſhall ſee.

41. In this Machine the great Wheel 1 is placed in the Centre, or upon the Axis of the Movement, and the other Wheels and Parts towards one Side, which therefore would prove a Biaſs to the Body of the Clock, and cauſe it to move, even on a horizontal Plane, for ſome ſhort Diſtance; this makes it neceſſary to fix a thin Plate of Lead at C on the oppoſite Part of the Hoop, to reſtore the *Equilibrium* of the Movement.

42. This being done, the Machine will abide at reſt in any Poſition on the horizontal Plane H H; but if that Plane be changed into the Inclined Plane D E, it will touch it in the Point D; but it cannot reſt there, becauſe the Centre of Gravity at M acting in the Direction M T, and the Point T having nothing to ſupport it, muſt continually deſcend and carry the Body down the Plane.

43. But now if any Weight P be fix'd on the other Side the Machine, ſuch as ſhall remove the Centre of Gravity from M to the Point V in the Line L D, which paſſes thro' the Point D, then it will reſt upon the Inclined Plane, as we have ſhewn in the Caſe of the *Rolling-Cylinder*; for this Inſtrument is founded on the ſame Principle.

44. If now the Weight P be ſuppoſed not fix'd, but ſuſpended at the End of an Arm, or *Vectis*; which Arm or Lever is at the ſame Time faſten'd to a Central Wheel 1, moving on the Axis M of the Machine, which Wheel by its Teeth

ſhall communicate with the Train of Wheels, *&c.* on the other Side; I ſay, in this Caſe if the Power of the Weight P be juſt equal to the Friction or Reſiſtance of the Train, it will remain motionleſs all one as before when it was fix'd; and conſequently the Clock alſo will be at reſt on the Inclined Plane.

45. But ſuppoſing the Weight P has a Power ſuperior to the Reſiſtance of the Train, it will then put it into Motion, and of courſe the Clock likewiſe, which will then commence a Motion down the Plane, while the Weight P, its *Vectis* PM, and the Wheel 1, all retain conſtantly the ſame Poſition they at firſt have when the Clock begins to move.

46. Hence 'tis eaſy to underſtand that the Weight P may have ſuch an intrinſic Gravity as ſhall cauſe it to act upon the Train with any required Force, ſo as to produce a Motion in the Machine of any required Velocity, as ſuppoſe ſuch as ſhall carry it once round in twenty four Hours; then if the Diameter of the Plates A B C be four Inches, it will deſcribe the Length of its Circumference, *viz.* 12,56 Inches in one natural Day; and therefore if the Plane be of a ſufficient Length, ſuch a Clock may go ſeveral Days, and would be a *perpetual Motion*, if the Plane were of an infinite Length.

47. Let S D be drawn through M perpendicular to the Inclined Plane in the Point D. Alſo let L D be perpendicular to the horizontal Line H H, paſſing through D. Then is the Angle H D E = L D S = D M T. Whence it follows, that the greater the Angle of the Plane's Elevation is, the greater will be the Arch D T, and conſequently the farther will the common Centre of Gravity be removed from M; therefore the Power of P will be augmented, and of courſe the Motion of the whole Machine accelerated.

48. Thus it appears, that by duly adjuſting the intrinſic Weight of P at firſt to produce a Motion ſhewing the mean Time as near as poſſible; the Time may afterwards be corrected, or the Clock made to *go faſter or ſlower* by raiſing or depreſſing the Plane, by means of the Screw at S. The Angle to which the Plane is firſt raiſed is about ten Degrees.

49. We have given the Theory of a Clock moving *down an Inclined Plane:* Let us now conſider how a Clock may be made *to aſcend* on the ſame Inclined Plane. To this End let A B D be the Machine on the Inclined Plane E D E, and let it be kept thereon at Reſt, or *in Equilibrio*, by the Weight P, at the End of the Lever P M. The circular Area C F is one End of a Spring-Barrel in the Middle of the Move- Plate X. Fig. 8.

In the ORRERY and COMETARIUM, the whole Machinery is a Compages of Wheels of various Sizes, ſuitably contrived and adapted to produce circular and elliptical Motions of Bodies repreſenting the *Planets* and *Comets*, in ſuch Periods of Time as are exactly proportional and correſpondent to the reſpective Motions of the Heavenly Bodies which they repreſent; alſo their ſeveral Phaſes, Poſitions of the Orbits, and other Affections, ſo as to be a perfect *Microcoſm*, or *Solar Syſtem* in Miniature (XLIII, XLIV.)

ment, in which is included a Spring, as in a common Watch.

50. To this End of the Barrel the Arm or Lever PM is fix'd upon the Centre M; and thus when the Clock is wound up, the Spring moves the Barrel, and therefore the Lever and Weight P, into the Situation *P* M. In doing this, the Centre of Gravity is conſtantly removed farther from the Centre M of the Machine, and therefore it muſt determine the Clock to move upwards; which it will continue to do, ſo long as the Spring is unbending itſelf; and thus the Weight and its Lever PM will preſerve the Situation they firſt have, and do the Office of a Chain and Fuſee. This is the Contrivance of M. *de Gennes*. See *Phil. Tranſ.* N°. 140.

(XLIII.) It would be too great an Undertaking here to give an Account of the Mechaniſm of the larger Sort of Orreries, which repreſent the Movements of all the Heavenly Bodies; nor indeed can it be done either by Diagram or Deſcription, to render it intelligible to the moſt diſcerning Reader; but inſtead of that, I ſhall exhibit an Idea of the Theory and Structure of an uſeful, conciſe, and portable *Planetarium*, which any Gentleman may have made for a ſmall Expence, and will exhibit very juſtly the Motions of all the Primary Planets about the Sun by Wheel-Work; and thoſe that have Secondaries or Moons, may have them placed about their Primaries, moveable by the Hand, ſo that the whole ſhall be a juſt Repreſentation of the *Solar Syſtem*, or true State of the Heavens, for any given Time of the Year.

2. In order to this we muſt firſt compare, and find out the Proportion, which the Periodical Times or Revolutions of the Primary Planets bear to that of the Earth; and they are

ſuch as are expreſs'd in the Tablet below, where the firſt Column is the Time of the Earth's Period in Days, and Decimal Parts; the ſecond that of the Planets; the third and fourth are Numbers in the ſame Proportion to each other.

As 365,25 : 88 ☿ :: 83 : 20, for *Mercury*.
365,25 : 224,7 ♀ :: 52 : 32, for *Venus*.
365,25 : 686,9 ♂ :: 40 : 75, for *Mars*.
365,25 : 4332,5 ♃ :: 7 : 83, for *Jupiter*.
365,25 : 10759,3 ♄ :: 5 : 148, for *Saturn*.

Plate XII. Fig. 1.

3. If now we ſuppoſe a Spindle or Arbor with ſix Wheels fix'd upon it in an horizontal Poſition, having the Number of Teeth in each, correſponding to the Numbers in the third Column, *viz.* the Wheel AM of 83 Teeth, BL of 52, CK of 50 (for the Earth,) DI of 40, EH of 7, and FG of 5; and another Sett of Wheels moving freely about an Arbor, having the Number of Teeth in the fourth Column, *viz.* AN of 20, BO of 32, CP of 50 (for the Earth,) DQ of 75, ER of 83, and FS of 148; then if thoſe two Arbors of *fix'd and moveable Wheels* are made of the Size, and fix'd at the Diſtance from each other, as here repreſented in the Scheme, the Teeth of the former will take thoſe of the latter, and turn them very freely when the Machine is in Motion.

4. Theſe Arbors, with their Wheels, are to be placed in a Box of an adequate Size, in a perpendicular Poſition; the Arbor of fix'd Wheels to move in Pivots at the Top and Bottom of the Box; and the Arbor of moveable Wheels to go through the Top of the Box to a proper Height, on the Top of which is to be placed a round Ball, gilt with Gold, to repreſent the Sun. On each of the moveable Wheels is to be fix'd a Socket or Tube aſcending above the Top of the Box, and having on the Top a Wire fix'd, and bent at a proper Diſtance into a Right Angle upwards, bearing on the Top a ſmall round Ball repreſenting its proper Planets.

5. If then on the lower Part of the Arbor of fix'd Wheels be placed a Pinion of Screw-Teeth, a Winch turning a Spindle, with an endleſs Screw, playing in the Teeth of the Arbor, will turn it with all its Wheels; and theſe Wheels will move the others about with their Planets in their proper and reſpective Periods of Time very exactly. For while the fix'd Wheel CK moves its equal CP once round, the Wheel AM will move AN a little more than four Times round, and ſo will nicely exhibit the Motion of *Mercury*; and the Wheel FG will turn the Wheel FS about $\frac{1}{29,5}$ round, and ſo will

truly repreſent the Motion of *Saturn*. And the ſame is to be ſaid of all the reſt.

6. If on the Top of the Box be placed a Circle with the Signs of the Ecliptic, including all the Planets, it will be eaſy by this Machine to repreſent the Motions and various Phænomena of the Heavenly Bodies, by thoſe who have Skill in ſuch Things. Having thus ſhew'd the Reaſon, Structure, and Parts of this portable *Planetarium*, the reſt muſt be left to the Fancy and Pleaſure of the Gentleman, and the Ingenuity of the Workman.

7. Since the above was wrote, I have had the Pleaſure to find that Mr. J. NEALE (*Watch-Maker* in *Leaden-Hall-Street, London*,) has invented and made an Orrery upon the ſame Plan with mine above deſcribed; the Diſpoſition of the Parts, and even the very Number of the Wheels (all but two) being the ſame, except that in mine the Wheel of 148 Teeth for *Saturn* he thought too large, and therefore by breaking the Quotient of 5) 148 (=29,6 into two, he ſubſtituted two Wheels (as there is Room enough ſo to do) in its Stead, and has thus brought the Work into ſo ſmall a Compaſs, that for ſuch Gentlemen as require it, he can make an Orrery in the Compaſs of a Watch, that ſhall ſhew the Motions of all the Primary Planets in the Solar Syſtem very nicely, which, as ſuch, may be properly call'd POCKET ORRERIES. See the laſt Article of the following Annotation.

(XLIV.) 1. I ſhall here give an Account of the Mechaniſm of that Inſtrument, which I call a COMETARIUM; for though Dr. *Deſaguliers* has (the firſt of any I know of) deſcribed it under the Character of Part of a PLANETARIUM, and by it repreſented the Motion of the Planet *Mercury*; yet I conceive it is much more adapted to repreſent the Motion of a Comet than a Planet, becauſe none of the Planets deſcribe Orbits ſenſibly *Elliptical*, whereas thoſe of the Comets are very much ſo. I have therefore alter'd the Doctor's Contrivance, and adapted it to the Motion or Theory of the Comet of the Year 1682, whoſe Period is 75½ Years; and have moreover alter'd the Mechaniſm of the Parts, by which the Inſtrument is render'd much more elegant and uſeful.

Plate XII. Fig. 2.

2. The Structure and Rationale of this Inſtrument is as follows. When the Lid is taken off of the Box, the internal Parts appear as in the Figure. NO and QT are two Elliptic Wheels turning each other about their Foci I and S, by means of a Cat-Gut String in a Groove on their Edges, croſſing at K. Theſe oval Wheels are fix'd on Arbors or Axes which paſs through the ſame Focus S and I in each; the Oval NO is moved by the circular Wheel I, fix'd alſo upon the ſame

Axis, but above it on the Bar or long Piece G V; which Wheel is itſelf moved by another equal Wheel G, and that by an endleſs Screw, turn'd by a Winch, on the Out-ſide of the Box. All which is evident in the Figure.

3. The Perimeter of the Oval Q T, where it touches that of N O, will have a Velocity always proportional to the Diſtance from I, that is, in the Points K, 4, 3, 2, 1, &c. the Velocities will be as the Lines I K, I 4, I 3, I 2, I 1, &c. which may be conſider'd as Levers acting upon and moving the Oval Q T in thoſe Points. Now if the Ovals are ſuch that S K is to S V, or I K to I 1, as 6 to 1, then will the Point K have 6 times the Velocity turn'd by the Lever I K, as the Point V will have when it has made ½ a Revolution, or is come under the Point S, where it is turn'd by the Lever I *s*, than in the Situation I S.

4. If we take S P = S K = I S, and upon the Point S, *s*, as Foci, you deſcribe the Ellipſis P L I M, that will repreſent the Orbit of the Comet, or the Figure of the Groove on the Lid of the Box, in which a round Braſs Ball, repreſenting the *Comet*, is made to ſlide along on a Piece of Wire, call'd the *Radius Vector*, fix'd at one End into the Top of the Arbor at S, where we ſuppoſe the Sun to be, and is accordingly repreſented by a ſilver'd Plate at Top.

5. The Place of the Comet at P is call'd the *Perihelion*, as being there *neareſt the Sun*; as I is its *Aphelion* or Point of *greateſt Diſtance*. Since S P = S K, the Velocity of the Comet will be in the Point P equal to that of the Point K; and were the Comet's Aphelion at *s*, its Velocity there would be equal to that of the Point V when under *s*, *viz.* ſix times leſs than before; but ſince the Comet's Aphelion is at I, and ſince the greater Arch deſcribed in the ſame Time muſt have a greater Velocity, the Velocity of the Comet at I will be about ⅕ of that at P.

6. The ſilver'd Ellipſis on the Lid of the Box is divided into 100 Parts, ſhewing the *Anomaly* of the Comet; and about the Axis of the Wheel G is placed a Silver Circle, divided into 75½ equal Parts, repreſenting the Years or Period of the Comet, with a proper Index, pointing to thoſe Diviſions, by which Means 'tis eaſy to ſhew the ſeveral Particulars relating to the Theory of Elliptic Motions, whether of a Planet or Comet.

7. The ingenious Artiſt (before mention'd) Mr. J. NEALE, has in his larger *Planetarium* placed the Orbit of the Comet here deſcribed; by which Means the Motions of all Sorts of Bodies in the Solar Syſtem are now repreſented in one Machine, which thing has never been done by any one. It is finiſh'd in a Man-

In WATER-MILLS, the *Momentum* of the falling Water is the Power; the Force to be overcome is the great Attrition of the two Stones in grinding the Corn, &c. which is effected wholly by a Complication of Wheels and Axles. A Query may here be put, Why, ſince the Power conſtantly acts upon the Wheel, the Motion of the Wheel ſhould be equable, and not accelerated? The Anſwer is, The Increments of Velocity keep riſing, till their *Momentum* is equall'd by the Reſiſtance of the Machine; after which Equilibrium the Wheel goes on with an uniform Motion (XLV.)

a Manner equally grand and elegant; and is of ſo ſimple a Structure as to render the Purchaſe as eaſy as can be deſired; it is therefore but reaſonable he ſhould meet with Encouragement equal to his Merit and Ingenuity.

(XLV.) The Mechaniſm of a Water-Mill depends upon the following Principles.

1. The Action or Power of the Water which drives the Great Wheel. Here it will be neceſſary to determine its Force iſſuing out of the Aperture of the Sluice or Pen-Stock, and alſo the Velocity of its Motion when the Height is known; and the Height neceſſary to produce a given Velocity.

2. As to the Force of Water iſſuing through the Sluice, that is, its momentary Impulſe, we ſhall ſhew from Hydroſtatic Principles, that it is always proportional to the Altitude of the Water above the Centre of the Hole thro' which it paſſes. And ſince we know by Experiment that a cubic Foot of Water weighs very nicely 1000 Ounces *Averdupois*, or 62,5 *lb*; if we find the Area of the Aperture in Feet and Parts, and multiply that by the Number of Feet the Water has above the Centre, and laſtly, you multiply that Product by 62,5 *lb*, this laſt Product will be the Force of the Water expreſs'd in Pounds *Averdupois*.

3. For Example; ſuppoſe the Width of the Sluice 12 Inches, or 1 Foot, and that it is drawn up to the Height of three Inches, or 0,25 of a Foot, the Area of the Aperture will then be $1 \times 0,25 = 0,25$; if the Height of the Water be

7,5 Feet above the Orifice, then 7,5 × 0.25 will be 1,875, which multiplied by 62,5 gives 116,1875, or about 116,2 *lb.* for the inſtantaneous Preſſure of the Water on any Obſtacle.

5. To find the Velocity, and conſequently the Quantity of Water expended at the Orifice in a given Time, we muſt conſider that a Body falls 16,2 Feet in the firſt Second, and acquires a Velocity which in an horizontal Direction is at the Rate of 32,4 Feet *per* Second; now let S be any other Space and the horizontal Velocity V; then we ſhall have 16,2 : S :: $\overline{32,4}^2$: V^2; therefore $\overline{32,4}^2\ S = 16,2\ V$ and ſo $\frac{\overline{32,4}^2}{16,2} \times S = V^2$, that is $\sqrt{64,8\ S} = V$. Now ſince S = 7,5 = Height of the Water above the Centre of the Orifice, therefore $\sqrt{64,8 \times 7,5} = 22$ Feet *per* Second, the Velocity of the iſſuing Water as required.

6. This Velocity might alſo have been found by finding the Time of the Fall thro' 7,5 Feet; thus 16,2 : $\overline{60'''}^2$ :: 7,5 . t^2 = 1666,6, the Square Root of which is 40''',8; now ſince the Velocity of ſpouting Water is uniform, there will paſs twice 7,5 or 15 Feet in 40''',8; and therefore if 40''',8 : 15 F. :: 60''' : 22 F. and a little more, the ſame as before.

7. If the Velocity be given, and it be required to find the Height of the Fall neceſſary to produce it; we have from the foregoing Theorem ($\sqrt{64,8\ S} = V$,) $S = \frac{V^2}{64,8}$; ſo that if the given Velocity be 22 Feet *per* Second, then V = 22, and $S = \frac{V^2}{64,8} = 7,5$ nearly, the Height of the Water above the Centre of the Aperture. For the Velocity of Water is the ſame in the Fall thro' any Space or Height, and in the Orifice of a Sluice at the ſame Depth below the Surface.

8. The Quantity of Water iſſuing out is thus determined for any given Time; ſince a Column of Water = 22 Feet is produced in 1 Second, we ſhall have 60 × 22 = 1320 Feet in 1 Minute, or 158400 Inches; and the Area of the Aperture being 12 × 3 = 36 Inches, we have in that Column 158400 × 36 = 5702400 Cubic Inches; which divided by 282, gives 2022 Gallons, or 32 Hogſheads 6 Gallons, which ſpouts on the Wheel in a Minute.

9. If we now ſuppoſe this Column of Water flowing into the Buckets of an Overſhot-Wheel, which is 16 Feet in Diameter, and has 30 Buckets on its Periphery, (as is the Caſe of

that at the *Bar-Pool*, by the Abbey in *Nun-Eaton* in *Warwickſhire*, which is reckon'd the beſt in *England)* then the Water will act on the Wheel by two Forces, *viz.* of Impulſe and Weight. The Impulſe or Stroke, were it made in a Tangent-Direction, and perpendicular to the Sides of the Buckets, would be equal to 116 *lb.* (by *Art.* 4.) But ſince it runs ſome Diſtance in a *Trough* before it comes to the Wheel, and the Buckets are placed not at Right Angles, but nearly at the Angle of 45 Degrees with the Circumference of the Wheel, and ſo the Water muſt ſtrike them very obliquely; I ſay, on theſe Accounts we muſt abate more than one half of the Force of Water, and may take about 50 *lb.* for a Medium.

10. The Weight of the Water is more or leſs in twelve or thirteen of theſe Buckets on the Fore-part of the Wheel, but moſt of all on that Bucket at the End of the horizontal Diameter, becauſe there no Part of the Water reſts upon the Perimeter of the Wheel or Sides of the Bucket, but gravitates with its whole Weight in a Tangent to that Part of the Wheel. What that Weight in that Bucket is, may be thus found. It was ſhewn that when the Engine is in the greateſt Perfection, the Velocity of the Wheel ought to be equal to $\frac{1}{3}$ of that of the Water. (See *Annotat.* XL. 6.) Now ſince the Diameter is 16 Feet, the Circumference will be 50,3; and the Velocity of Water being 1320 Feet *per* Second, $\frac{1}{3}$ of that is 440, which divided by the Circumference gives $8\frac{7}{10}$ Revolutions of the Wheel in a Minute. The Aperture of the Sluice gives 2022 Gallons *per* Minute, which divided by 8,7 gives 232,4 Gallons, which again divided by 30 (the Number of Buckets) is 7,7 Gallons for a Bucket.

11. Now a Gallon of Water weighs 10,2 *lb.* (for 1728 : 62,5 :: 282 : 10,2); therefore 7,7 × 10,2 = $78\frac{1}{2}$ *lb.* for the Weight of Water in each Bucket. But as part of this Water runs out of the Buckets in the lower Quarter of the Wheel, and what remains gravitates in various Degrees of Obliquity to the Radii of the Wheel, as does all the Water in the full Buckets above the Horizontal one, we muſt allow for the total Weight about 450 *lb.* (as will be found near the Truth by Calculation) to which if we add the 50 *lb.* for Impulſe, the whole Force of the Water on the Wheel will be 500 *lb.*

12. If the Cog-Wheel be 3,5 Feet Radius, or 7 Feet Diameter (as in the Mill above-mention'd) then, as 7 : 16 :: 500 *lb.* : 1143 *lb.* = the Force of the Cogs to turn the Wallower or Trundle, which Force is applied to overcome the Reſiſtance ariſing from the Weight of the Stone, the Friction of the Gear in general, and the great Friction of the Stones and Corn in grinding.

13. Suppoſe the Diameter of the Trundle 1,5 Foot, or 18 Inches, and that of the upper Stone 6 Feet; then as 6 : 15 :: 1143 : 285,7 *lb.* = the Force at the Periphery of the Stone. If there be 48 Cogs in the Cog-Wheel, and nine Rounds in the Trundle, then $\frac{48}{9} = 5,33 =$ the Turns or Revolutions of the Trundle for one of the Water-Wheel; wherefore ſince the Wheel makes 8,7 in a Minute, the Trundle, and of courſe the Stone, will make (8,7×5,33=) 46½ Turns in a Minute. The Circumference of the Stone is 18,84 Feet; whence 18,84×46,5=876 Feet *per* Minute, for the Velocity of the Stone's Periphery.

14. Suppoſe the Stone contain 22,5 Cubic Feet, or its Weight 1912 *lb.* then the *mean Velocity* of the Stone being that Point of the Radius ⅔ of its Length, conſequently ⅔ of the Velocity 876 at the Periphery, *viz.* 584, will be the mean Velocity of the Stone, which multiplied by 1912, *viz.* 1912×584=1116608 *lb.* would be the Expreſſion of the *Momentum* of the Stone *per* Minute, were it to preſs upon the Corn with its whole Weight, which it does not; for nearly the whole Weight being ſupported by the Pivot of its perpendicular Spindle, a very ſmall Part thereof is concern'd in the Triture of the Corn, for that is principally effected by the violent Rotation of the Stone producing a Centrifugal Force.

15. This circular Motion of the Stone brings the Corn out of the Hopper by Jerks; and cauſes it to recede from the Centre to the Circumference in a ſpecial Manner; the Corn, while whole, cauſing the Stone to riſe a little higher above the fix'd Stone than it would otherwiſe do, begins to be cruſh'd by the Weight of the Stone gently preſſing upon it, and the more ſo, as it approaches the Circumference, where being quite reduced to Flour, it is thrown out of the Mill by the Centrifugal Force of the Stone through a Hole for that Purpoſe.

16. When I ſay, *the Stone preſſes gently, and but with a very ſmall Part of its Weight on the Corn,* it is implied, that the Piece of Wood which ſupports the Stone on the Pivot of its Spindle muſt be ſomewhat elaſtic or ſpringy, in order to admit of an eaſy and ſmall Degree of Motion upwards and downwards, as the Stone is more or leſs reſiſted by the Grains of Corn, by which means the Surfaces of the two Stones will have always a varying Diſtance, and ſo be adapted for Trituration and Comminution of the Corn in any Degree; whereas, if the ſupporting Piece were perfectly rigid or fix'd, the Stones muſt always be at an equal Diſtance, and the Flour very coarſe, becauſe it can't be affected by the up-

per Stone after the whole Grain has receiv'd its firſt Impreſſion, or ſuffer'd its firſt Comminution. And the nicer this *Point of Support* is adjuſted, ſo much the more nice and exact will be the Work done by the Mill, or ſo much the better will the Flour be that is produced thereby.

17. As the Water acts upon an *Overſhot-Mill* both by Impulſe and Weight, ſo does it likewiſe on a *Breaſt-Mill*, or that where the Water comes upon the Breaſt or middle Part of the Wheel; and here, though the Weight of the Water on the Wheel is not ſo great as before, being contain'd in the Buckets of the lower Quarter only, yet the Impulſe of the Water is much greater, the Height of the Water being increaſed nearly the Semidiameter of the Great-Wheel, all other Things being equal.

18. If the Height of the Water remain the ſame, the Aperture of the Pen-ſtock muſt be enlarged to nearly twice the Area, that the Force of Water may be the ſame; and in this Caſe there will be twice the Expence of Water as before; ſo that as much more Water is neceſſary for a Breaſt-Mill than for an Overſhot one, every Thing elſe being the ſame.

19. Since the Spout of Water is in the Curve of a Parabola, the Orifice through which it iſſues ſhould not be juſt againſt the Middle of the Wheel, for then, if it were very near the Wheel, great Part of the Force would be ſpent in ſtriking the Wheel directly againſt its Axis, and ſo would make the Motion round more difficult; and if it were not very near the Wheel, the Spout would reach only ſome of the lower Buckets, and none go to thoſe on the Middle, where the Effect would be rather greater. Therefore the Orifice of the Pen-ſtock ought to be ſituated ſome little Height above the Middle or Horizontal Diameter of the Wheel.

20. In order that the Water may have the greateſt Effect on the Wheel, Dr. *Barker* has conſtructed it with 24 Ladle-Boards (inſtead of Buckets) of 18 Inches Square; theſe Ladle-Boards paſs through a quadrantal Channel of the ſame Dimenſions, nearly, or a little more than 18 Inches
Pl. XIII. Fig. 1. ſquare, that the Motion may be free. The Water entering on the Top of this Channel is kept wholly on the Boards, till it comes to the Bottom, where it goes off without interrupting the Motion of the Wheel. The Contrivance is very uſeful, and may be eaſily apprehended by the Scheme, where ADBE repreſents the Wheel with its Ladle-Boards, C the Axis, F the Jet or Spouting-Water, GHI the ſquare Channel or Trough in which the Ladle-Boards with the Water deſcend to I, where the Water is diſcharged from the Wheel.

21. In *Fig.* 2. you have a Section of the Sole or Perimeter of the Wheel, and the Channel with the Ladle-Boards in it; thus *abcd* is the Sole of the Wheel, *fghi* is the Section of the Channel open on the Part next the Wheel by a narrow Slit, in which *ek* is the Part call'd the Tongue, which projects from the Sole to fill up that Slit, that the Water may not run through it; *lmno* is one of the Ladle-Boards put on upon the Supporter *pq*, which goes through the Tongue and Sole of the Wheel, with a Hole behind at *q*, through which a Key or Wedge, like a Piece of Wood, being driven, faſtens it with the Board to the Wheel. Thus you ſee by this Means how great the Force of the Water muſt be on the Wheel, and how much leſs will ſuffice here than in the common Way. Pl. XIII. Fig. 2.

22. As to an *Underſhot-Wheel*, 'tis evident there can be only the Force of the Impulſe from the Water on ſuch a Wheel; and therefore the Height of Water remaining the ſame, there muſt be a larger Aperture of the Penſtock, that ſo a greater Quantity of Water may come upon the Float-Boards in the ſame Time, to have an equal *Momentum*, or to produce the ſame Effect as in the *Overſhot* or *Breaſt-Wheel*. Whence a greater Expence of Water will be made here, than in any other Sort of Mill, and can only be ſupplied for a Conſtancy by a River; and where there is not a Want of Water, this is the eaſieſt, cheapeſt, and moſt ſimple Structure a Mill is capable of.

23. In this Caſe, the Float-Boards are to be placed perpendicular to the Sole or Periphery of the Wheel, becauſe when they come into a vertical Poſition at the Bottom or loweſt Part of the Wheel, they will then receive the horizontal Impetus of the Water directly, and therefore with the greateſt Force. If the Water-Courſe be ſufficient to cover the Float-Boards, that is enough; all that is more, runs waſte, either by the Sides or under the Floats, or both.

24. Let XYZ be an Underſhot-Wheel, and HC the Water coming upon it in the Direction HC. Now if there were ſo few Float-Boards, that when one, as DE, became vertical, the next on each Side IK, LM, ſhould but juſt touch the Surface of the current Water, then would the Water ſtrike the ſaid Float-Board DE with all its Force. But when the two Floats K and E come into the Situation C and G, then will the Water ſtrike but a Part of the Float C, *viz.* BC, and that too obliquely, which is to be eſtimated by AC, the Sine of the Angle of Incidence; and ſince we ſuppoſe the Section of the Water equal to the Area of a Float-Board, 'tis plain the Float C will intercept the Water from the Float G, ſo Fig. 3.

that none can ſtrike it; and therefore the Force of Water will be every where leſs than upon the Float-Board in the Site DE. So that the Force upon the Wheel will always be fluctuating between AC and DE, which will be the two Extremes.

25. If the Float-Boards are ſo many, that while one is Vertical as DE, others on each Side, as C and G, are alſo partly in the Water-Way; then in this Caſe it is beſt to have the Stream of Water larger than the Floats, that ſome may run beſide, and fill up the Space between, that ſo the Floats C, E, G, may all be impell'd at once; for the Back-Water, here, having a greater Velocity than that of the Wheel, will ſtill keep the Float G forward; and though it be an oblique Force, it will always avail ſomething ſo long as it touches the Float; and cannot be eſteem'd a Negative Quantity, as ſome have aſſerted.

26. Since many may be curious to know how the Force of Impulſe and Gravity of the Water in the Buckets of an *Over-ſhot Wheel* is to be computed or eſtimated to a mathematical Exactneſs, I ſhall here give the Method, and illuſtrate it by a Scheme. Let A, B, C, *&c.* be the Buckets of an Overſhot-Wheel, (having twenty-four in all) inclined to the Periphery in an Angle of forty-five Degrees: Thoſe with Dots or Points repreſent the Buckets with the different Quantities of Water they contain, among which the largeſt Dots in the Middle repreſent the Centres of Gravity of the ſeveral Bodies of Water. Now ſuppoſe the Water comes upon the uppermoſt Bucket A in the horizontal Direction X A, though by its Curvature at entering the Bucket it cannot ſtrike the Side of the Bucket directly, yet, ſince the Side of the Bucket obſtructs and ſuſtains the ſaid whole horizontal percuſſive Force, and that under an Inclination of forty-five Degrees, we may conclude that half that percuſſive Force is ſpent in turning about the Wheel.

27. As to the Force ariſing from every deſcending Bucket of Water, it may be eaſily determined by finding the Bulk of the Water in each Bucket, and multiplying that into the perpendicular Diſtance of the Line of Direction of the Centre of Gravity from the Centre of Motion. Hence, with reſpect to the firſt Bucket A, ſince its Centre of Gravity acts in the Direction V N, that is, perpendicularly on the Centre of Motion, that Product, or the Weight of that Bucket of Water, will avail nothing to move the Wheel round, as being wholly ſupported on the Axis of Motion.

28. But the Water in the ſecond Bucket B gravitating in the Direction R O, at the perpendicular Diſtance NO from

the Centre, if we multiply its Maſs or Weight into the Diſtance NO, we ſhall have its *Momentum* or Force to move the Wheel; and ſo of any other. Here we may obſerve, that as the Water in the Buckets decreaſes, the Diſtances increaſe from the Centre, in the upper deſcending Quadrant; and ſince the Diſtances increaſe much faſter than the Quantities decreaſe, the Forces will increaſe till we come to the horizontal Bucket F, which is half full of Water; from thence it decreaſes to the Bucket I, where all the Water runs out, as being parallel with the Horizon, or Diameter KM. And hence we ſee four Buckets on the lower deſcending Quadrant carry no Water, when they are inclined to an Angle of 45 Degrees.

29. I ſhall here ſubjoin a Specimen of the Computation, ſuppoſing the Wheel and Buckets ſuch as in the Scheme, and the Quantities of Water, and Diſtances of their Centres of Gravity the ſame as in the Table below.

Water in the Bucket.		*Diſtances from the Center* N.	*Momenta of each Bucket.*
A = 1,00	×	O	= 0,00
B = 0,9	×	NO = 0,3	= 0,27
C = 0,8	×	NP = 0,6	= 0,48
D = 0,7	×	NQ = 0,85	= 0,595
E = 0,6	×	NL = 0,95	= 0,57
F = 0,5	×	NM = 1,00	= 0,5
G = 0,4	×	NM = 1,00	= 0,4
H = 0,2	×	NL = 0,95	= 0,19

The Total of all the Buckets = 3,005

That is, the *Momentum* of Water in all the Buckets is equal to the *Momentum* of three Times the Water contain'd in the Bucket A, and hanging at the End of the horizontal Diameter KM.

30. I ſhall conclude this Theory of *Water-Mills*, with a Deſcription of Dr. *Barker*'s new-invented Mill, of the moſt ſimple Structure of any yet made, performing its Effect without any Wheel, Trundle, Cog, or Round; the Nature of the Machine, and Manner of its Operation will be eaſy to underſtand from the following Account of its ſeveral Parts.

31. ABCD is an upright Frame ſtanding on a proper Baſe; Pl. XIII. EF is the wider Part of GH an upright hollow Pipe Fig. 5. or Tube, fix'd at the Bottom to an horizontal Square Trunk IK; which Trunk, together with the Tube, is fix'd to an upright Spindle or Axis RS, by means of a Nut and Screw at S. The lower End of the Axis on a fine Point moves in the Pivot-Hole in the Part of the Frame at T; on the upper Part of the Frame is a Hole thro' which the Spindle paſſes, as

alſo thro' the round circular Piece P fix'd on the ſaid Frame; on the upper Part of the Spindle is fix'd another round circular Piece Q, which repreſents the upper moveable Stone of the Mill. Q is a Spout of Water filling the Tube or Trunk, and giving Motion thereto, and conſequently to the Axis and upper Stone, by the horizontal Jets of Water from each End of the Trunk I K, thro' Holes made at each End on contrary Sides.

32. While the Holes continue ſtopp'd, the Trunk will be at reſt, becauſe then the Preſſure is equal over all the Parts; but when the Holes are open, the Preſſure of the Water (by its having Liberty to iſſue out) will be leſs on that Part where the Hole is than on the other Part oppoſite to it; which ſtronger Preſſure will prevail, and carry round the Trunk and Tube with the Axle and Stone, in a contrary Direction; and each Hole contributes to produce this Motion, which will be greater or leſſer in proportion to the *Momenta* of the Jets of Water, or greater or leſſer Aperture of the Holes.

33. For 'tis eaſy to underſtand, that the Power of this Machine is derived from, or depends upon three Things: (1.) The Velocity of the Spouting-Water; (2.) The Quantity thereof; and (3.) The Diſtance at which the Water ſpouts from the Axis of Motion. The two firſt make the *Momentum* ariſing from the Preſſure of the Fluid, which is proportional to the Altitude, or Height of the Tube; the laſt is of a Mechanical Nature, for the Trunk is in this reſpect exactly of the Nature of the *Lever*.

34. In the Lecture of Hydroſtatics it will be ſhewn, that the Velocity of the Spouting Fluid will be as the Square Root of the Altitude of the Fluid; whence this Part of the Force will be *in the Subduplicate Ratio of the Height of the Tube*. The Quantity of the Spouting Fluid will be alſo in the ſame Ratio, while the Apertures are the ſame; but if theſe vary their Magnitude, it will be directly as the Aperture, the Altitudes being given. Therefore the whole *Momentum* ariſing from theſe hydroſtatical Principles will be *as the Altitude of the Tube, and Sum of the Apertures conjointly*; and if this be multiplied by the Diſtance of the Aperture from the Centre or Axis, we ſhall have the Expreſſion of the whole Force of the Machine.

35. That is, if $H =$ the Height of the Fluid, $A =$ Sum of both the Apertures, and $D =$ Diſtance of each from the Axis; then we ſhall have the *Momentum* M expreſſed by $M = H \times A \times D$. Hence we ſee that the Trunk I K is analogous to the great Wheel of a common Water-Mill, whoſe Force is in the ſame Manner computed from the Height of the Fluid

H,

H, the Aperture of the Sluice A, and the Diſtance or Radius of the Wheel D.

36. For both in the common Mill and this, if the Aperture A and Diſtance D from the Centre be the ſame, the Force of the Jet will vary with the Height of the Fluid H. Alſo if the Height of the Fluid H and Length of the Trunk or Diameter of the Wheel D remain the ſame, the Force will be as the Aperture A, or as the Quantity of the Fluid flowing out in a given Time. Laſtly, if the Height of the Fluid H, and the Aperture A continue the ſame, the Force will be as the Diameter D of the Wheel, or Length of the Trunk of this new Mill.

37. I ſhall now give a Calculation of the Power of this Machine; and in the firſt Place, let us ſuppoſe the Height of the Tube to be 9 Feet, and always full of Water; the Velocity of the Spouting Water will be the ſame that a Body will acquire in falling 9 Feet, *viz.* an uniform Velocity of 18 Feet in the *Time of the Fall*; which Time is thus found, as $16 : 9 :: 1^2 : \frac{9}{16} =$ the Square of the Time, whoſe Square Root is $\frac{3}{4}$, that is, $\frac{3}{4}$ of a Second. Therefore ſay, as $\frac{3}{4} : 18 :: 1 : 24$; the uniform Velocity is at the Rate of 24 Feet *per* Second, in the Spouting Water, from 9 Feet Altitude.

38. Let the Aperture at each End of the Trunk be 6 Inches Square, then in both there will be 72 ſquare Inches, or half a ſquare Foot in Area. The Water iſſuing out in a Second at both Orifices will be equal to a Column 24 Feet long, and $\frac{1}{2}$ a Square Foot in Baſe; therefore the whole Water will be equal to 12 Cubic Feet. The Weight of one Cubic Foot is $62\frac{1}{2}$ *lb*; wherefore $62\frac{1}{2} \times 12 = 750$ *lb.* is the Force acting on the Trunk, ſuppoſing the Depth of Water only one Foot; but ſince it is at the Depth of 9 Feet, we have $9 \times 750 = 6750$ *lb.* acting upon the extreme Parts of the Trunk *per* Second.

39. Let us now ſuppoſe the Length of the Trunk to be 6 Feet; then the Motion is made by a Power of 6750 *lb.* acting at the End of a Lever three Feet long; conſequently $6750 \times 3 = 20250$ *lb*, the *Momentum* of the Wheel *per* Second.

40. Now the Velocity of the Trunk at the Orifices will be the ſame as that of the Water nearly, while the Mill is not charged; but that of the Water is 24 Feet *per* Second, and the Circumference which each Orifice deſcribes is 18,84, or 9 Feet nearly; therefore the Trunk turns round a little more than once *per* Second. But when the Mill is charged with the Stone, Corn, *&c.* the Velocity will not then be ſo great; and

we have ſhewn (*Annot.* XL. 6.) that it ought not to be above $\frac{1}{3}$ Part of that of the Water, *viz.* at the Rate of 8 Feet *per* Second; but let us ſuppoſe it a little more, *viz.* 9 $\frac{1}{2}$ Feet; then will the Trunk move round once in two Seconds.

41. If we ſuppoſe the Stone 6 Feet in Diameter, its Velocity in the Periphery will be the ſame with that of the Trunk; *viz.* 9 $\frac{1}{2}$ Feet *per* Second; but $\frac{2}{3}$ of this is the mean Velocity of the Stone, which is therefore but 6 $\frac{3}{10}$ Feet *per* Second. Suppoſe the Weight of the Stone 1912 *lb*, and that $\frac{1}{3}$ of this be allow'd for Friction, *viz.* 637 *lb*; then $6{,}3 \times 637 = 4015$ *lb.* = the Moment of the Stone by its Reſiſtance ariſing from $\frac{1}{3}$ of its Weight; but this is only when the Stone is firſt put into Motion; after it is in Motion, this *Momentum* is greatly leſſen'd by the Centrifugal Forces; and therefore if we allow 4000 *lb.* for the *Momentum* ariſing from the Weight of the Stone, Attrition of the Corn, &c. it will be but $\frac{1}{5}$ Part of the *Momentum* of the Trunk or Mill, which we ſhew'd was 20256 *lb.*

42. Here we have ſuppoſed the Mill to carry the largeſt Stone that is uſed in any Mill, and that the Power of the Mill is five times greater than the Reſiſtance to be overcome; the Quantity of Water therefore in the Jets may be diminiſh'd a *Fifth Part*, and conſequently the Apertures; which therefore, inſtead of being 72 Square Inches, need be only 15 or 16. If therefore at each End of the Trunk there be an Aperture of four Inches long and two wide, the Jets from the Altitude of nine Feet, at three Feet from the Axis, will have a *Momentum* ſufficient for turning the largeſt Stones.

43. I ſhall only obſerve further, that there are many mechanical Purpoſes to which this Invention might be made ſubſervient, as it has ſo great a Power of Motion in ſo eaſy and ſimple a Structure: And alſo, that the Water-Wheel of a common Mill, if placed in a horizontal Situation, and the Sluice ſo order'd as to throw a Side-Jet in a Tangent-Direction on the Ladle-Boards, ſuch a Wheel would be in the ſame Circumſtances nearly as the Trunk of this New Mill, and therefore might be made to anſwer all the ſame Intentions, and capable of nearly all the ſame Advantages.

44. I need not obſerve to the Reader, that whatever Quantity of Water is expended at the Jets, as that muſt be ſupplied at the Top or Ciſtern of the Tube, ſo the Stream which ſupplies it muſt be as much longer than the Jets, as its Velocity is leſs; becauſe what is deficient in one Reſpect muſt be made up in the other. There are ſeveral other Circumſtances and Particulars, which may deſerve to be conſider'd, when it ſhall be found to anſwer any valuable End, beyond that of mere Speculation.

IN WIND-MILLS the Mechaniſm is the ſame, only the Vanes or Sails are to be conſider'd as a *Wheel on the Axle*, actuated by the Power of the Wind. In this Machine we have only to conſider the Poſition of the Sails, and the Power of the Wind. As to the former, if the Sails ſtand *right* before the Wind, it cannot affect them at all; if they ſtand *direct* to the Wind, the Mill will be blown down, at leaſt the Sails can have no Power to move round; they muſt therefore be placed *oblique* to the Wind, and that under an Angle of 54 Degrees and 44 Minutes, for the greateſt Advantage (XLVI.)

(XLVI.) The internal Parts of a Wind-Mill are the ſame with thoſe of a Water-Mill nearly, of which I ſhall take no farther Notice, but ſhall confine myſelf to the Theory of the Sails in regard of their Poſition, Motion, and Figure.

1. In regard of the Poſition of the Sails, we muſt conſider that if they are placed *Direct to the Wind*, or at Right Angles to the Axis of the Mill, they will receive the whole Force of the Wind, which in this Caſe will tend to blow them forward, and conſequently to blow down the Mill; which Poſition of courſe cannot be admitted.

2. If the Sails are ſet *Right to the Wind*, or parallel with the Axis of the Mill, 'tis plain that in that Poſition the Wind cannot act upon them at all, and therefore they cannot be turn'd round, nor the Mill put into Motion; which Poſition of the Sails muſt likewiſe be rejected.

3. Since neither the *Direct* nor *Right Poſition* of the Sails will do, an *Oblique Poſition* muſt, as there can be no other. Now to ſhew that an Oblique Poſition of the Sails will turn the Mill, let AB be the Axis, CD a Sail, and its Angle of Obliquity (*viz.* that which it makes with the Axis) be ECG; then if GC be the Force of the Wind in the Direct Poſition of the Sail, GE will be the Force of the Wind in its Oblique Poſition (as being the Sine of the Angle of Incidence GCE). But the Force GE is reſolvable into two others, EF and GF; of which the latter, being parallel to the Axis, avails nothing Pl. XIII. Fig. 6.

thing in turning the Sail about it; but the other, E F, being perpendicular thereto, is wholly ſpent in compelling the Sail to turn round; which was the Thing to be ſhewn.

4. The Force of the Wind on the Sail will be as the Square of the Sine of Incidence, or as $\overline{GE}^2$; for the Force of each ſingle Particle of Air will be as the Sine GE, (by *Annot.* XXIV. 6.) and it will be alſo as the Number of Particles which ſtrike it at the ſame Time, which Number of Particles is alſo as the Sine of Incidence GE. For let CD repreſent the Section of the Sail in a Direct Poſition, and CG the ſame in an Oblique Poſition, 'tis plain the Number of Particles ſtriking it in the former Caſe will be to the Number ſtriking it in the latter as CD to CF, which is equal to GE the Sine of Incidence; for all the Particles between AD and BF will not come upon the Sail in the Oblique Poſition CG. Since then the Force of the Wind on the Sail is on two Accounts as GE, it will be as the Square of the ſaid Line GE.

Pl. XIII.
Fig. 7.

5. If we ſuppoſe the Velocity of the Wind to vary, the Force thereof will be as the Square of the Velocity; for the greater the Velocity, the greater will be the Stroke of each ſingle Particle, and alſo the greater will be the Number of Particles coming upon the Sail in the ſame Time; the Force will be therefore as the Squares of the Velocity.

6. Again, if the Area of the Sail be variable, the Force of the Wind will be directly as the Area or Superficies of the Sail, becauſe the Number of Particles of the Air coming upon it will always be proportional thereto, and conſequently the Force with which they ſtrike it. Hence, if A, S, and V repreſent the *Area*, *Sine* of Incidence, and *Velocity* of the Wind on one Sail, and a, s, v, thoſe on another; the Force compelling the former to turn round will be to that compelling the latter, as $A \times S^2 \times V^2$ to $a \times s^2 \times v^2$.

7. When the Area of the Sail and its Poſition in reſpect of the Wind continue the ſame, the Force which turns the Sail will be as the *Squares of the Velocity*; and ſince the Wind ſcarce ever blows with one uniform Velocity, but varies with almoſt every Blaſt, the Force upon the Sail will be much more variable and unequal; and therefore the Action or Working of a Wind-Mill cannot be ſo equal, uniform, and ſteady as that of a Water-Mill, whoſe Power is always of the ſame Tenor, while the Jet of Water is ſo.

8. If the Area of the Sail and the Velocity of the Wind be ſuppoſed conſtant, the Force of the Wind in the Direct Poſition will be to that in the Oblique one as $\overline{GC}^2$ to $\overline{GE}^2$, as we have before ſhewn; and it has been alſo ſhewn that

that Part of the Force which turns the Sail is repreſented by EF, when GE is the whole Force: But GE : EF (:: GC : CE) :: $\overline{GE}^2$: $\frac{CE \times \overline{GE}^2}{GC}$ = to the Force which turns the Sail, when the whole Force is repreſented by $\overline{GE}^2$, as is here the proper Expreſſion of it.

9. This Expreſſion $\frac{CE \times \overline{GE}^2}{GC}$ begins from Nothing, when the Angle of Incidence begins to be oblique, and increaſes with the Obliquity of the ſaid Angle to a certain Number of Degrees; becauſe that Part of the Force which is parallel to the Axis becomes leſſer in proportion to that which is perpendicular to it; but after it has paſs'd this Limit, it again decreaſes, and becomes Nothing, when the Angle of Incidence vaniſhes; as is eaſy to underſtand by conſidering that the Quantity of Wind on the Sail does in this Caſe continually decreaſe.

10. There is therefore one certain Poſition of the Sail, in which the Force of the Wind is greateſt of all upon it, or a *Maximum*; and to find it, put Radius GC = a, EC = x, and we have $\overline{GE}^2 = aa - xx$, and conſequently the Force $\frac{CE \times \overline{GE}^2}{GC} = \frac{aax - xxx}{a}$, which muſt be a *Maximum*: Therefore its Fluxion $aa\dot{x} - 3xx\dot{x} = 0$; whence $aa = 3xx$, and ſo $x = \sqrt{\frac{aa}{3}}$, which in Logarithms is $\frac{20,000000 - 0,477121}{2}$ 9,761439, which is the Logarithm Sine of the Angle 35° 16′ = Angle CGE; and therefore the Angle ECG is equal to 54° 44′, when the Force of the Wind is a *Maximum*, as required.

11. The Angle now found, is only that which gives the Wind the greateſt Force to put the Sail in Motion, but not the Angle which gives the Force of the Wind a *Maximum* upon the Sail when in Motion. What this Angle is Mr. *Mac Laurin* has ſhewn in his Book of *Fluxions*, to which I refer the Reader.

12. Mr. *Parent* has alſo ſhewn that an Elliptic Form of the Sails is better than the Parallelogram or long Square; and that the beſt Poſition of the Sail is not that which is common, *viz.* with its longeſt Side or Diameter parallel to the Axis of the Sail; but, on the contrary, it ought to be perpendicular to it; that is, they ought to be of ſuch a Form, and placed in ſuch a Manner,

a Manner, as repreſented in the Figure; and after the four Plate X. Fig. 9. Sails B, C, D, E, are thus placed on the Axis or Arm A, they are then to be turn'd about, and fix'd under the proper Angle of Obliquity above-mention'd.

13. There are three Things yet wanting to the Perfection of a Wind-Mill. The *Firſt* is, ſome Contrivance in the Nature of a *Fly* to regulate the Motion of the Train, under the irregular and unequal Impulſe of the Wind. The *Second* is, ſome other Contrivance to ſupply the Hopper or Stones with more or leſs Corn, in Proportion to the greater or leſs Strength of the Wind. And, *Thirdly*, a Method of altering the Angle of the Sail's Obliquity from its *Maximum* of 54° 44′ at the Beginning of the Motion to its *Maximum* when in Motion.

14. By means of an *Anemoſcope* (hereafter to be deſcribed) it will be eaſy to prove by Experiment what Form of the Sails, that is, whether Rectangular or Elliptical, whether the Vertical or Horizontal Poſition of the longeſt Diameter, and what Angle of Obliquity is beſt; alſo whether the Surface of the Sails ſhould be plain or concave, with many other Things of this Kind. But this muſt be done by as many particular Anemoſcopes, or in other Words, you muſt have an Anemoſcope for every particular Experiment; and all diſpoſed together in one common Frame; the Reaſon is evident, becauſe they all require the ſame Strength of Wind, which cannot otherwiſe be had.

15. That theſe Things may be better underſtood, I ſhall premiſe the following Definition of ſome Geometrical Lines and Figures, which are abſolutely neceſſary to a compleat Knowledge of the modern Mechanical Philoſophy.

16. I take it for granted, that the Reader knows, that if Plate IX. Fig. 6. on any Point C, taken in the Right Line A B, a Circle A D E F be deſcribed, the Point C is call'd the *Centre*, and A E the *Diameter* of the Circle: To which I ſhall add, that A C, or C E, is call'd the *Radius* of the Circle, which is the ſame thing as the *Semidiameter*.

17. If the Circle be divided into four equal Parts, A D = D E = E F = F A, by the two Diameters A E, D F; then each of the Areas A C D, D C E, E C F, F C A, are call'd *Quadrants*, or Quarters of the circular Space; and the Parts of the Circle A D, D E, E F, F A, are call'd *Quadrantal Arches*, or Quarters of the Circle.

18. Every Circle, great or ſmall, is ſuppoſed to be divided into 360 equal Parts, call'd *Degrees*; conſequently each Quarter, A D, D E, &c. will contain 90 of thoſe Degrees, as is evidently repreſented by the large Half-Circle of *Fig.* 7.

19. If two Lines B C and F C meet in a Point C, the Space F C B included between them is call'd an *Angle*; and the Meaſure of that Angle is the Number of Degrees contain'd in an Arch E I of a Circle deſcribed on the angular Point C, and included between the ſaid two Lines B C and F C. Thus the Angle in the Figure contains 40 Degrees.

20. If a Line, as G C, be drawn through the 90th Degree on the Point C, it will make the Angle on one Side G C B equal to the Angle G C A on the other Side, becauſe each is equal to 90 Degrees. Such an Angle is call'd a *Right Angle*; and the Line G C is then ſaid to be *perpendicular* to the Line A B.

21. The Angle F C B, which is leſs than a Right Angle or 90 Degrees, is call'd an *Acute Angle*; and the Angle H C B, which is greater than a Right Angle or 90 Degrees, is call'd an *Obtuſe Angle*. Again; the Arch I D is call'd the *Complement* of the Arch E I to a Quadrant E D, or 90 Degrees; and the Arch A K is call'd the *Supplement* of the Arch E K to a Semicircle E D A, or 180 Degrees.

22. If from the Point I be let fall the Perpendicular I L to the Line or Radius E C, then is that Line I L call'd the *Sine* of the Angle I C E or F C B. In the ſame manner the Line I M is the Sine of the Complement-Arch I D, or Angle I C D. But inſtead of *Sine-Complement*, we ſay, in ſhort, *Co-Sine*: Thus we ſay that I L is the *Sine*, and I M the *Co-Sine*, of the Angle I C E. The Angle F C B is call'd the *Inclination* of the Line F C to the Line B C; and the Angle F C D is the *Inclination* of the Line F C to the Perpendicular D C: That is, F C is inclined to B C in an Angle of 40 Degrees, and to D C in an Angle of 50 Degrees. Hence I L and I M are call'd the *Sines of Inclination* reſpectively.

23. Hence, when we ſay, *The Force of a direct Stroke is to that of an oblique one as Radius is to the Sine of Inclination*, we only mean, that thoſe Quantities are to each other as the Radius I C to the Sine I L, or I M, according as the Inclination is 40 or 50 Degrees. Alſo, when 'tis ſaid that *the centrifugal Force decreaſes from the Equator towards the Poles, in proportion of Radius to the Co-Sines of the Latitude*; no more is meant than this, that if the Radius C E repreſent the ſaid Force in the Equator E, and E I be any given Latitude, then will I M, the Co-Sine of the Latitude, repreſent the Force in that Latitude: Or, the Force decreaſes with the Length of the Line I M, as the Point I moves on from E to D.

24. In ſeveral Books we have Tables of Numbers which expreſs the Length of the Sine of every Degree and Minute

of the Quadrant, in ſuch equal Parts as the Radius CE or CI contains 100000. And ſince it is of the greateſt Uſe to know the Proportion of Radius to the Sine of every Degree, I have here ſubjoin'd a Table thereof, and a Specimen of its Uſe.

Deg.	Parts.	D.	Parts.	D.	Parts.	D.	Parts.	D.	Parts.
SINE of 1	1745	19	32556	37	60181	55	81915	73	95630
2	3483	20	34202	38	61566	56	82903	74	96126
3	5233	21	35836	39	62932	57	83867	75	96592
4	6975	22	37460	40	64278	58	84804	76	97029
5	8715	23	39073	41	65605	59	85716	77	97437
6	10452	24	40673	42	66913	60	86602	78	97814
7	12186	25	42261	43	68199	61	87461	79	98162
8	13917	26	43837	44	69465	62	88294	80	98480
9	15643	27	45399	45	70710	63	89100	81	98768
10	17364	28	46947	46	71933	64	89879	82	99026
11	19080	29	48480	47	73135	65	90630	83	99254
12	20791	30	50000	48	74314	66	91354	84	99457
13	22495	31	51503	49	75470	67	92050	85	99619
14	24192	32	52991	50	76604	68	92718	86	99756
15	25881	33	54463	51	77714	69	93358	87	99862
16	27563	34	55919	52	78801	70	93969	88	99939
17	29237	35	57357	53	79863	71	94551	89	99984
18	30901	36	58778	54	80901	72	95105	90	100000

25. The Uſe of this Table will be obvious from two or three Examples. It was obſerved, that the Power is to the Weight it ſuſtains on any Inclined Plane IC, as the Height of the Plane IL to the Length thereof IC; that is, as the Sine of the Plane's Inclination to the Radius. Suppoſe the Angle of Inclination ICE = 40 Degrees, then will the Sine IL be equal to 64278, and the Radius CI equal to 100000, which Numbers are as 64 to 100; therefore 100 Pounds will be ſuſtain'd on the Inclined Plane by a Power equal to 64 Pounds nearly.

26. Again; ſince EC = 100000 repreſents the *centrifugal Force* under the Equator, then will IM = 76604 (the Sine of 50 Degrees, and Co-Sine of 40) be as the ſaid Force in the Latitude of 40 Degrees: Which Numbers are as 1000 to 766; and ſuch is the Proportion of the Forces in thoſe two Places.

27. In the ſame Manner, if the Radius CD = 100000 expreſs the Force of any *direct Stroke*, then will the Sine IL = 64278 be expreſſive of the Force of an *oblique Stroke* in the Direction FC, every thing elſe being equal.

28. Again; ſince the Force of a direct Stroke is expreſs'd by CD = 100000, if it were required to find the Angle of Obliquity, ſuch that the Force of the Stroke ſhall be but one fourth Part ſo great, or 25000, look in the Table for the Number neareſt to this, and you will perceive it lie between 14 and 15 Degrees, and therefore about 14 Degrees and a half is the Angle required.

29. In the laſt Place: It was ſaid, That *the Force of the Wind on the Sail is proportional to the Squares of the Sines of the Angle of Incidence.* This may be illuſtrated by Numbers, thus: If the Sail be turn'd to the ſame Wind, firſt under an Angle of 45 Degrees, and then under an Angle of 30 Degrees; the Sine of the firſt Angle is (by the *Table*) 70710, and of the latter 50000, the Squares of which are 4999904100 and 2500000000, which Squares are as 50 to 25, or as 2 to 1; and therefore the Power of the Wind is twice as great upon an Angle of 45 Degrees, as upon an Angle of 30.

30. Becauſe the Square of the Sine of 45 Degrees is 5000000000, twice that Square will be 10000000000, which is equal to the Square of Radius 100000; 'tis evident the Sum of the Squares of the Sines of any two Angles above 45 Degrees will be greater than the Square of Radius; and therefore the Force of the Wind upon two oblique Sails, in that Caſe, will be greater than upon one Sail ſet direct before the Wind.

31. After the ſame manner, the Table of Sines may be applied to Calculation in any other Caſe of the like Nature, where the *Proportion of Radius and Sine of an Angle* is required to be expreſs'd or ſtated in Numbers. And ſince each Degree is divided into 60 equal Parts or *Minutes*, therefore the Sine of any Number of Degrees and Minutes alſo may be eaſily found by the foregoing Table, by thoſe who underſtand the Rule of Proportion.

32. I ſhall only further obſerve here, that as IL is the *Sine*, and LM the *Co-Sine*, of the Arch IE; ſo if on the extreme Point E of the Radius CE we raiſe a Perpendicular which ſhall cut the Line CI, continued, in F, then is the ſaid Perpendicular EF call'd the TANGENT of the Angle ICE, and the Line FC the SECANT of the ſame Angle. In like manner, the Perpendicular DG is the *Tangent* of the Angle DCI, and GC the *Secant* thereof; and therefore DG is the *Co-Tangent*, and GC the *Co-Secant* of the Angle ICE. I thought it neceſſary to acquaint the Reader with theſe Definitions, becauſe they ſometimes occur in Treatiſes of this Kind.

It is to be obſerved, that in order to turn a Ship about in the leaſt Time, or with the greateſt Celerity poſſible, the Rudder ought to make an Angle with the Stern of 54 Degrees, 44 Minutes: And alſo, that this is the Angle which the *Gates of a Lock* upon a River ought to make with the Sides of the River, in order to reſiſt the Water with the greateſt poſſible Force (XLVII.)

Pl. XIII. Fig. 8. (XLVII.) 1. If ABF be the Rudder of a Ship, AH placed in the oblique Situation FC, and the Water ſtriking againſt it in the Direction GC; let CE be Radius, then the Sine of the Angle of Incidence will be FE, and ſo the Force of the Water againſt the Rudder will be as $\overline{FE}^2$; but EF is reſolvable into the two Forces FD and DE, of which the latter is parallel, the other perpendicular to the Direction of the Ship's Courſe, and therefore FD is the only Part of the Force that compels the Ship to turn round. But EF is to FD (:: CE : CF) :: $\overline{EF}^2 : \frac{CF \times \overline{EF}^2}{CE}$, that is (putting $CE = a$, $CF = x$) as $\frac{aax - x^3}{a}$, and ſo x will be found (by the Method *de Maximis & Minimis*) equal to $\sqrt{\frac{aa}{3}}$; and therefore the Angle of Incidence ECF = 54°-44′, as before, when the Force of the Water againſt the Rudder to turn the Ship is a *Maximum*.

2. After a like Manner we determine the Angle of Poſition of the Gates AE, BE, of a Lock upon a River, (*viz.* the Angle BAE = ABE) in which the ſaid Gates ſhall reſiſt the Preſſure of the Water with the greateſt Force poſſible. For if upon AB we deſcribe the Semicircle ADB, and continue AE to D; then the Preſſure of the Water againſt the Gate AE will increaſe with the Length of the Gate, and the Reſiſtance of the Gate will decreaſe as the Preſſure increaſes, and therefore it will be on this Account inverſely as the Length of the Gate, or as $\frac{1}{AE}$.

3. Again, the Reſiſtance will be diminiſh'd as the Length of

Pl. XIII. Fig. 16. of the Gate increaſes, inaſmuch as the Strength of the Gate will be diminiſh'd in that Proportion, therefore it will be on this Account alſo as $\frac{1}{AE}$, conſequently the Reſiſtance of the Gate on both theſe Accounts will be as $\frac{1}{\overline{AE}^2}$.

4. Join BD and EC perpendicular to AB; then we have $\overline{AE}^2 : \overline{AC}^2 :: \overline{AB}^2 : \overline{AD}^2$; and here, becauſe $\overline{AD}^2$ decreaſes as $\overline{AE}^2$ increaſes, we ſhall have the Reſiſtance expreſs'd by $\overline{AD}^2$.

5. But this Reſiſtance is augmented by the Oblique Preſſure of the other Gate, which let be repreſented by BE; this Oblique Force BE is reſolvable into two Forces BD and DE; which latter, as it is parallel to the Gate AE, avails nothing, but the other BD being at Right Angles thereto, is wholly ſpent in reſiſting it; therefore the whole Reſiſtance the Water meets with from the Gate AE, is as $\overline{AD}^2 \times BD$.

6. This Expreſſion, therefore, is to be determined to a *Maximum*; in order to which, let $AB = a$, $BD = x$, and then $\overline{AD}^2 = aa - xx$, and ſo $\overline{AD}^2 \times BD = aax - x^3$, whoſe Fluxion $aa\dot{x} - 3xx\dot{x} = 0$, gives $x = \sqrt{\frac{aa}{3}}$, which ſhews the Angle BAE = 35°, 16′, as in the Examples above.

7. Since we are upon the Subject of *Maximums*, I ſhall here add Examples of two or three other Caſes of the ſame Kind, which 'tis hoped will be acceptable to the Curious, and yet not beſides the Purpoſe of Mechanical Gentlemen. Fig. 9. Let BB be a Piece of Wood placed horizontally, and ſupported by the Pieces AB, AB, which make a given Angle ABC with the former; it is required to find the Poſitions of two other Pieces AC, AC, given in Length, ſuch that they ſhall ſupport the Piece BB with the greateſt Force poſſible.

8. The Pieces AC, are fix'd in A and C ſo as not to ſlip, they are ſuppoſed to have no conſiderable Weight. Then make $BH = \frac{1}{2} AC$, and from the Points A and H draw the Lines AG, HK, at Right Angles to BB. If AC expreſſes the abſolute Strength of the Piece AC, then AG will expreſs the Strength with which it ſupports the Piece BB, as being perpendicular thereto. Now AG multiplied by the Lever (or Diſtance) BC from the Centre of Motion B, (which expreſſes the *Momentum* or Force of the Piece AC) ought to be a *Maximum*.

9. To this End, put AC $= a$, AG $= x$; alſo HK $= n$, and KB $= m$; then GC $= \sqrt{aa - xx}$, and becauſe of the ſimilar Triangles HKB, AGB, we have HK : KB :: AG : GB $= \frac{m}{n}x$, and ſo BC $= \sqrt{aa - xx} - \frac{m}{n}x$, and AG × BC $= x\sqrt{aa - xx} - \frac{m}{n}xx$, whoſe Fluxion made equal to nothing is $\dot{x}\sqrt{aa - xx} - \frac{xx\dot{x}}{\sqrt{aa - xx}} - \frac{2m}{n}x\dot{x} = 0$.

10. Therefore $\sqrt{aa - xx} - \frac{xx}{\sqrt{aa - xx}} - \frac{2m}{n}x = 0$;

Whence $aa - 2xx - \frac{2m}{n}x\sqrt{aa - xx} = 0$.

And ſquaring each Part } $a^4 - 4a^2x^2 + 4x^4 - \frac{4m^2}{n^2}a^2x^2 + 4\frac{m^2}{n^2}x^4 = 0$.

And multiplying by n^2 } $n^2a^4 - 4n^2a^2x^2 - 4m^2a^2x^2 + 4n^2x^4 + 4m^2x^4 = 0$.

But it is $4n^2 + 4m^2 = aa$, becauſe $\overline{BH}^2 = \overline{HK}^2 + \overline{KB}^2$,

Therefore $n^2a^4 - 4n^2a^2x^2 - 4m^2a^2x^2 + a^2x^4 = 0$.

And dividing by a^2 } $n^2a^2 - 4n^2x^2 - 4m^2x^2 + x^4 = 0$.

Whence again $n^2a^2 - a^2x^2 + x^4 = 0$, (becauſe $-4n^2 - 4m^2 = -aa$)

Hence by Tranſpoſition } $x^4 - a^2x^2 = -a^2n^2$.

And compleating the Square } $x^4 - a^2x^2 + \frac{1}{4}aa^4 = \frac{1}{4}aa^4 - a^2n^2$.

But it is $\frac{1}{4}aa^4 - a^2n^2 = aam^2 + a^2n^2 - a^2n^2 = a^2m^2$.

Therefore $x^4 - a^2z^2 + \frac{1}{4}aa^4 = a^2m^2$.

And extracting the Square Root } $x^2 - \frac{1}{2}aa = \mp am$.

Wherefore $x^2 = \frac{1}{2}aa \mp am$; and ſo $x = \sqrt{\frac{1}{2}aa \mp am}$.

11. Hence 'tis evident if m (= BK) $= 0$, in which Caſe the Angle ABC is a right one, then $x = \sqrt{\frac{1}{2}aa}$, and therefore the Angle ACB will then be half a right one, or 45 Degrees.

12. Since the two Values of x, *viz.* $\sqrt{\frac{1}{2}aa - am}$, and $\sqrt{\frac{1}{2}aa + am}$ being ſquared, and the Sum of thoſe Squares are equal to the Square of the Radius, *viz.* $aa - am + \frac{1}{2}aa$ $+ am$

$+am = aa$, therefore the two Angles, of which x is the Sine, are Complements to each other. Thus ſuppoſe the Angle A B C obtuſe, and $=$ 120 Degrees, or the Angle A B G $=$ 60 Degrees; then B K $= m = \frac{1}{4}a$, and ſo A G $= x = \sqrt{\frac{1}{2}aa - am} = \sqrt{\frac{1}{2}aa - \frac{1}{4}aa} = \frac{1}{2}a$, therefore the Angle A C B is $=$ 30 Degrees. But if the Piece A C be placed on the other Side A B, then $x = \sqrt{\frac{1}{2}aa + am} = \sqrt{\frac{3}{4}aa}$, and ſo the Angle A C B would in that Caſe be $=$ 60 Degrees, and conſequently equal to the Angle A B G.

13. If the Angle A B C, inſtead of being obtuſe, were acute, and the Complement to this, we ſhould have A G $\times$ B C $= x\sqrt{aa - xx} + \frac{m}{n}xx$ a *Maximum*, which would in Fluxions, give the ſame Value for x as before, *viz.* $\sqrt{\frac{1}{2}aa \mp am}$, which if ſubſtituted for x in the above Expreſſion, will give $\frac{2ann + aam \mp 2amm}{2n}$, where 'tis plain if $x = \sqrt{\frac{1}{2}aa - am}$, the greateſt Force will be $\frac{2an^2 + a^2m - 2am^2}{2n}$; but if $x = \sqrt{\frac{1}{2}aa + am}$, then the greateſt Force will be $\frac{2an^2 + a^2m + 2am^2}{2n}$. But $x = \sqrt{\frac{1}{2}aa - am}$ is the Sine of 30°, and $x = \sqrt{\frac{1}{2}aa + am}$ is the Sine of 60°; therefore the Piece B B is ſupported with the greateſt Force by the Piece A C when placed on that Side B on which the acute Angle is.

14. If A G be a Piece of Wood of an indefinite Length, and fix'd in A, ſo as to make a given Angle G A D with the horizontal Line A D; let it be required to find the Poſition of another Piece D E, given in Length, ſuch that it ſhall ſupport the Piece A G with the greateſt poſſible Force. To this End make A C $= \frac{1}{2}a$, and from B let fall the Perpendicular C B; then ſince the Angle A is given, the Ratio of C B to A B is given alſo, which let be as n to m; that is, let C B $= n$, and A B $= m$; from the Point D draw the Perpendicular D F $(= x)$ to the Piece A G. Then if D E $(= a)$ expreſſes the whole Force of the Piece D E preſſing perpendicularly, D F will expreſs that with which it ſupports A G; therefore D F multiplied by the Diſtance from the Fulcrum A, or Lever A E, ought to be a *Maximum*. Pl. XIII. Fig. 10.

15. Now from the ſimilar Triangles A C B, A D F, we have C B : B A :: D F : F A $= \frac{m}{n}x$, alſo F E $=$

$\sqrt{aa-xx}$, therefore $AE = \sqrt{aa-xx} + \frac{m}{n}x$, which multiplied by $DF = x$, is $AE \times DF = x\sqrt{aa-xx} + \frac{m}{n}xx$, whoſe Fluxion made equal to nothing will give $x = \sqrt{\frac{1}{2}aa \pm am}$, as before. If the Angle ADE be acute, the Point E will fall between F and A, and we ſhall have $AE = \frac{m}{n}x - \sqrt{aa-xx}$ and $AE \times DF = \frac{m}{n}xx - x\sqrt{aa-xx}$, which fluxed will give the ſame Value of x as before found.

16. In each Caſe, 'tis plain $x = \sqrt{\frac{1}{2}aa + am}$ = Sine of 60°, muſt be the *Maximum*, which if ſubſtituted in the Expreſſion $x\sqrt{aa-xx} + \frac{m}{n}xx$, (where the Angle ADE is oblique,) will give $\frac{2ann + aam + 2amm}{2n}$, = Force of DE which ſupports the Piece AG. But if $\sqrt{\frac{1}{2}aa + am}$ be put for x in the other Expreſſion $\frac{m}{n}xx - \sqrt{aa-xx}$, we ſhall have for the ſaid Force (when the Angle is acute) $\frac{aam + 2amm - 2ann}{2n}$, which is leſs than the other Forces; and therefore the latter Poſition is leſs advantageous than the former.

17. I can't conclude this Speculation of the *Maxima* and *Minima* of Quantities, without obſerving to the Reader, that though the Method of diſcovering thoſe by Fluxions is a Part of Knowledge which the *Mathematicians* have but lately acquired, and which they eſteem the Sublimity of human Science, yet this very Thing was imparted to the *Inſect Tribe* at the firſt Creation of Things; for by this very Method it is that Bees conſtruct the Cells of their Combs in which they depoſit their Honey.

Plate XI. Fig. 5. 18. Each Cell conſiſts of ſix plain Sides, which are all *Trapeziums*, but equal to each other. The Bottom of the Cell is contrived with three Rhombus's HKDI, DEFI, and FIHG, ſo diſpoſed as to conſtitute a Solid Angle at I, under the three equal Angles DIH, DIF, and HIF; each of which is double the *Maximum* Angle of 54°—44′ = DIK = DKI. Hence it comes to paſs, that a leſs Quantity of Surface is ſufficient to contain a given Quantity of Honey,

I SHALL conclude this Lecture with a few Words concerning WHEEL-CARRIAGES, the whole Doctrine whereof (as it ſtands on a *Mathematical Theory*) may be reduced to the following Particulars, *viz.* (1.) Wheel-Carriages meet with leſs Reſiſtance than any other. (2.) The larger the Wheels, the eaſier is the Draught of the Carriage. (3.) A Carriage upon four large Wheels, of equal Size, is drawn with leſs Force than with two of thoſe Wheels, and two of a leſſer Size. (4.) If the Load be laid on the Axle of the larger Wheels, it will be drawn with leſs Force than if laid on the Axis of the leſſer Wheels; contrary to the common Notion of *loading Carriages before.* (5.) The Carriage goes with much leſs Force on *Friction-Wheels*, than in the common way; all which will be confirmed by Experiments (XLVIII.) (XLIX.)

Honey, than if the Bottom had been flat, in the Proportion of 4658 to 5550, as I have found by Calculation; that is, $\frac{892}{4658}$, or $\frac{1}{5}$ Part of the whole, ſo far as the Figure of the Ends of the Cells extends in each, which fifth Part of Wax and Labour ſaved amounts to a vaſt deal in the whole Structure of the Comb. And if thoſe Creatures knew their Advantage, they could not more nicely keep up to the Rules of this ſublime Geometry.

19. The laſt Thing among the *Maxima*, that I ſhall mention, is, that if a Chain A B C be ſuſpended by its two Ends, it will ſink down in ſuch a Manner, by its Gravity, as to form the Curve A B C, call'd the *Catenaria*, which if inverted, would exhibit the beſt Form for an Arch of any other whatſoever. For the Demonſtration of this, we muſt refer the Reader to the Inventor Dr. GREGORY, in *Philoſ. Tranſ.* N°. 231. Plate XI. Fig. 6.

(XLVIII.) The THEORY of Wheel-Carriages is as follows: Let A P G E M be a Wheel, N D the horizontal Plane on which it moves, E F the Height of an Obſtacle over which it is to be drawn; the Wheel arriving at the Obſtacle, and touching the Top E, ſtands upon the Point G, and preſſes it with its whole Weight. Draw OEK, a Tangent to the Wheel in the Point E, and meeting the vertical Diameter AG produced, in O. Draw the Radius EC, and EH perpendicular to AG; and MC, *m r*, perpendicular to CE, and conſequently parallel to the Tangent OK. Laſtly, draw the Radius C*m*.

2. Since the Wheel gravitates in the Direction CO, let CO expreſs its Weight preſſing the Point G; this may be reſolved into two others CE and OE; of which CE preſſes the Top of the Obſtacle, and is wholly ſuſtain'd by it; the other Weight OE draws the Wheel down in a Direction parallel to the Tangent OK. Now let $W = CO$ Weight of the Wheel, $R =$ Radius, $H = EF$, the Height of the Obſtacle, and $x = OE$; then ſince $OE : CO :: HE : CE$, we have $x : W :: HE : R$, whence $x = \frac{W \times HE}{R}$; but, from the Nature of the Circle, $HE = \sqrt{AH \times HG} = \sqrt{AH \times EF} = \sqrt{2RH - H^2}$; therefore $x = \frac{W \times \sqrt{2RH - HH}}{R}$.

3. A Force juſt equal to this Weight x, and acting in Oppoſition to it, that is, drawing the Wheel upwards in the Direction CM parallel to EK, will juſt be able to make the Wheel reſt on the Top of the Obſtacle at E, without ſuffering any Part of its Weight to reſt on the horizontal Plane at G.

4. Now this Force muſt be increaſed if it acts in any other Direction but that of CM; for let it draw the Wheel in the Direction C*m*, between M and E, then the Force may be reſolved into two others C*r*, and *rm*, of which C*r* draws the Wheel directly againſt the Top of the Obſtacle E, and ſo is deſtroy'd by equal Re-action of the Point E; what therefore remains to draw it upwards in a Direction parallel to OK, is *mr*, which is leſs than C*m* or CM; and to be made equal thereto, (as it muſt be to ſupport the Wheel on the Top of the Obſtacle E) it muſt be increaſed in the Ratio of C*m* to *rm*, which let be as R to S (or as *Radius to the Sine* of the Angle which the Direction of the Force makes with CE). But 'tis plain, the Force *rm* cannot be increaſed, but the whole Force CM muſt be increaſed in the ſame Proportion; that is, when *rm* becomes $\frac{R}{S} \times rm$, CM will become

come $\frac{R}{S} \times CM = \frac{R}{S} \times \frac{W \times \sqrt{2RH - H^2}}{R} = \frac{W \times \sqrt{2RH-H^2}}{S}$.

5. In order that the Wheels may be drawn over the Obſtacle FE, it is neceſſary the Direction of the Force ſhould lie between CE and CA; for if it were in the Direction CE, it could only draw the Wheel upon or againſt, but not over the Obſtacle; and if it acted in the Direction CA, it would not make it preſs againſt the Obſtacle, and conſequently, could never draw it over.

6. Let $F = \frac{W \times \sqrt{2RH - H^2}}{S}$ = the Force ſufficient to ſuſtain the Wheel on the Top E of the Obſtacle, 'tis evident if W, R, H continue the ſame, $F = \frac{1}{S}$; that is, the Force will always be leſs as the Sine of the Angle E C*m* is greater, till *rm* = CM, when the ſaid Force will be leaſt of all.

7. If W and H be given, or always the ſame; then $F = \frac{\sqrt{2R-1}}{R}$ (for here we ſuppoſe the Force applied to draw in the moſt advantageous Direction, *viz.* CM, where S becomes equal to R.) If therefore the Radii of four Wheels be 1, 2, 3, 4, then will $\frac{\sqrt{2R-1}}{R}$ be 1, $\frac{\sqrt{3}}{2}$, $\frac{\sqrt{5}}{3}$, $\frac{\sqrt{7}}{4}$, or as the Numbers 1000, 866, 745, 661. From hence 'tis evident how much leſs Force is neceſſary to draw a large Wheel over any Obſtacle than a leſſer one, when the Weight of the Wheels are the ſame.

7. If the Height of the Obſtacle H be indefinitely ſmall and given, in which Caſe the Tangent OK will coincide with the Horizontal Line ND, and the Point E with the Point G, very nearly; and the Direction of the Force be parallel to ND; then becauſe H^2 is inconſiderable we reject it, and the Expreſſion for the Force will be $F = \frac{W \times \sqrt{2R}}{R}$, (for H is given, and therefore not expreſs'd). And if W be alſo given, the Force will be $F = \frac{\sqrt{2R}}{R}$, or $F = \frac{\sqrt{R}}{R}$, becauſe 2 is a given Quantity; but $\frac{\sqrt{R}}{R} = \frac{1}{\sqrt{R}}$: therefore

F =

$F = \frac{1}{\sqrt{R}}$; that is, in caſe of rough uneven Surfaces the Force to draw the Wheel will be inverſely as the Square Root of the Radius or Diameter of the Wheel. Thus if three Wheels are in Diameter as 1, 4, 9; the Force to draw them will be as 3, 2, 1.

8. If $H = o$, that is, if the horizontal Plane on which the Wheel moves be perfectly ſmooth or plain, then the Quantity $\frac{W}{S}\sqrt{2RH - H^2} = o$; whence it appears that no Force is required to move an heavy Body on an horizontal Plane which is perfectly even.

9. If the Height H of the Obſtacle be proportional to the Radius of the Wheel, that is, if H be as R, and the Force draw in a Direction parallel to O K; then becauſe $\frac{\sqrt{2RH - H^2}}{R} = \frac{\sqrt{2RR - RR}}{R} = \frac{\sqrt{R^2}}{R} = 1$, therefore $F = W$, *or the Force will be proportional to the Weight of the Wheel.*

10. If the Direction of the Force be parallel to the horizontal Plane, that is, if Cm be parallel to N D, then becauſe the Angle mCE is (in that Caſe) equal to the Angle CEH, their Sines will be equal, that is $rm = CH = R - H$; therefore the Expreſſion of the Force (*Art.* 6.) will become $F = \frac{W \times \sqrt{2RH - H^2}}{R - H}$; and if the Height H be given it will be $F = \frac{W \times \sqrt{2R - 1}}{R - 1}$.

11. From the Expreſſion $F = \frac{W \times \sqrt{2RH - H^2}}{S}$, we have this Equation $\frac{F}{W} = \frac{\sqrt{2RH - H^2}}{S}$, which gives the following Analogy $F : W :: \sqrt{2RH - H^2} : S$. That is, *The Force is to the Weight as the Sine of the Angle* ECH (*viz.* EH) *is to the Sine of the Angle* mCE, *which the Line of Direction makes with the Line* EC.

12. If the Obſtacle is capable of being depreſs'd or born down by the Wheel; the larger the Wheel the greater will be the Force to do this; for ſince CE repreſents the whole Force with which the Wheel bears upon the Obſtacle, and this is reſolvable into the two Parts CH and HE, of which the former CH being parallel to EF tends to preſs it down, it will be expreſs'd by $R - H$, and ſince H is given, the depreſſing Force will be as $R - 1$, and therefore will increaſe

with R, or the Radius of the Wheel.

13. If the Obſtacle be ſuch, as that it can neither be ſurmounted nor depreſs'd, but muſt be driven forward, the Force to do that will be expreſs'd by $HE = \sqrt{2RH - H^2}$, which, ſince H is given, will be as $\sqrt{R-1}$; but $\sqrt{R-1}$ will be greater in proportion to R when R is ſmall than when it is greater. Thus if $R = 2$, then $\sqrt{R-1} = 1 = \frac{1}{2}R$; but if $R = 5$, then $\sqrt{R-1} = 2$, which is leſs than $\frac{1}{2}R$; and if $R = 10$, then $\sqrt{R-1} = 3$, which is leſs than $\frac{1}{3}R$; ſo that in this Reſpect ſmall Wheels have the Advantage of large ones. But this Caſe ſeldom happens.

Pl. XIII. Fig. 12.

14. The principal Advantage of ſmall Wheels is, that in them the Line of Traction is not parallel to the Horizon as CK, but inclined thereto in a certain Angle, as CM, making with the Horizon the Angle MCK; now if CM be parallel to the Tangent OK, the whole Force will be employ'd to draw the Wheel over the Obſtacle EF; whereas, if the Line of Traction were parallel to the Horizon, the Line CK might then expreſs the Force, which being reſolved into the two Forces CE and KE, ſhews that the Part CE draws the Wheel directly upon the Obſtacle, and is therefore loſt by its Re-action; and only the Part KE remains to draw rhe Wheel over the ſaid Obſtacle; and conſequently the horizontal Direction is not the beſt, unleſs upon a ſmooth and even Plane, where no Obſtacles and Aſcents are to be ſurmounted.

15. From what we have ſaid, it is evident that a ſmall Wheel, whoſe Radius is KE, and the Line of Traction parallel to OK, is equivalent to a large Wheel whoſe Radius is CK, and the Line of Traction parallel to the Horizon ND; but EK : CK :: HB : CB :: CI : CE; that is, the Radius of the ſmaller Wheel is to that of the larger, as the Co-ſine of the Angle ECG to Radius.

Fig. 13.

16. Though the Force employ'd be never wholly ſpent in drawing, but when the Direction is CN, parallel to the Plane on which the Carriage moves; yet if it be applied in that oblique Direction CM, where the Breaſt of the Horſe is higher than the Axle of the Wheel C, in which Caſe only the Part BM is employ'd in drawing, the other Part CB is not however wholly loſt, but is acting contrary to the Gravity of the Carriage, and by that means leſſens ſomewhat of the Weight of the Load, by lifting it (as it were) along, for in this Caſe the Horſe not only *draws*, but alſo *carries* along (in ſome meaſure) the Load.

17. On the contrary, if the Axle of the Wheel be higher

from the Plane than the Breaſt of the Horſe, that is, if the Power be applied in the oblique Direction CO, then the Part DO draws the Load along, but the Part CD acting perpendicular on G draws the Load directly againſt the Plane, and thereby increaſes the Weight of the Load, or the Difficulty of drawing it; and is therefore the worſt Direction in which the Force can be any how applied in drawing. Hence it follows, that (*cæteris paribus*) where the Wheels of a Carriage have their Radius equal to the Height of the Horſe's Breaſt, or Traces, the Draught will be eaſieſt of all; and Wheels, whoſe Radius's are leſs than that, are better than thoſe Wheels whoſe Radius's exceed it.

Pl. XIII. Fig. 14. 18. A ſmall Wheel BDC will deſcend farther down between two Obſtacles DF and CE than a larger Wheel ADC, as is evident from the Figure; and therefore the Draught is more difficult, and ſubject to greater Shocks or Jolts, with the ſmall Wheel, inaſmuch as its Axis, and conſequently the Weight of the Load, muſt be raiſed to a greater Height in order to get from between them.

Fig. 15. 19. Alſo in ſoft or yielding Ground, a ſmall Wheel will ſink deeper than a larger Wheel charged with the ſame Weight. Thus ſuppoſe ABC be the Plane of the Road, which is ſo ſoft as to permit the ſmall Wheel to ſink down to E, then the Weight muſt overcome the Reſiſtance of as much Earth as the Wheel in ſinking has diſplaced, that is, as much as is equal to the Segment HED; if now the larger Wheel were to ſink to the ſame Depth, it muſt overcome the Reſiſtance of ſo much Earth as is equal to the Segment AEC, which is greater than HED, which is impoſſible, becauſe the ſame Weight can overcome but an equal Reſiſtance in either Wheel, therefore the larger Wheel will not ſink ſo deep as the ſmaller, and ſo will be drawn more eaſily.

20. Since the Ends of the Axles, and the Holes in the Naves of large and ſmall Wheels are equal, and ſince, in paſſing along, the ſmall Wheel (to meaſure the ſame Length of Road) muſt turn round upon its Axis oftner than a large one; it follows, that there will be a greater Quantity of Friction in the ſmall Wheel than in the larger, and that in the ſame Proportion as it is leſs, or as its Velocity is greater. Hence on Account of this, and ſeveral other like Cauſes, ſmall Wheels are much more ſubject to be out of Repair, to be at Fault, and to be worn quite out, than larger ones.

21. Next to the Conveniency mention'd *Art.* 14. that of turning the Carriage in a ſmaller Compaſs, with ſmall Wheels, than can be done by large ones, has made them more ne-

ceſſary in Waggons and Coaches, for becauſe of their Smallneſs they can be brought near to, and partly under the Sides of the Carriage, and ſo their Axles lying more obliquely under the Bed of the Carriage, admit it to be turn'd about with greater Eaſe.

(XLIX.) 1. To conclude theſe Mechanical Lectures, I ſhall give the Reader a ſhort View of the famous Controverſy that has ſo long (about 60 Years) ſubſiſted between the *Engliſh* and *French* Philoſophers on one Side, and the *Dutch*, *Germans*, and *Italians* on the other. The Subject was, *Whether the Force of Bodies in Motion, ſtriking each other, be proportional to the Simple Velocity of the Motion, or to the Square of the Velocity?* The *Engliſh* and *French* maintain the former; the other Gentlemen the latter.

2. This Diſpute firſt commenced between Mr. *Huygens* and the Abbot *Catalan*, about the Force of Oſcillating Bodies; it continued ſome Time between theſe two Gentlemen; at laſt, another Subject of the ſame Kind engaged the ſaid Abbot with the famous *Leibnitz*, who is to be eſteem'd the firſt Author, that plainly declared, in expreſs'd Words, *That the Forces of Bodies were as their Maſſes multiplied by the Square of the Velocity*. *Catalan*, and afterwards Mr. *Papin*, anſwer'd *Leibnitz*; he reply'd again, and ſeveral Papers were written on the Subject.

3. It then became a Matter of general Enquiry, and the Philoſophers of every Nation began to conſider which Side to be of, and whether they ſhould declare for the *old* or for the *new Opinion*. Hovever, they did here, as they do in Relilion, go by a whole Nation together; the common Herd of Philoſophers following the Dictates of their Leaders. Thus *Leibnitz*, *Polenus*, *S'Graveſande*, and *Muſchenbroek* lead the *German* and *Dutch*; *Papin*, *Mairan*, &c. the *French*; and *Pemberton*, *Eames*, *Deſaguliers*, *Clarke*, &c. the *Engliſh*.

4. In this Controverſy alſo, as in thoſe of Religion, the Opponents diſputed with very great Warmth, wonder'd at each other's Slowneſs of Apprehenſion, or Backwardneſs of Belief; and I wiſh I could ſay, that they had always obſerv'd ſuch an impartial, free, and generous Behaviour and Stile of Expreſſion, as the Dignity of Philoſophy demands. We ſhould then perhaps have found Dr. *Deſaguliers* better employ'd than in blaming Dr. *Samuel Clarke* for *uncivil Treatment*, for *rude and impertinent Expreſſions*, &c. in regard to thoſe who defend the new Opinion of the *Square of the Velocity*.

5. As it uſually falls out in other Caſes, ſo here when Men find themſelves preſs'd with Difficulties and Abſurdities

in their Schemes and Notions, they have recourſe to the Subtilties of Metaphyſical Diſtinctions, though ſeldom to any good Purpoſe. Thus when it appear'd too plainly by all Experiments, and even to common Senſe, that the natural Force of Bodies was proportional to the ſimple Velocity and Maſs of Matter conjointly; we were told 'twas neceſſary to diſtinguiſh the Force of Bodies into two Kinds, *viz.* the *Vis Viva* or *Living Force*; and the *Vis Mortua*, or the *Dead Force*.

6. By this *Vis Viva*, or living Force, we were to apprehend that which reſulted from the viſible Action of one Body upon another, as that of a falling Weight; but the *Vis Mortua* or dead Force was to be underſtood of that which was deſtroy'd by a contrary Agent, as a Weight in one Scale of a Balance is kept from deſcending by a Counter-Poiſe in the other Scale. But unluckily for the Author of this Refinement, both thoſe Forces appear, even by the Balance, to be in the Ratio *of the ſimple Velocity into the Maſs*.

7. Thus for the living Force, if on the proportional Balance you place 4*lb.* at the Diſtance of 6 Inches on one Side, and 2*lb.* at the Diſtance of 12 Inches on the other Side, and if the Balance be put into Motion, they will each of them have a *Vis Viva*, or an active Force, becauſe they will keep the Beam in Motion for ſome time, till, by militating, they murder or kill each other; and then ſurely enough they become *Vires Mortuæ*, or *Dead Forces*.

8. But let us ſee how they exerted their Power while living. One (A) acted againſt the other (B), with the Gravity of every Particle in a Maſs of 4*lb.* and with the Velocity in each Stroke that 6 Inches diſtance could give; but ſince all allow the Weight to be as the Maſs of Matter, and the Velocity as the Diſtance from the Centre of Motion, therefore all the Force which A exerted was as $4 \times 6 = 24$. In the ſame Manner it is ſhewn the whole Force of B acting againſt A, was as $2 \times 12 = 24$; that is, in each Caſe, the Force was as the Velocity into the Maſs of Matter. And becauſe they were equal they deſtroy'd each other.

9. But had thoſe Combatants A and B been arm'd with Forces proportion'd to the Squares of the Velocities, that of A would have been but $4 \times 6 \times 6 = 144$; whilſt B would have had a Force equal to $2 \times 12 \times 12 = 288$; by which he would have demoliſh'd A at one ſingle Stroke, and been the ſurviving Conqueror.

10. Let us now conſider theſe two Bodies A and B as dead, and ſee what Forces they exert (*Verbo detur Venia*) in that State. In order to this they muſt be hang'd upon the Arms of the Balance, one on each Side, till they are

dead, *i. e.* motionleſs; but this will not happen till their Diſtances from the Centre of Motion become reciprocally proportional to their Maſſes of Matter; and then 'tis plain the Caſe is the ſame as before; for the dead Force of A will be as $4 \times 6 = 24 = 2 \times 12 =$ dead Force of B. Whereas if theſe Forces were as the Maſſes into the Square of the Velocities, A of 4 *lb.* and B of 2 *lb.* ought to die at the Diſtances 6 and 8,4 Inches from the Centre reſpectively; but if the Experiment be tried there, ſuch ſtrong Symptoms of Life will be found in A, as manifeſt the Falſity of this Hypotheſis.

11. Being driven from this Subterfuge, they ſeek another in a critical Diſtinction between *Force and Preſſure*. *Preſſure*, ſay they, is the Power with which Bodies act by Means of Inſtruments; thus a Weight in one Scale acts againſt the Weight in another by Preſſure; but the Power by which Bodies act on each other alone, is properly call'd *Force*, as when one Stone ſtrikes another by falling on it, or a Hammer ſtrikes an Anvil. The former they allow is proportional to the Velocity and Maſs of Matter conjointly; but the latter, they ſay, is as the Maſs multiplied by the Square of the Velocity.

12. But this *Eclarciſſement* boots them as little as the former, if they mean a momentary Impact or Stroke, or ſuch whoſe Effect is produced in a Moment of Time. For in ſuch a Caſe we ſay, the Stroke is in Proportion to the Maſs of Matter, and alſo to the Degree of Velocity, and therefore as both conjointly; but we deny there is any other Source of Power from whence a Body can derive any Force for producing a momentary Effect: Nor have any of thoſe Gentlemen been able to ſhew there is, though ſome (bewilder'd in the Labyrinth) have attempted it, but in how weak and ridiculous a Manner may be ſeen in a Piece intitul'd, *De Conſervatione Virium vivarum*, &c. And 'tis worth the Reader's while to ſee the jocular Confutation of this ludicrous Piece by *Phileleutherus Londinenſis*.

13. If they ſay, they would not be underſtood of momentaneous Effects, but ſuch as are produced in Time, then they have no Antagoniſts. And 'tis plain from their Experiments, that this is their Meaning after all. For the Experiments which they ſo much inſiſt on, are ſuch as are made by letting Bodies fall on ſoft Subſtances, as Clay, Butter, *&c.* or by the Action of Bodies on yielding Springs, or ſpringy Bodies; in all which Caſes, the Effect is not produced inſtantaneouſly, but in Time; the Clay takes Time to recede, the Spring to bend, *&c.*

14. If the Time be taken into Conſideration, then every Body muſt know that the Effect will be proportional to the Intenſity of the Cauſe, and the Time of its Continuance: Thus, for Example, the Effect of Gravity on Bodies left to themſelves cauſes them to deſcend; their Deſcent therefore will be proportionate to the Power of Gravity upon them, and therefore greater in thoſe which fall freely, than in others which deſcend on Inclined Planes in a given Time, where Part of the Force of Gravity is deſtroy'd by the Re-action of the Plane. But ſince the Action of Gravity is not inſtantaneous, but continual, therefore its Effect, *i. e.* the Deſcent of the Body will be greater in Proportion to the Time of its continuing to act on the Body, ſeparately conſider'd from its intrinſic Force. But this Force is as the Velocity produced by it in a given Time; alſo the Time is as the Velocity, when the Force is given; therefore the Effect, *viz.* the Deſcent of the Body, is as the Square of the Velocity.

15. Or in Symbols thus; let $E =$ the Effect produced by any Power $= P$, acting in any Space of Time $= T$, upon any Body $= Q$, moving with any Degree of Velocity V. Then 'tis plain, E will be as $Q =$ Maſs of Matter in the Body, when P, T, and V, are given; alſo the Effect E will be as the Intenſity of the Power P, when T, Q, and V are given; again, when P, Q, T are given, E muſt be as V; and laſtly, we ſhall have E as T, when P, Q, V, are given. Therefore when neither of theſe are given $E : Q \times P \times V \times T$; and in caſe of any different Effect, let $e : q \times p \times v \times t$.

16. Then ſuppoſing the Bodies equal, *viz.* $Q = q$; we have $E : e :: PVT : pvt$. Thus the ſame or two equal Bodies, deſcending on two Planes unequally inclined, will deſcend thro' Spaces, which will be as the different Powers of Gravity P and p, as the different Velocities V and v in any Point of Time, and as the different Times T and t of their Deſcent, conjointly.

17. If not only the Bodies Q and q, but alſo the Powers P and p which actuate them, be ſuppoſed equal, or $P = p$; then $E : e : TV : tv$. Thus in equal Bodies falling freely by Gravity, the Effect, or the Space which they deſcribe, will be as the Time T and t of the Falls, the Velocities V and v at any Point of Time in the Fall. And becauſe in this Caſe the Velocity is always proportional to the Time, that is, $T : t :: V : v$; if we multiply each Ratio by the ſame Ratio $V : v$, the Analogy will ſtill be the ſame, *viz.* $TV : tv :: V^2 : v^2$; whence $E : e :: V^2 : v^2$; or the Effect of falling Bodies in deſcribing the Space, will be as the Square of the Velocities.

18. If we ſuppoſe the Times given, that is, $T=t$, then (the Reſt as before) we have $E:e:V:v$; that is, the Forces or Effects of equal Bodies, falling in equal Times, are proportional to the *ſimple Velocities*.

19. If $V=v$, or the Velocities given; then $E:e::T:t$. Thus if two equal Bodies lying on an horizontal Plane, receive a Stroke each from Springs of equal Force, then will the Velocities be equal in every Part of the Motion, and the Effects E and e, in this Caſe, being the Spaces deſcribed, will be as the Times of their Motion.

20. Hence we have ſeen every poſſible Caſe wherein the Forces of Bodies or their Effects can be ſuppoſed to vary; and 'tis plain there is none where the Force is as the Square of the Velocity but where the Time is concern'd; or where ſome one of the Factors Q, P, V, T, is proportional to the Velocity; thus in Spouting-Water, becauſe Q is always as V, therefore, tho' T be given, E will be as QV, or as V^2; for P alſo in this Caſe is given, the Power of Gravity which gives Motion to the Particles of the Fluid being always conſtant.

21. Nor have they any Experiment which ſhews the Force or Effects of Bodies proportional to the Squares of the Velocities, but where the Time in which the Effect is produced ought to be conſider'd. Thus in their famous Experiment of Cavities, form'd in ſoft Clay by falling Bodies, 'tis true, thoſe Cavities are the Effect of the falling Bodies, and proportional to the Squares of the Velocity. But what is this to the Purpoſe, unleſs they will ſay, thoſe Pits are inſtantaneouſly produced, which I believe none will pretend to do?

22. 'Tis evident, this Cavity muſt be proportional to the Quantity of Matter put in Motion in the Clay, by the falling Body; but this is proportional to the Velocity on two Accounts; the firſt is, that every Particle which comes in Contact with the Striking Body, receives a Stroke proportional to the Velocity, therefore alſo the Number of Particles which each of theſe can move, will be as the Force of the Stroke, or as the Velocity of the falling Body. Secondly, among yielding Particles, the Number which the falling Body can apply to in a given Time, will be as the Velocity; conſequently the whole Number of Particles which can be moved, both immediately by the Body itſelf, and by the Motion communicated to the Particles, will be as the Square of the Velocity; and therefore the Cavity will be in the ſame Ratio alſo.

23. We hence obſerve, the ſame Method of Reaſoning may be applied to ſoft and yielding Subſtances, as has always

vity, and Preſſure of Fluids. A Fluid is a Subſtance whoſe Particles yield to the leaſt partial Preſſure, or Force impreſs'd. And hence 'tis reaſonable to infer, that thoſe Particles muſt be exceeding ſmall, ſmooth, round, and ponderous Bodies; and obſerve all the Laws of Motion and mutual Action in common with thoſe of all other Matter (L). The following are the Laws and Properties of Fluid Bodies.

(L) 1. That the Characters of a Fluid intended in this Definition are all of them neceſſary, will appear from what follows. The firſt is, that they are *exceeding ſmall*; for that the Smallneſs of the Particles conduces to *Fluidity*, is evident from hence, that the Points of Contact between Spheres are in Proportion to their Diameters, and therefore grow leſs with the Spheres, though not in the ſame Proportion. And the leſs the Points of Contact, the leſs will be the Coheſion, and of courſe the greater the Diſpoſition of the Particles to Fluidity.

Plate XV. Fig. 1.

2. To illuſtrate what I have ſaid, let ABCD be a Sphere inſcribed in a Cylinder EFGH; it is demonſtrated by Geometers, that the Superficies of the Sphere is equal to the curved Surface of the Cylinder, and that the Superficies of any Segment of the Sphere LDM is equal to the correſpondent Surface IKGH of the Cylinder; if therefore the Line IK move on till it coincides with HG, the Superficies of the Segment of the Sphere LDM will become the Point of Contact with the Plane HG, and that of the Segment of the Cylinder IKGH will become the Periphery of the Baſe of the Cylinder; but this Periphery of the Baſe will be as the Diameter HG of the Cylinder, which is equal to the Diameter of the Sphere AC; therefore the Point of Contact of the Sphere with the Baſe of the Cylinder, or the Plane which it touches, will be proportional to its Diameter; whence it follows, that the Point of Contact between two Spheres will be in Proportion to their Diameters.

3. The *ſecond Characteriſtic* of the Particles of a Fluid is, that they are *ſmooth*; by this means they become lubricous, and apt to ſlide or move by each other with the greateſt Fa-

I. *All Fluids are incompreſſible, except Air*; or, *they cannot by any Force be compreſs'd into a leſs Space than what they naturally poſſeſs*; as is proved by the *Florentine Experiment* of filling a Globe of Gold with Water, which, when preſs'd with a great Force, cauſes the Water to tranſude or iſſue through the Pores of the maſſy Gold, in Form of Dew, all over its Surface. (LI)

cility and Freedom; and therefore their Diſpoſition to *Fluidity* is proportionably promoted and augmented; for want of this Quality, though the Particles of Matter were poſſeſs'd of all others mention'd in the Definition, yet would they never conſtitute a Fluid; ſince nothing is a greater Obſtruction thereto, than Aſperity or Roughneſs of the Superficies.

4. A *third Characteriſtic* is their *Roundneſs or Sphericity*; for the more ſpherical the Particles, the fewer Points they touch each other by, and the leſs the Points of Contact are; on both which Accounts the Attraction of Coheſion will be diminiſhed, and their Diſpoſition to Fluidity or Volubility will be increaſed.

5. The *fourth Characteriſtic* in the Definition of the Particles of a Fluid is, that they are *ponderous or heavy Bodies*. By this Property, Fluids do not only gravitate in common with all other Sorts of Bodies, but alſo they derive from hence a Power or Force of Preſſure peculiar to themſelves; and by that means act upon Solids in a very different Manner than that in which Solids act upon each other, as will be ſhewn farther on.

6. The ſurprizing Subtilty or Minuteneſs of the Particles of Fluids deſerves our farther Speculation and Remark. So ſmall are they, as to eſcape the Sight aſſiſted by the beſt of Glaſſes; that they freely paſs through the Pores of the denſeſt Matter, as *Gold*, &c. that they become inviſible in Vapour; and ſo light as to riſe in Air, yea, in Air that is greatly rarified and attenuated. And it is well known, that Water has its Particles ſo very ſubtile as to paſs by Ways that Air will not, and perhaps is exceeded by nothing in this wondrous Property, but the Particles of Fire and ardent Spirits. Pl. XV. Fig. 2.

(LI) 1. This Experiment muſt be made with a Globe, for this Reaſon, becauſe a Sphere contains a greater Quantity of

II. *All Fluids gravitate, or weigh, in proportion to their Quantity of Matter*; and that not

Matter, under the ſame Superficies, than a Body of any other Figure whatſoever. This may be made appear in the following manner; let A B, B C, C D, and A D be four Lines, of which the three firſt are of a given Length, and the fourth variable; and let it be required to diſpoſe them in ſuch a Manner as to comprehend the greateſt Area poſſible.

2. Let A B and B C make a given Angle A B C; then will the triangular Space A C B be given. Upon C, with the Radius C D, deſcribe the Semicircle *a* D *e*, then 'tis plain the other Part of the Area, or the Triangle made by the Lines A C, C D and A D, will be greater or leſſer, according to the Poſition of the Line C D; for as 'tis made C D or *c d*, the variable Line will be A D or A *d*; and the Triangle form'd will be A C D or A C *d*, whoſe Area will be $\frac{AC}{2} \times$ C D, or *c d*, (*c d* being drawn perpendicular to A C produced.)

3. Now 'tis evident this Triangle will be greateſt, when *c d* is a *Maximum*, that is, when it becomes C D; and conſequently the Triangle A C D is the greateſt poſſible when C D makes a Right Angle with A C; and in this Caſe the Point C is in a Semicircle deſcribed on A D as a Diameter. Therefore, alſo the other Angle B, being in the Semicircle deſcribed on the ſame Diameter A B, will make the other Part of the Trapezium a *Maximum*; and ſo the whole Trapezium A B C D inſcribed in a Semicircle, will be greater than any other, whoſe three Sides A B, B C, C D, are the ſame.

4. Since what has been demonſtrated of the Trapezium, is true of any other Polygonal Figures (becauſe they may be reſolved into *Trapezia*) and ſince the Sides of a Polygon, when infinitely ſmall, do coincide with the Circle; therefore the Circle is the moſt capacious Figure, or contains the greateſt Area under the ſame Periphery.

5. For Example; ſuppoſe a String C were diſpoſed into the Form of a Circle; then as 22 : 7 :: C : $\frac{7C}{22}$ = Diameter of the Circle; the Radius therefore is $\frac{7C}{44}$, and ſince the Area of a Circle is equal to the Periphery multiplied into half

only in the Air, or *in Vacuo*, but *in proprio Loco*; or, a Fluid weighs the ſame communicating with a Quantity of that Fluid, as *in Vacuo*; which all Philoſophers (till very lately) have denied (LII).

the Radius, therefore $C \times \frac{7C}{88} = \frac{7CC}{88} =$ Area of the Circle. Again, ſuppoſe the ſame String diſpoſed in the Form of a Square; the Side would be $\frac{C}{4}$, and the Area $= \frac{CC}{16}$; hence the Area of the Circle would be to that of the Square as $\frac{7CC}{88}$ to $\frac{CC}{16}$, or as $\frac{7}{88}$ to $\frac{1}{16}$, that is, as 14 to 11.

6. Now ſince a Sphere may be conſider'd as made up of circular Areas, it plainly follows, that a Sphere will contain a greater Quantity of Matter under the ſame Surface than any other Solid. Thus a Sphere will be found by Calculation to be more than twice as big as a Cube of the ſame Surface; and therefore the Water contain'd in the Globe muſt, when the Figure of the Globe was alter'd, be either compreſs'd into a leſs Space, or make its Way through the Pores of the Globe, as in the Experiment we find it will; which therefore evinces its *Incompreſſibility*.

7. Some Philoſophers are inclined to think that Water is not abſolutely incompreſſible, or that the Particles thereof do not touch each other; but are kept at a Diſtance by a Centrifugal Force ſuperior to any Force we can apply in compreſſing them. But becauſe this is an *Hypotheſis* that has ſcarce any thing more than bare Poſſibility to recommend it, at leaſt nothing is offer'd to render it probable or neceſſary; therefore we are not to admit it among the Principles of the *Newtonian* Philoſophy.

(LII) 1. That Fluids gravitate, or are heavy, in the ſame Manner with Solids, is evident, becauſe the Earth's Attraction, which is the Cauſe of Gravity, equally affects the Particles of all Sorts of Matter; and therefore excites the ſame Endeavour or Tendency towards the Centre of the Earth in the Particles of a Fluid, as in thoſe of a ſolid Body; and this is what we call their *Abſolute Gravity*.

2. Now ſince in Fluids of the ſame Kind, as Water, all the Particles are reaſonably ſuppoſed equal and alike in all Circumſtances, they will be all equally affected by Attraction, and therefore have among themſelves an equal Tendency to-

III. *From the Gravity of Fluids ariſes their Preſſure, which is always proportional thereto*; and ſince we may ſuppoſe all the Particles of a Fluid to have equal Bulk and Weight, the Gravity of the Fluid, and conſequently its Preſſure, *will be*

wards the Earth's Centre. Whence, ſince they gravitate equally, if they are equally obſtructed in their Deſcent (as by the Bottoms of Veſſels, *&c.*) they will all retain the ſame Poſition among themſelves, as if they were affected by no Kind of Power at all; and thus they are ſaid to be *relatively at Reſt*, or in a State of *Quietus* among themſelves.

3. Since no one Particle of the ſame Fluid has a greater Share of the attracting Power than another, no one will tend to deſcend before another; and therefore among the Particles of the ſame Fluid, there is no ſuch Thing as we call *Relative* or *Reſidual Gravity*, which is nothing but *the Exceſs of Gravity, by which one Body tends downwards more than another*, as will be more fully explain'd hereafter (See *Annotat.* LVIII.)

4. Now becauſe Philoſophers found that a Bucket of Water in Water weigh'd nothing (that is, that there was *no Relative Gravity in Water*) they very ſtrangely inferr'd there was no *Abſolute Gravity* of any Part or Particle of Water, whilſt it remain'd in Water, but only became heavy when taken out, or ſeparated from the reſt. But their Miſtake is eaſily evinced by the following Experiment; let a Bottle or Vial with Shot in it to make it ſink in Water (when cloſe corked) be hung at the End of a nice Balance, and then immerſed into a Jar of Water; while thus hanging in Water, let it be counterpoiſed very exactly by Weights put into the Scale at the other End. Then pulling out the Cork the Water will ruſh into the Bottle and deſtroy the *Equilibrium*, by cauſing the End of the Balance to deſcend; which will be a plain Proof *that Water has Weight in Water.*

5. But to carry this Experiment farther; let now the *Equilibrium* be again reſtor'd, by adding more Weights to thoſe in the Scale; then taking the Bottle out of the Water, let the Water be pour'd out, and weigh'd, and the Weight will be found exactly equal to that which was added to the Scale to reſtore the *Equilibrium* above-mention'd. This ſhews *that Water (and all Fluids) weigh the ſame in their own Element as out of it.*

always proportional to the Altitude or Depth thereof. Whence the Weight and Preſſure of Fluids on the Bottoms of Veſſels, *&c.* muſt be equal.

IV. *The Preſſure of Fluids upwards is equal to the Preſſure downwards*, at any given Depth. To illuſtrate this, and the foregoing Propoſition, let ABCD be a Veſſel of Water, whoſe Altitude EF ſuppoſe to conſiſt of a Column of 10 aqueous Particles: Then, 'tis evident, the firſt or uppermoſt Particle 1 can affect the next Particle 2 only by its Weight or Preſſure, which therefore is as 1; and ſince that Particle 2 is immoveable, and Action and Re-action equal and contrary, the ſaid Particle 2 will re-act upwards upon the Particle 1 with a Force which is as 1. In the ſame manner the Particle 2 acts on the Particle 3 by Preſſure downwards, with 2 Degrees of Force, ariſing from its own Weight and that of the Particle above it; and accordingly it is preſs'd upwards with an equal Force by the Re-action of the Particle 3: And ſo of all the reſt. Wherefore the Propoſitions are manifeſt. Pl. XVI. Fig. 1.

V. *The Preſſure is upon all Particles of the Fluid at the ſame Depth equal in every Part*; or, *the Particles of a Fluid, at the ſame Depth, preſs each other every way, and in all Directions, equally.* For if any Particle were preſs'd more on one Part than another, it muſt give way, or yield, till the Preſſure became every way equal; otherwiſe an inceſſant inteſtine Motion of the Particles would enſue, which is abſurd, and contrary to Experience (LIII).

(LIII) 1. There are ſeveral Ways to ſhew that the Preſſure of Fluids is every Way equal; but the moſt ſimple and evident, I think, is by taking a very long Glaſs Tube hermetically ſeal'd at one End, and immerſed in a perpendicular Poſition with the open End in another tall Tube or Jar of Water; the Air in the Tube being compreſſible will yield to the Preſſure of the Water below, and admit the Water to riſe in the Tube, to Heights which will be always proportional to the Altitudes of the Fluid above it; and this will be the Caſe when the lower Part of the immerſed Tube is bent into a Right Angle, that the Water may come upon it laterally; and thence it will appear *that the Preſſure of Fluids is every Way equal, and proportional to the Altitudes.*

Pl. XV. Fig. 3.

2. This likewiſe may be ſhewn from mechanical Principles, for let the Sphere A be ſupported by two others in an horizontal Poſition B and C; join the Centres a, b, g; and draw the Perpendicular af; let ee be a Tangent to the Sphere B in the Point c, where the Sphere A touches and preſſes it, interſecting the Perpendicular af in e; and let ae expreſs the Force of A downwards; then ſince ae is reſolvable into two Forces ac and ce, of which the latter being in the Tangent to B, does not at all affect it, and the other ac being perpendicular to the Surface; this other Force ac is that alone by which the Body A preſſes B.

3. But ſince ac is an oblique Force, that is, neither perpendicular nor lateral, let it be reſolved into the two Forces ad and dc; of which the former is perpendicular, and ſo does not affect the Body B in preſſing it Sideways; but the other dc being in an horizontal Direction is all the Force with which A preſſes B in a lateral Direction; but ſince dc is parallel to bf, we have $ac : dc :: ab : fb$; and ſince the Particles of a Fluid are equal, we have $fb = bc = ac$; therefore $fb = \frac{1}{2} ab$, and conſequently $dc = \frac{1}{2} ac$; that is, *the Force with which A preſſes B laterally is juſt half the Force with which it preſſes it directly.*

4. And ſince by the third Law of Motion the Preſſure of the Sphere D upwards is equal to that of A downwards, and ſo the Force upon the Bodies B and C in a lateral Direction is the ſame, that is, *half the direct Force*; therefore the whole Force with which the Body B or C is urged la-

VI. FROM the mutual Preſſure and equal Action of the Particles it follows, *that the Surface of a Fluid muſt be perfectly ſmooth and even*; for ſhould any Part ſtand higher than the reſt by any Force, as Attraction, *&c.* it would immediately ſubſide to a Level with the other Part by the Force of its own Gravity, when that Force is removed.

VII. THE Figure of the Surface of all Fluids is ſpherical or convex; for all the Particles equally gravitating towards the Centre of the Earth, will take their Places from it at equal Diſtances at the Surface, and ſo form a Part of the Superficies of a Sphere, equal to the Bulk of the Earth (LIV).

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terally by the joint Action of A and D *is equal to the whole Force with which either of them act upon them ſingly.* From what has been ſaid, it appears, that the Force with which the Body A or D preſſes the Body B or C, is leſs than the Force of Gravity, or that by which they act perpendicularly, that is, nc is leſs than ae.

(LIV) 1. Beſides the Reaſon of the Thing, we know by Experiment, that the Surface of large Waters, as thoſe of the Sea or Ocean, is convex; for a Perſon ſtanding on the Shore, and viewing a Ship under Sail, directly before him, will loſe Sight thereof by Degrees, the Hull or Body of the Ship firſt diſappearing, then the lower Parts of the Maſts, then the Tops of the lower Maſts, and laſtly, the Top of the talleſt Maſt; as repreſented at A, B, C, where the Ship gradually deſcends below the horizontal Line H O, or Line which bounds the Sight, as it proceeds on the convex Surface of the Earth from A to C. Pl. XV. Fig. 4.

2. The Reaſon why we ſee not this Convexity in the Surface in Fluids of ſmall Extent, as in Veſſels, Ponds, *&c.* is becauſe the Superficies of a Sphere ſo large as that of the Earth will ſo nearly co-incide with a Plane, for a certain Space, as not to be diſcernable from it; and therefore the

VII. SINCE

VIII. SINCE Fluids preſs equally every way, the Preſſure of each Particle againſt the Side of a Veſſel will be proportional to its Altitude; and conſequently the Preſſures of the Particles 1, 2, 3, 4, &c. of a perpendicular Column againſt the Side BC will be as a Series of Numbers in Arithmetical Progreſſion, whoſe firſt Term is 0; therefore *the Sum of all the Preſſures is equal to the Number of*

Surface of Fluids within that Space, or Extent, will appear plain or nearly ſo; and what that Extent of Surface is, as the Reader may be curious to know, I will ſhew by the following Calculation.

Pl. XV. Fig. 5.

3. Let A C be the Semidiameter of the Earth = 4000 Miles, or 21120000 Feet; (for 5280 make one Mile,) A E a Part of the Earth's Surface, A F a Tangent thereto at the Point A. Now it is found by Experience, that nothing is diſtinctly viſible to the Eye that does not ſubtend an Angle of one Minute at leaſt. Let B D then be the leaſt Diſtance between the Convex Surface A E, and the Tangent Line A F, that is diſcernable to the Eye at A; then will D A B, contain'd between the Tangent and the Chord A B, be one Minute. If now on A B we let fall the Perpendicular C G, then in the right-angled Triangle A G C, we have all the Angles given, and the Side A C, to find the Side A G. For ſince D A C is a Right Angle, and D A B = 1 Minute, the Angle G A C = 89° 59′; and therefore the Angle A C G = 1 Minute alſo. Therefore we ſay,

As Radius 90°	—	10,000000
Is to the Angle A C G 1 Minute	—	6,463726
So is the Side A C = 21120000	—	7,324694
To the Side A G = 6144	—	3,788420

4. But 2 A G = A B = 12288 Feet, which is a little more than two Miles and a Quarter; and therefore, unleſs the Surface of Water be more than two Miles extent, it will not appear different from a Plane; the Diſtance D B being in any leſs Extent inſenſible. Hence a Ship on the Sea will depart about two Miles and a Quarter before we ſenſibly loſe Sight of any Part, becauſe till then ſhe will ſeem to be ſailing on a Plane.

Preſſures multiplied by half the greateſt Preſſure: But the Number of Preſſures is as the Number of Particles, or Altitude of the Fluid BC; alſo the greateſt Preſſure is as the ſame Altitude: *Wherefore the total Preſſure againſt the Side of a Veſſel is as the Square of the Altitude of the Fluid* (LV).

(LV) 1. That the perpendicular Preſſure of Fluids on the Bottoms of Veſſels is eſtimated by *the Area of the Bottom multiplied by the Altitude of the Fluid*, is evident, I ſuppoſe, to every Reader. For ſuppoſe a Veſſel were 2 Feet wide, 3 Feet long, and 4 Feet deep, and fill'd with Water to the Brim; then the Area of the Bottom is $2 \times 3 = 6$ ſquare Feet, and every ſquare Foot being preſs'd by the Column of Water, containing 4 cubic Feet, 'tis plain the whole Bottom will be preſs'd by $6 \times 4 = 24$ cubic Feet, that is, by the whole Number of cubic Feet which the Veſſel contains.

2. But the Quantity of the lateral Preſſure, or that againſt the Side of the Veſſel, is not ſo obvious; yet among the ſeveral Ways of computing it, I think none ſo eaſy, natural, and univerſal, as that I have mention'd, *viz.* by conſidering it in a ſingle Column of Particles, as a Series of Quantities in *Arithmetical Progreſſion*, becauſe this depends entirely on the ſingle Principle already eſtabliſhed, viz. *that the Action or Preſſure of Fluid Particles is every way equal.* By which I ſhall endeavour farther to illuſtrate this Matter, and facilitate the Method of Computation.

3. Upon the uppermoſt Surface of the firſt Particle 1, which coincides with the Surface of the Fluid, there is no Gravity, and conſequently no Preſſure; therefore the Beginning of the Series, or firſt Term is 0, or nothing. The Body of the firſt Particle preſſes the ſecond Particle 2 with its own Weight which is as 1, and it preſſes the Side of the Veſſel with the ſame Force, and therefore the *ſecond Term* of the Series will be 1. The ſecond Particle preſſes the third Particle 3 with the Force of its own Weight, and the Weight of that above it, that is, with a Force as 2; and ſince it preſſes the Side of the Veſſel with the ſame Force, the *third Term* of the Series will be 2. After the ſame Manner it is ſhewn, that the *fourth Term* will be 3, the *fifth Term* 4, and ſo on. Whence 'tis evident the ſeveral Preſſures will be as the Series 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, &c.

4. Now that the Sum of ſuch a Series is equal to the

Pl. XVI. Fig. 1.

IX. Hence if the Veſſel AC be of a Cubical Form, the Preſſure againſt a Side BC is half that upon the Bottom CD; and conſequently, *the total*

greateſt Term, multiplied by half the Number of Terms, is known to every Perſon verſed in common Arithmetic, and may be eaſily ſhewn by an Example. For ſuppoſe the Series were 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, then the Number of Terms is 12, and the half thereof 6; alſo the greateſt Term is 11; but $6 \times 11 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 = 66 =$ the total Preſſure againſt the Side of the Veſſel. Now this is manifeſtly but half the Preſſure upon a Line of the ſame Length in the Bottom of the Veſſel, *viz.* a Line of 12 Particles, for ſince each Point ſuſtains the Preſſure of 11 Particles, the whole muſt be $12 \times 11 = 132$.

5. It may here be objected that I have taken 12 Particles in a Line at the Bottom, whereas there is but 11 at the Side, and therefore the Length of the Side and Bottom is not the ſame, as in the Suppoſition we make it. But it is to be conſider'd that when the Particles are ſuppos'd indefinitely ſmall and numerous, as is the real Caſe of Fluids, the Difference of Length occaſion'd by one ſingle Particle will be infinitely ſmall, and therefore will make no Error in the Computation.

6. Let the Sum of all the Preſſures, or the total Preſſure againſt the Side of a Veſſel, be repreſented by S, the Number of Terms in the Series by N, and the greateſt Term by G; then ſince $S = \frac{1}{2} NG$, and ſince N and G are ever proportional to each other, we ſhall have S as $\frac{1}{2} NN$, or 2 S as N^2; and ſince Halves are in the ſame Ratio with their Wholes, we have S as N^2; that is, *the Sum or total Preſſure will be as the Square of the Number of Terms, or Altitude of the Fluid.*

7. For Example, in the above Series, if we take the Sum of 4 Terms, it will be $0 + 1 + 2 + 3 = 6$; and then of 8 Terms, the Sum will be $0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$, which is 4 times 6, and 4 over. Now theſe Sums ought to be as 1 to 4, becauſe the Altitudes were as 1 to 2; and tho' the Sum of any Number of Terms will always be a little more than 4 times the Sum of half that Number of Terms in this Way of computing, yet when the Number comes to be exceeding great, the Exceſs will become indefinitely ſmall, and therefore may be neglected in the Caſe of Fluids.

Preſſure againſt the Sides and Bottom is equal to three times the Weight of the Fluid on the Bottom of ſuch a Veſſel.

Pl. XVI. Fig. 2.

X. THE *Weight, Preſſure, or Effect of a Fluid upon the Bottom* DE *of any Veſſel* ACDEF, *is proportional to the Altitude* AE *only, and not to the Quantity of the Fluid in the Veſſel:* For every

8. Or yet more clearly and accurately in Symbols thus; ſince S is always as $\frac{1}{2}$ GN or as GN; and in the Series above adapted to Fluids, $G = N - 1$, therefore $GN = NN - N$; and ſo S will always be as $NN - N$; but when N is indefinitely great, N^2 will be infinitely greater than N, which therefore will vaniſh in that Caſe in the Expreſſion $NN - N$; and ſo S will ever be as N^2, that is, the Sum or total Preſſure will be as the Square of the Altitude of the Fluid.

9. This Way of conſidering the Quantity of lateral Preſſure by the *Arithmetical Series* is univerſal, whereas the common Method reſtrains it to the Property of an equicrural right-angled Triangle, and to a Veſſel of a cubical Form; which I ſhall here give for the Sake of ſuch as would ſee the Demonſtration of a Thing in ſeveral Ways, ABCD is a Veſſel of a cubical Form, that is, whoſe Side BC is equal to the Length of the Bottom CD; if then the Diagonal BD be drawn, we ſhall have the Lines 1*a* = B1, 2*b* = B2, 3*c* = B3, *d*4 = B4, *&c.* but B1, B2, B3, B4, *&c.* being as the Altitudes of the Fluid, will repreſent the lateral Preſſures in the Points 1, 2, 3, 4, *&c.* therefore alſo the Lines *a*1, *b*2, *c*3, *d*4, *&c.* will repreſent the ſame lateral Preſſures; hence when the Diſtances B1, 12, 23, 34, *&c.* are indefinitely ſmall, the Lines *a*1, *b*2, *c*3, *d*4, *&c.* will be infinitely near each other; and ſo all thoſe Lines drawn in the Triangle BCD will make the Area of that Triangle. Therefore the Sum of all the lateral Preſſure againſt the Side BC, will be as the Area of that Triangle. But the Area of the Triangle BCD is as the Square of the Side BC, as we know from the Elements of *Geometry*. Conſequently, the Sum of all the lateral Preſſures is as the Square of the Altitude of the Fluid BC. *Q. E. D.*

10. Now the Preſſure on the Bottom of the Veſſel CD being equal to $CD \times BC$, and the Area of the Triangle BDC being equal to $BC \times \frac{1}{2} CD$; 'tis plain, the Sum of

Column of Particles G H, which preſſes downwards on the Side of the Veſſel E F, has its Force deſtroy'd by the equal Re-action of the ſubjacent Particle H in the Side, and ſo cannot at all affect the Bottom of the Veſſel. Again; the Preſſure of any Column of Particles L M upwards, againſt the Side of any Veſſel C D, is e-

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all the lateral Preſſures againſt any one Side, is equal to but half the Preſſure on the Bottom of ſuch a Veſſel; and ſo the Sum of all the Preſſures againſt the four Sides is twice that upon the Bottom; and that the whole Force of the Fluid is equal to three Times the Force of its Gravity.

11. Hence we ſee how very differently Fluids act from Solids, which act upon each other by their Gravity only, whereas Fluids act by Gravity, and by Preſſure jointly. Solids act only downwards, but Fluids every way equally. Solids act with a Force proportional to their Quantity of Matter, but the Force of Fluids is not according to their Quantity, but their Altitude only.

12. The Quantity of Preſſure on the Bottom of Veſſels, or Planes oblique to the Horizon, is eaſily eſtimated from what has been ſaid above; for let AK be ſuch an inclined Plane, and AE the Surface of the Water; then will the Preſſure upon the Points A, G, H, I, K, be as the Altitudes BG, CH, DI, EK. If now on the ſaid Points we erect the Perpendiculars Gb, Hc, Id, &c. equal to thoſe Altitudes reſpectively, and ſuppoſe the ſame Thing done for every other Point in the Line AK, 'tis evident the Triangle AeK will be as the whole Preſſure on the Line AK. But the ſaid Triangle is equal to $AK \times \frac{1}{2}eK$; and ſuppoſing $AG = GH = HI = IK$, and cf drawn parallel to AK; we ſhall have $\frac{1}{2}eK = fK = cH = CH$; therefore $AK \times CH$, will be as the total Preſſure on the Line AK.

13. If AK were the Section of a Plane, then the Surface of that Plane multiplied by CH, will be the Expreſſion for all the Preſſure on that Plane, provided CH be the Depth of the Centre of Gravity H, from the Surface of Water. But to give a more general Theorem of the above Rule, let a, b, be two Weights hanging from an horizontal Plane, at the Diſtances ac, bd; and join their Centres by the Line ab, and let x be their common Centre of Gravity, and xz its Diſtance from the Plane cd, perpendicular to which draw

qually re-acted by the Particle of the Veſſel over it; and ſo its Force or Preſſure on the

xy and bz; then ſince $a : b :: bx : ax$, by the Property of the common Centre of Gravity; and by ſimilar Triangles we have $bx : ax :: xz : xy$. Therefore $a \times xy = b \times xz$; but $xy = xo - yo = xa - ac$; and $xz = zo - xo = bd - xo$; therefore $a \times \overline{xo - ac} = b \times \overline{bd - xo}$; that is, $a \times ac + b \times bd = a + b \times xo$. That is in Words, *The Product of the Weights multiplied by their Diſtances from the Plane is equal to their Sum multiplied by the Diſtance of their common Centre of Gravity from the Plane.*

14. Now that this holds true in Lines and Planes is evident, becauſe the indefinitely ſmall Particles of Lines and Planes may be conſider'd as very ſmall Weights, and as what has been demonſtrated of two, holds equally for all, therefore the above Rule is applicable to all Sorts of Surfaces, or the Preſſure upon the Bottoms of Veſſels, however poſited or figured, may be exactly computed thereby.

15. Thus ſuppoſe A B C D E F repreſent a Veſſel of a Priſmatic Form, whoſe Bottom is an oblique triangular Plane D E F, in which let F G be drawn perpendicular to the Baſe D E, then if we make $Fx : xG : 2 : 1$, the Point x will be the Centre of Gravity of the Plane, and xo its perpendicular Depth from the Surfuce of the Fluid A B C; when the Veſſel is fill'd with Water. Now ſuppoſe A F = 1 Foot; F G = 6 Feet; E D = 2 Feet; C E = H G = 3 Feet; and let the Plane Fbc be parallel to the Surface of the Fluid A C B. Then we have $FG : Fx :: Gb : xa$; that is $6 : 4 :: 2 : 1\frac{1}{3} = xa$; therefore $xa + ao = xo = 2\frac{1}{3}$ Feet, the Depth of the Centre of Gravity of the Triangle F E D whoſe Area is $FG \times EG = 6 \times 1 = 6$ ſquare Feet. Therefore $6 \times 2\frac{1}{3} = 14$ Cubic Feet of Water, whoſe Weight is equal to the Preſſure on the Bottom of the Veſſel.

Plate XVI. Fig. 7.

16. Thus in caſe of a Cylindric Veſſel whoſe Bottom is either an Ellipſis, a Cone, a Segment of a Sphere, &c. if the Quantity of the Surface, and Centre of Gravity be found, the Content of the Veſſel is then known by the Rule above. Thus alſo if the Surface of the Side of any Veſſel in any Form or Poſition be multiplied by the Depth of the Centre of Gravity, the Product will be the Number of Cubic Feet, whoſe Weight is equal to the Preſſure on ſuch a Side or Surface. And ſince a cubic Foot of Water weighs 1000 Ounces, or $62\frac{1}{2}$ *lb.* Averdupoiſe Weight, the Quantity of Preſſure may be eaſily expreſs'd in any Denomination of Weight, as will be more particularly ſhewn hereafter.

Bottom muſt be the ſame as that of another Column of Particles A B of equal Altitude

17. One Thing more the Reader ſhould be appriz'd of, and that is the *Centre of Preſſure,* which is that Point in which we may conceive the whole Preſſure to be concenter'd or united; or to which if a Force were applied equal to the whole
Pl. XV. Fig. 8. Preſſure, and acting in a contrary Direction, it would exactly ballance or reſtrain the Effect of the Preſſure; or it would keep the Plane (otherwiſe free to move) in *Equilibrio* with the total Preſſure, unequally diſtributed over all its Parts. Thus if A B C D be a Veſſel of Water, and the Preſſure on the Side A B be equal to twenty Pounds; if *a* be the Centre of Preſſure, and a String *a b c* were fix'd thereto going over the Pulley *b*, and ſuſtaining a Weight *c* of twenty Pounds, then would the Side A B be kept in *Equilibrio* by thoſe two equal and oppoſite Powers.

18. Now the *Centre of Preſſure* is the ſame as the *Centre of Percuſſion*, as is evident from hence, that in the Line A K the
Fig. 5. percuſſive Force of every Point G, H, I, K, is as the Velocity, that is, as the Diſtance A G, A H, A I, A K from the Point A conſider'd as the Point of Suſpenſion. But thoſe Diſtances are as the Altitudes G B, H C, I D, K E, which are as the Preſſures on the aforeſaid Points: Therefore ſince the Preſſures and percuſſive Forces are always alike, the Centre of one muſt be the ſame with that of the other, but the Centre of Percuſſion of the Line A B is two Thirds of its Length diſtant from the Point of Suſpenſion A, therefore alſo the Diſtance of the Centre of Preſſure *a* is $\frac{2}{3}$ of the Side A B below the Surface of the Water. See *Annot.* XXIX.

SCHOLIUM.

19. The Reaſon why a greater Force of Preſſure is allow'd to an oblique Plane or Bottom of a Veſſel than what is equal to the Gravity of the Fluid, is this, that every Point in the Line A K is preſs'd with a *perpendicular* and *lateral Force* at the ſame Time, and by both *obliquely*; and of thoſe two oblique Forces, or Preſſures, one *direct Force* is compounded, which is greater than either, and acts in a Direction perpendicular to the Plane. But ſince both the oblique Preſſures are always as the Depth of the Fluid, the direct compound Preſſure will be in the ſame Ratio likewiſe. Hence as there is a greater Number of Points in the Line A K than in the Line A E, and each is preſs'd perpendicularly with a Force proportional to the Depth, 'tis neceſſary that we eſtimate the whole Force by the Area of the Triangle A *e* K, and not of A E K, as we muſt have done if Fluids preſs'd like Solids by their Gravity only.

with the Fluid: Whence the Propoſition is evident (LVI).

(LVI) 1. There is nothing in the Explication of the Hydroſtatic Paradox difficult to be underſtood, but the Manner how the Point M comes to be preſſed with the ſame Force as the Point B, which has the whole Height of the Fluid above it. But this Difficulty will ſoon vaniſh when we conſider *that the Preſſure among the Particles of a Fluid at the ſame Depth is every Way equal*; therefore if L H be parallel to the Bottom D E, 'tis plain there will be the ſame Preſſure upwards in every Particle H, I, Q, L, as there is downwards, and every where as the Altitude A I. The Particle at L therefore preſſes upwards againſt a Particle in the Side of the Veſſel with a Force equal to the Weight of the Column of Particles A I; which Force is deſtroy'd by the equal Re-action of that Particle in the Side of the Veſſel. The ſame Fluid Particle L alſo preſſes the Particle immediately beneath it in the Column M L with the ſame Force, which added to the Weight of the Column L M makes the ſame Preſſure on the Point M, as there is on the Point B ariſing from the Preſſure on the two Parts A I + I B.

2. That the Preſſure upward at the Point L is equal to the Weight of the Column A I, is evident from hence, that if it be removed (that is, if a ſmall Hole be there made) the Fluid will from thence be thrown up in Form of a *Jet d'Eau* to the Height of T nearly, which is in the ſame horizontal Line with the Point A, or Level of the Fluid. All which well conſider'd is, I preſume, ſufficient to evince that the Preſſure on the Point M and every other Point in the Line D E is the ſame with that on the Point B.

3. From what we have ſaid, 'tis alſo plain, that the Top of the Veſſel C N R F is preſs'd upwards with a Force every where proportional to the Altitude of the Fluid A P, which Force of Preſſure upon the whole Surface is equal to the Weight of a Body of Fluid of the ſame Baſe, and whoſe Altitude is A P. The Sides alſo are preſs'd every where outwards with a Force proportional to the Altitude of the Water, and in the ſame Manner as they would be at the Depth were the Fluid every where of the ſame Bulk as in the Body of the Veſſel, and of the ſame Altitude as at A.

4. The Conſequence of this will be, that if an Inſtrument be contrived with a Bottom A B and Top C D connected with pliant prepared Leather E F, ſo as to be Water-tight, and if in the Middle of the upper Part at G be inſerted a ſmall Tube

Plate XV. Fig. 9.

Pl. XVI. Fig. 2. XII. Hence *a very ſmall Quantity of a Fluid, as* APRS, *may be made to counter-balance or be*

GH, and Water pour'd in ſo as to raiſe the upper Part DC that it may float freely on the Surface of the contain'd Water, then any Weight I laid thereon will cauſe the Water to riſe ſo high in the ſmall Tube as will be equal to the Height of a Cylinder of Water of the ſame Baſe DE, and whoſe Weight will be equal to that laid on; and if another Weight K be added, it will raiſe the Water to twice the Height in the Tube; if a third Weight L be added, it will riſe to three Times the firſt Height, and ſo on. Whence it appears that ſince the Bore of the Tube may be exceeding ſmall, the greateſt Weight may be ſuſtain'd or counterballanced by the leaſt aſſignable Quantity of Water in the Tube.

5. But this Matter will be beſt explain'd by Calculation and Example. Suppoſe then $DC = 6$ Inches in Diameter, the Area of the Circular Surface will be 28,27 Square Inches; and ſince a Square Inch of Water weighs $\frac{578}{1000}$ of an Ounce Averdupois, therefore a Pound will contain 27,7 Cubic Inches; conſequently as this Number is but half a Cubic Inch leſs than the other, 'tis evident that a Cylindric Column of Water whoſe Baſe is ſix Inches in Diameter, and Height one Inch, will weigh nearly one Pound; if therefore I be one Pound, the Water by its Preſſure will riſe one Inch in the Tube; if K be another Pound, then $I + K$ will raiſe the Water two Inches, and ſo on. If the Bore of the Tube be $\frac{1}{10}$ of an Inch Diameter, the Quantity in the Tube one Inch high is to that in the Veſſel one Inch high, as the Square of $\frac{1}{10}$ to the Square of 6, that is, as $\frac{1}{100}$ to 36, as 1 to 3600; and in that Proportion are their Weights alſo; whence 'tis plain the Water in ſuch a Tube will ſuſtain 3600 Times its own Weight.

6. In like Manner we may conſider how great an Effect might be produced by only blowing with one's Breath thro' the Tube HG into the Veſſel ABCD. By this Means, if the Dimenſions of the Veſſel were large, and the Tube long and ſmall, a Man with the Breath of his Mouth might raiſe any given Weight, how great ſoever. Suppoſe, for Inſtance, the Area of the Top DC were 1000 ſquare Inches, and the Diſtance between the Top and Bottom $AD = \frac{1}{4}$ of an Inch, then will there be contain'd in the Veſſel 250 Cubic Inches of Air, of the ſame Denſity with the external Air. If now more Air be blown thro' the Tube into the Veſſel, it will condenſe the internal Air, and by increaſing its Spring cauſe it to raiſe the moveable Top DE, though charged with a very great Weight.

equivalent to the Weight or Force of any given Quantity T K G V, *how great ſoever.*

XIII. WHEN any Body is immerſed in a Fluid, *it loſes juſt ſo much of its Weight as is equal to the Weight of an equal Bulk of the Fluid*;

7. Thus ſince the Weight of Air preſſes upon every Square Inch with a Force of 15 *lb.* (as will be ſhewn hereafter) if we place on the ſaid Top D C, 1000 x 15 = 15000 *lb*, and then blow into the Inſtrument 250 Cubic Inches of Air, it will double the Denſity of the internal Air, and conſequently its Spring, which will then hold the great Weight in *Equilibrio*, and if ever ſo little more be blown in it will raiſe it; for in that Caſe the Air will preſs upwards with a greater Force than that of the Weight downwards.

8. If 500 Cubic Inches of Air were blown into the Inſtrument it would ſuſtain 30000 *lb*, and ſo on. But in ſuch Caſes we muſt ſuppoſe the Parts E F ſufficiently ſtrong to bear ſuch a Preſſure from within; for ſince the Air is a Fluid it preſſes every Way equally. Now becauſe the Bore of the Tube is ſmall (ſuppoſe the 100th Part of a ſquare Inch,) a Perſon blowing through the ſame would meet with no more Reſiſtance than the Preſſure of the Air on the Area of the Bore, which when the Air is of a double Denſity, or when 250 Cubic Inches are crowded in, will be but $\frac{15}{100}$ of a Pound which may be eaſily overcome by the Force of the Muſcles in blowing, eſpecially if we uſe a Stop-cock to ſhut off the Air in the Tube every Time we take in freſh Air to our Mouths.

9. If the Top of the Inſtrument D C were fix'd, and a Wire paſſing thro' the Tube G H were fix'd to the Bottom, and ſuſpended at the End of a Ballance, then if the Bottom be nicely ballanced by Weights in the Scale of the other End, and after that Water pour'd in till the Top and Bottom were ſeparated to their utmoſt Diſtance; to ballance this Water will require one Pound weight for every Inch in Height in the Veſſel, and it will after that require one Pound weight for every Inch that it riſes in the Tube, which plainly ſhews that the Force of Fluids is always proportional to the Height, and not the Quantity of the Fluid. *Note*, I have ſuppoſed in this Article that the Diameter of the Veſſel is ſix Inches, as before in *Art.* 5. Another remarkable Inſtance of this Hydroſtatic Paradox, I ſhall give when I come to conſider the *Swimming of an heavy Body in a lighter Fluid*, and which will be a very clear Illuſtration of the Matter. See *Annot.* LXI.

but the *Weight loſt by the Body is gain'd by the Fluid*, which will be ſo much heavier than before (LVII.)

(LVII) This is the fundamental Principle of every Hydroſtatic Proceſs, particularly of the whole Doctrine of *Specific Gravities*; which therefore cannot be made too plain and eaſy to be underſtood. To this End, let ABCD be a Veſſel fill'd with Water to the Height EF; and let I be a cylindric Body (heavier than Water) to be immerſed therein, as at L. By this Immerſion of the Body I, a Quantity of the Fluid *a b c d*, equal in Bulk to the Body, will be diſplaced by the ſuperior Force or greater Gravity of the Solid: And this Quantity of Fluid muſt aſcend, (as being confined towards the Bottom and Sides) and ſo raiſe the Surface of the Liquor from EF to GH; and then will the Quantity EFGH be equal to the Bulk of the immerſed Solid *a b c d*. Plate XV. Fig. 10.

2. But as the Solid comes to enter the Fluid, each Particle of the Fluid by its *Vis Inertiæ* will reſiſt the Solid, or endeavour to oppoſe its Deſcent with all its Power; and ſo the whole Body of the Fluid that is removed or diſplaced by the Solid will reſiſt it by the united Force of all the Particles: But this Force is equal to the Gravity of the Fluid removed, as is evident from hence, that the Fluid ſo removed is obliged to aſcend or move in a Direction quite contrary to Gravity: *Wherefore the Solid in its Deſcent will be reſiſted by a Force equal to the Gravity of an equal Bulk of the Fluid.*

3. A Conſequence of this will be, that, ſince Action and Re-action are equal, there will be juſt as much of the Gravity of the deſcending Solid deſtroy'd, as is equal to the renitent Force, that is, to the Gravity or Weight of an equal Bulk of the Fluid. Whence 'tis evident that all Bodies immerſed in Fluids will weigh lighter if ſuſpended therein at the End of a Balance, than in the Air; and lighter in the Air than *in Vacuo*, *where only the true, real, or abſolute Weight of Bodies can be ſhewn or known.*

4. And ſince the Force which reſiſts the Deſcent of Solids is proportion'd to their Bulk only, it follows, that equal Bodies immerſed in Fluids loſe equal Parts of their Weight; and therefore a lighter Body loſes more of its abſolute Weight, than a heavier one of the ſame Bulk: And conſequently if two Bodies of unequal Bulk are *in Equilibrio* in the Air, that Equilibrium will be deſtroy'd on their being immerſed in the Fluid; becauſe that which has the largeſt Bulk will loſe moſt Weight in the Fluid.

XIV. In

XIV. IF any Body E could be found without Weight, *it would, if placed on the Surface of a Fluid* A B, *float thereon without any Part im-* Pl. XVI. Fig. 3.

5. Again; 'tis plain the Weight of the Fluid is augmented in the ſame Proportion, as that of the immerſed Solid is diminiſhed; for the Force or Action of the Fluid on the Bottom of the Veſſel CD is before Immerſion to that afterwards as the Altitudes CF to CH, or to the Bulks of the Fluid EFCD and GHCD. And ſince theſe Bulks act only by their Gravity, 'tis plain the Action of the Fluid is increaſed only by the additional Gravity of the Quantity GHFE, which is equal to that which the Solid loſes by Immerſion, as was before ſhewn in *Art.* 2. All which Caſes anſwer very exactly by Experiments.

6. From what has been premiſed we may eaſily apprehend what is meant by *Specific Gravity*, viz. *that which is peculiar to any Sort or Species of Matter or Body, when conſidered or compared in any given Bulk or Magnitude*, as a *Cubic Inch*, for Inſtance. Thus we ſay, the Specific Gravity of common Water is to the Specific Gravity of Lead, as the Weight of a Cubic Inch of Water to the Weight of a Cubic Inch of Lead. And as the *Abſolute Weight* of Bodies is their whole Power of Gravity *in Vacuo*, ſo their *Relative Gravity* is that which they have in any reſiſting Medium, as Air, Water, *&c.* and is equal to the Abſolute Weight diminiſhed by the Weight of an equal Bulk of the Medium.

7. What relates to the *Abſolute and Specific Gravities*, the *Magnitude*, *Denſity*, *&c.* of Bodies, will beſt be underſtood by ſymbolical Computation; in order to which let A and B be two Bodies of equal Bulk, but different Quantities of Matter; and let B and C be two other Bodies with equal Quantities of Matter, but of different Bulks: Plate XV. Fig. 11.

And let $\left\{\begin{array}{l} \text{D} = \text{Denſity} \\ \text{B} = \text{Bulk} \\ \text{M} = \text{Quantity of Matter} \end{array}\right\}$ in the Body A.

Alſo $\left\{\begin{array}{l} D = \text{Denſity} \\ B = \text{Bulk} \\ M = \text{Matter} \end{array}\right\}$ in the Body B.

And $\left\{\begin{array}{l} d = \text{Denſity} \\ b = \text{Bulk} \\ m = \text{Matter} \end{array}\right\}$ in the Body C.

8. Then, becauſe the Denſity of any Body is proportional to the Quantity of Matter under equal Bulks, we ſhall have $D : \text{D} :: M : \text{M}$; and, becauſe when the Quantities of Mat-

merſed; for being devoid of Gravity, it could have no Force to diſplace any Particles of the Fluid, and ſink therein.

XV. If *any heavy Body* F, *lighter than an equal Bulk of the Fluid, be placed on its Surface, it will ſink or deſcend therein, till it has removed or diſplaced ſo much of the Fluid whoſe Weight is equal to that of the Body*; For then the Preſſure upwards and downwards on the under Surface of the Body is equal; and conſequently the Body will be there quieſcent, or in *Equilibrio* with the Fluid. Hence *the whole Solid is to the*

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ter are equal, the Bulks muſt be reciprocally as the Denſities, therefore we have $D : d :: b : B$. Whence $D = \frac{DM}{M} = \frac{db}{B}$; conſequently $DMB = dbM$. But $B = B$, and $M = m$; therefore $DBm = dbM$. Whence we have $D : d :: bM : mB$; and $B : b :: dM : Dm$; and $M : m :: DB : db$.

9. The *Specific Gravity* of Bodies being as the Weights, that is, as the Quantities of Matter, in equal Bulks, will be as the Denſity. Therefore $D : d :: S : s$; and by Subſtitution of Ratios we have the general Theorem above become $SBm = sbM$. And ſince the Abſolute Weights (A, a,) of any two Bodies are as the Quantities of Matter, we have $SBa = Asb$. Wherefore $S : s :: Ab : aB$; that is, the Specific Gravities will be as the Abſolute Weights directly, and the Bulks inverſely, or as the Abſolute Weights divided by the Bulks.

10. Alſo $A : a :: SB : sb$; that is, the Abſolute Weights of Bodies are in the compound Ratio of their Specific Gravities and Bulks. Or the Abſolute Weight of any Body is had by multiplying its Bulk and Specific Gravity together.

11. Again, becauſe $B : b :: As : aS$, it appears that the Bulk or Magnitudes of Bodies will be as the Abſolute Weights directly, and Specific Gravities inverſely. Or the Magnitude of any Body is had by dividing its Abſolute Weight by its Specific Gravity.

immerſed Part, as the ſpecific Gravity of the Fluid to that of the Solid (LVIII.)

XVI. If *a Solid, as* G, *equal in Weight to an equal Bulk of the Fluid, be immerſed therein, it*

(LVIII) 1. This Caſe is not ſtrictly true but *in Vacuo*; for in the Air ſuch a Body may be conſider'd as ſuſtain'd in two Mediums, *viz.* Air and Water; in one of which it will ſink or deſcend, and in the other riſe. And therefore to repreſent the true State of this Matter univerſally, we muſt raiſe a general Theorem in the Manner following: Let ABCD be a Veſſel fill'd firſt with an heavy Fluid to the Level EF, and from thence with a lighter Fluid to AB. Suppoſe the Solid X ſuſtain'd by thoſe two Fluids; let the Part in the heavier be call'd A, and that in the lighter B; and let the ſpecific Gravities of the heavier and lighter Fluids be as *a* and *b*.

Plate XV. Fig. 12.

2. Then ſince the Part A diſplaces a Bulk of the Fluid equal to A, the abſolute Weight of that Bulk of Fluid will be Aa, (by *Annot.* LVII. *Art.* 10.) and for the ſame Reaſon the abſolute Weight of a Bulk of the lighter Fluid equal to B will be Bb. Let c be the ſpecific Gravity of the Solid X; then the Sum of the Weights of the two Portions of the Fluids muſt be equal to the Weight of the Solid; otherwiſe it could not be ſuſtain'd by them: Therefore $Aa + Bb = Xc = \overline{A + B} \times c$. Hence $Aa - Ac = Bc - Bb$. Conſequently, $A : B :: c - b : a - c$; and compounding, $A : A + B \, (= X) : c - b : a - b$.

3. Theſe two Theorems are thus expreſs'd in Words:

1. As the Part of the Solid within the heavier Fluid is to the Part contain'd within the lighter: So is the Difference of the ſpecific Gravity of the Solid and lighter Fluid, to the Difference between the ſpecific Gravity of the Solid and the heavier.
2. The Part of the Solid in the heavier Fluid is to the whole Solid, as the Difference between the ſpecific Gravity of the Solid and lighter Fluid, to the Difference between the ſpecific Gravity of the two Fluids.

4. Hence, if $b = o$, we have $A : X :: c : a$; that is, the Part immerſed is to the whole Solid, as the ſpecific Gravity of the Solid to the ſpecific Gravity of the Fluid. And if the two Fluids were Water and Air, Water and Oil, or any other, and their ſpecific Gravities given, with that of the Solid, it will be eaſy to find the Parts of the Solid contain'd in either Fluid by the Theorems above-mention'd.

will take any Situation indifferently in any Part of the Fluid, as at G, H, I, *without any Tendency to aſcend or deſcend therein:* For being totally immerſed, it muſt remove a Parcel of the Fluid of equal Bulk and Weight; and conſequently the Preſſure upwards is equal to the Tendency downwards on the lower Surface every where; and therefore it can have no Power to ſink: Alſo the Preſſure downwards muſt be equal to the Preſſure upwards on the upper Surface, whence it can have no Tendency to riſe or ſwim; it will therefore remain at Reſt in any Poſition, G, H, I, whereſoever in the Fluid (LIX.)

XVII. LASTLY, *If a Body* K *or* L, *heavier than an equal Bulk of the Fluid, be immerſed therein, it will deſcend by the Exceſs of its Gravity above that of the Fluid:* For, when immerſed,

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(LIX) I have ſhewn, that while a Body is ſuſpended in a Medium, its abſolute Gravity is diminiſh'd by the Reſiſtance of the Medium; it is therefore only the *relative* or *reſidual Gravity* of the Body that we find in ſuch a Caſe: And this being equal to the Difference between the ſpecific Gravities of the Body and the Medium, it is plain, where that Difference vaniſhes, that is, where the ſpecific Gravities of the Body and Medium are equal, there the relative Gravity will become Nothing; whence ſuch a Body ſuſpended by us in ſuch a Fluid has no ſenſible Weight. And this is the Reaſon why a Bucket of Water, while in the Water, ſeems to have no Weight; becauſe the ſpecific Gravities of the Water in the Bucket, and of the Wood of the Bucket, being the ſame with that of the Water in which it is ſuſpended, there can be no relative Gravity of the Bucket of Water experienced, and therefore no Gravity or Weight at all. The Want of conſidering this has been the Occaſion of many abſurd Poſitions and puerile Concluſions in Philoſophy; as *abſolute Levity*, a *Diminution of abſolute Gravity*, &c.

it will be reſiſted by the Force of an equal Bulk of the Fluid, which therefore will deſtroy juſt ſo much of the Gravity of the Solid; and conſequently, the Reſidue or Exceſs of Gravity in the Solid is that alone by which it muſt deſcend (LX.)

(LX) 1. As thoſe Bodies ſwim which are ſpecifically lighter than Water, as above explain'd; ſo others ſink or deſcend in a Fluid by being ſpecifically heavier, that is, by their relative Gravity. Thus, if the ſpecific Gravity of the Solid be to that of the Fluid as 3 to 1, then $3 - 1 = 2$ is the relative Gravity by which it deſcends: If the ſpecific Gravities are as 7 to 1, then $7 - 1 = 6$ is the relative Gravity. Whence we obſerve,

2. That the Deſcent of Solids in a fluid Medium is the very ſame with that of Bodies deſcending on an inclined Plane, becauſe in both Caſes the abſolute Gravity is only diminiſh'd, by the Reſiſtance of the Medium in one Caſe, and by that of the Plane in the other; and therefore all the Properties of the Motion of a Body falling freely, belong to this Motion through a Medium likewiſe.

3. This relative Gravity of Solids, by which they ſink or ſwim, is uſually illuſtrated by the Deſcent and Aſcent of Glaſs Images and Bubbles included in a Jar of Water cover'd over with a Bladder, ſo as to include a ſmall Quantity of Air between the Bladder and Water. The Images, &c. have ſmall Holes in the Bottoms of their Feet, through which ſome Water is put into their Bodies, and that in ſuch Quantities as will render them but very little ſpecifically lighter than Water, but ſome more ſo than others, that they may not begin to move all together. Plate XV. Fig. 13.

4. The Images being thus put to float in Water, and the Bladder tied down, if the Hand be laid on the Bladder, and gently compreſſes the Air beneath, the Air by its Spring will act upon the Water, and cauſe it to compreſs the Air in the Bodies of the Images, by which means more Water will be driven into their Bodies; and when ſo much is got in as will make them ſpecifically heavier than the Water, then they will begin to deſcend one after another; and by varying the Degree of Preſſure you may keep them ſuſpended in any Part of the Fluid as you pleaſe.

5. It is to be obſerved, that the Matter of which the Bo-

FROM what has been premiſed of the Nature of Fluids, it will be eaſy to underſtand, *that*

dies of thoſe Images and Bubbles are made ought to be ſpecifically heavier than Water, that they may ſink when fill'd with Water, which otherwiſe they could not do: They are therefore made with Glaſs, which is about three times heavier than Water. Alſo the Holes in the Feet ought to be very ſmall, leſt the Water ſhould run out into the Air.

6. On the contrary, if the Images were but juſt heavy enough to ſink to the Bottom of the Jar, if then the Bladder, inſtead of being preſs'd down, were lifted up from the Surface of the Water, the Air would expand itſelf, and have a leſs Spring; and therefore the Air alſo in their Bodies would exert its Spring, (which is now greater than that under the Bladder) and drive ſome Water out of their Bodies, by which means they will become ſpecifically lighter than the Water, and riſe in it to the Top.

7. The Image or Bubble will ſink or ſwim without the Artifice of a Bladder, if nicely managed; for if Care be taken to have juſt ſo much Water in the Image as will render it but very little higher than the Water, it will then ſwim at the Surface; if then you put it into a proper Depth, the natural Preſſure of the Fluid upwards will force ſo much Water more into it, as to make it heavier than Water, and from that Place, it will of its own Accord, ſink down to the Bottom. Whence it appears the ſame Body will ſink or ſwim in the ſame Medium, according to the different Circumſtances it is under from the Medium, and not from any Thing in itſelf.

8. If any Veſſel AB be fill'd half full with ſalt Water to C, and a Bubble at B be made juſt heavy enough to ſink therein to the Bottom; and after that an Image at C, having ſo much Water put into its Body, that when the Hole in its Foot is ſeal'd up, it ſhall be juſt light enough to ſwim in the ſaid ſalt Water. Things being thus prepared; if ſo much hot freſh Water be put into the Jar AB as will fill it to the Top, the Conſequence will be, that the Fluid being now lighter, will not ſuſtain the Image C which will ſink to the Bottom; and the Heat of the Water will rarify the Air in the Bubble B, by which means the Water in it will be in part extruded, and the Bubble thereby becoming lighter, will riſe to the Top. Thus by this Artifice, the Image and Bubble will ſpontaneouſly (as it were) change Places; which pleaſant Experiment, or Hydroſtatic Problem, was firſt propoſed by Mr.

the lighteſt Body P *may be depreſs'd in the heavieſt Fluid*, by any Contrivance to keep the ſaid Fluid from preſſing on the under Surface of the light Body, by which means only light Bodies are made to ſwim. Thus *Cork* or *Wood* will abide at the Bottom of a Veſſel fill'd with Quickſilver.

AGAIN: *On the other hand, the heavieſt Body* M *may be made to ſwim in the lighteſt Fluid, by keeping the ſaid Fluid from preſſing on its upper Surface*, by means of the Tube N O: For when by this means it is immerſed ſo deep as to keep off an equal Weight of the Fluid, the Preſſure then of the Fluid acting upon its under Surface upwards will be equal to the Weight of the Solid tending downwards; and therefore if the Solid be ſunk ever ſo little deeper, it muſt ſwim by the ſuperior Force or Preſſure of the Fluid upwards (LXI.)

John Caſwell, Aſtronomy Profeſſor at *Oxford*, near 40 Years ago.

(LXI) Thus for Inſtance, if the Body M be 5 times heavier than Water of an equal Bulk, and if by means of the Tube N O, placed on its upper Surface, the Water be kept from preſſing thereon, that it be immerſed to 7 times its Thickneſs below the Surface of the Water, 'tis plain the Preſſure on the under Surface upwards will be as 7, but downwards only as 5; and therefore ſince there is the Exceſs of two Degrees of Preſſure upward, 'tis plain the Body cannot deſcend; but may very properly be ſaid to ſwim on the Water, and this will be the Caſe of all Bodies, however large or heavy, if placed at a proper Depth in the Water, with the above-mention'd Circumſtances.

2. But what is moſt remarkable, and at the ſame time the cleareſt Proof of the Hydroſtatic Paradox, is, that if the

HENCE alſo the Reaſon of *trying the different Gravity, Denſity, or Strength* (as it is commonly call'd) *of divers Fluids or ſpirituous Liquors by the* HYDROMETER *or Water-Poiſe*: For, ſince the ſtronger any Fluid is, the greater will be its Reſiſtance to any Solid immerſed, 'tis evident the Hydrometer cannot ſink ſo far into the heavy or ſtrong Fluids, as into thoſe which are lighter or weaker. The ſeveral Degrees of Strength, therefore, are eaſily ſhewn by the graduated Neck of this Inſtrument (LXII.)

Body M be immerſed to the Depth of 10 times its Thickneſs below the Surface, with a Tube to keep off the Water from its upper Surface, then whatever be the Form of the Tube upwards, Water pour'd into the ſame will in all Forms thereof, at the Altitude of 5 times the Thickneſs, make the Preſſure on the under Surface upwards and downwards equal, and conſequently, if the Altitude be ever ſo little increaſed, the Body will deſcend. If the Body be immerſed to a Depth, equal to 12 times its Thickneſs, then the Altitude of the Water pour'd in muſt be equal to 7 times its Thickneſs, becauſe $7 + 5 = 12$, and ſo on in all other Depths, let the Veſſel N O be ever ſo great or ſmall, provided its Baſe be equal to the Surface of the Body; the Preſſure of the Water pour'd in being proportional to the Altitude only, and not at all to the Quantity thereof.

(LXII) The HYDROMETER is one of the moſt uſeful Inſtruments of the Philoſophical Kind; for tho' the *Hydroſtatic Balance* be the moſt general Inſtrument for finding the ſpecific Gravities of all Sorts of Bodies, yet the Hydrometer is beſt ſuited to find thoſe of Fluids in particular, and with the greateſt Eaſe, Conveniency and Expedition, as will appear from the following Account, and Deſcription thereof.

2. This Inſtrument conſiſts of four principal Parts, *viz.* (1.) A large round hollow Ball or Sphere A B C D which is made of Ivory, Glaſs, or Metal, as Copper, Braſs, *&c.* (2.) An hollow Glaſs-Ball E C, of a ſmaller Size, partly fill'd with Quickſilver ſcrew'd on to the lower Part of the former,

The Hydrostatic-Balance is alſo an Inſtrument invented on the ſame Principle. By it *we have a moſt uſeful and ready Method of finding the various comparative or ſpecific Gravities of Fluids and ſolid Bodies, to the laſt Degree of Accuracy*; eſpecially in the *New Structure and Method of uſing it*, as repreſented in *Plate* XVI. *Fig.* 4. The Parts of which are as follow; A B, the Foot on which it ſtands; C D, a Pillar ſupporting a moveable Braſs Plate E F, faſten'd thereto by the Screw in the Knob *e*. In the End of this Plate is fix'd an upright Piece I K, ſupporting

in order to render it but little ſpecifically lighter than Water, that it may nearly ſink in it. (3.) A long ſmall Stem or Shank A F fix'd into it on the upper Part, which by its Weight cauſes the Body of the Inſtrument to deſcend in the Fluid with Part of the ſaid Stem. (4.) A ſmall Weight F, ſcrew'd on upon the Top of the Stem or Wire A F, to cauſe the Inſtrument to ſink ſo far, that the Surface of the Fluid may always cut the Stem in its middle Point G.

3. Tho' the above be the uſual Structure and Compoſition of this Inſtrument, it is certainly not the beſt; for if the Body or Globe A B C D be of Ivory, it will imbibe part of the Liquor, and ſo its ſpecific Gravity will be alter'd, and conſequently the Experiment cannot be juſt. If it be of Glaſs, it will be ſubject to break, and will ſtand in need of Screws, Solder, Leather, &c. to make the Parts tight, which will be found alſo both inaccurate and inconvenient. The beſt Way therefore is, to have it made of Metal, *viz.* of Copper; with a Braſs Stem above, ſolder'd on; and a Braſs Ball below to ſcrew on with a very nice Shoulder; then every Part will be durable, and very ſecure from any Alteration of its Weight.

4. When this Inſtrument is ſwimming in the Liquor, the Part of the Fluid diſplaced by it will be equal in Bulk to the Part of the Inſtrument under Water, and equal in Weight to the Weight of the whole Inſtrument. Suppoſe the Weight of the Whole were 4000 Grains, then 'tis evident we can by this means compare together the different Bulks of 4000

another Plate G H, which ſlides backwards and forwards thereon, and is moveable every way about it. In the End of this Plate, at H, is fix'd (by a Nut beneath) a Wire L M, tap'd with a fine Thread from one End to the other; upon this moves the *Swan-Neck Slip* of Braſs N O, to which a very exact Balance is hung at the Point N; to one of whoſe Scales P is appended the heavy Body R, by a fine Horſe-hair or Piece of Silk S: The Weight of the ſaid Body R in the Air is expreſs'd by the Weights put into the Scale Q, to make an *Equilibrium* therewith, which

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Grains of various Sorts of Fluids. For if the Weight F be ſuch as ſhall cauſe the Hydrometer to ſink in *Rain Water*, till its Surface comes to the middle Point of the Stem G; and if after this, it be immerſed in common *Spring Water*, and the Surface obſerved to ſtand $\frac{1}{10}$ of an Inch below the middle Point G, 'tis evident that the ſame Weight of each Water differs in Bulk only by the Magnitude of one Tenth of an Inch in the Stem.

5. Now ſuppoſe the Stem were 10 Inches long, and weigh'd 100 Grains; then every 10th of an Inch would be 1 Grain weight, and ſince the Stem is of Braſs, and Braſs is about 8 times heavier than Water, the ſame Bulk of Water will be equal to $\frac{1}{8}$ of a Grain; and conſequently to the $\frac{1}{8}$ of $\frac{1}{4000}$ Part, that is a 32000dth Part of the whole Bulk, which is a Degree of Exactneſs as great as can be deſir'd. Yet the Inſtrument is capable of ſtill greater, by making the Stem or Neck to conſiſt of a flat thin Slip of Braſs, inſtead of one that is round or cylindrical: By this means we increaſe the Surface, which is the moſt requiſite Thing; and diminiſh the Solidity, by which the Inſtrument is render'd more exact.

6. In order to adapt this Inſtrument to all Sorts of Uſes, there ought to be two different Stems to ſcrew on and off in a ſmall Hole at A. One Stem ſhould be ſuch a nice thin Slip of Braſs, or rather of Steel, like a Watch-Spring ſet ſtrait, as I have juſt mention'd, on one Side of which ought to be the ſeveral Marks, or Diviſions, to which it will ſink

being deſtroy'd by immerging the Solid in the Fluid T V, contain'd in the Glaſs W V, is again reſtored by Weights put into the Scale P. *So that the Weights in the Scale Q compared with*

in various Sorts of Waters, as *Rain-Water*, *River-Water*, *Spring-Water*, *Sea-Water*, *Salt-Spring-Water*, &c. And on the other Side you mark the Diviſion to which it ſinks in various lighter Fluids, as *Hot-Bath-Water*, *Briſtol-Water*, *Lincomb-Water*, *Cheltenham-Water*, *Port-Wine*, *Mountain*, *Madeira*, and various other Sorts of Wine. But in this Caſe, the Weight F on the Top muſt be a little leſs than before, when it was uſed for the heavier Waters.

7. But in Caſe of trying the Strength of ſpirituous Liquors, a common cylindric Stem will do beſt, becauſe of its Strength and Steadineſs; and this ought to be ſo contrived, that when immerſed in what is call'd *Proof-Spirit*, the Surface of the Spirit may be upon the middle Point G, which is eaſily done by duly adjuſting the ſmall Weight F on the Top, and making the Stem of ſuch a Length, that when immerſed in Water, it may juſt cover the Ball, or riſe to A; but when immerſed in pure Spirit, it may riſe to the Top at F; then by dividing the upper and lower Parts G A, G F into ten equal Parts each, when the Inſtrument is immerſed into any Sort of ſpirituous Liquor, it will immediately ſhew how much it is above or below Proof.

8. This Proof Spirit conſiſts of half Water, and half Alcohol, or pure Spirit, that is, ſuch as when pour'd upon Gunpowder, and ſet on Fire, will burn all away, and permit the Powder to take Fire, which it will, and flaſh as in the open Air. But if the Spirit be not ſo highly rectified, there will remain ſome Phlegm or Water which will make the Powder wet, and unfit to take Fire. This Proof-Spirit of any Kind weighs ſeven Pounds twelve Ounces *per* Gallon.

9. The common Method of ſhaking the Spirits in a Vial, and by raiſing a Crown of Bubbles, to judge by the Manner of their riſing and breaking away, whether the Spirit be Proof or near it, is very precarious, and capable of great Fallacy. There is no way ſo eaſy, quick, certain, and philoſophical as this by the *Hydrometer*, which will demonſtrate infallibly the Difference of Bulks, and conſequently ſpecific Gravities, in equal Weights of Spirits, to the 30, 40, or 50 thouſandth Part of the Whole, which is a Degree of Accuracy, beyond which nothing can be deſir'd.

thoſe in the Scale P, *ſhew at once the ſpecific Gravity of the Solid* R *to that of the Fluid* T V (LXIII.)

(LXIII.) The Reaſon why I have given no Account or Figure of the *Common Hydroſtatic Balance*, is, becauſe it is every where to be found in Books of this Sort; but principally, becauſe I would adviſe Gentlemen to the Uſe of that exhibited in the Lectures, which they will find far more expeditious and exact, and therefore much better fitted to anſwer the End of ſuch an Inſtrument. For this Method of ſuſpending the Solid to be weigh'd, by a Horſe-Hair, or fine Silk, requires not the large heavy Glaſs Bucket (as in the *Common Hydroſtatic Balance*) to find the ſpecific Gravities of Solids; or the heavy Glaſs Bubble to find thoſe of Fluids; and therefore the Weight being reduced, the Balance may be ſmaller and nicer, and conſequently ſuch as will turn with a leſſer Difference of Weight.

2. The different Gravities of Solids being firſt compared with that of Water, are then eaſily compared with each other; for let S be the Weight of one Solid, s the Weight of any other, and W the Weight of Water, all of equal Bulks; and let $S:W::5:1$, and $s:W::7:1$; therefore $5W=S$, and $7W=s$; conſequently $S:s::5W:7W::5:7$; that is, the ſpecific Gravities of the Solids S and s are as 5 to 7. And in the ſame Manner the ſpecific Gravities of Fluids are compared, being firſt compared with that of any one Solid.

3. As the very Notion of *ſpecific Gravity* implies Compariſon, ſo there muſt be ſome Sort of Body fix'd upon, whoſe Gravity muſt be made a Standard for the Gravities of other Bodies of equal Bulk to be compared with. This Standard Body muſt have two Properties; firſt, it muſt be eaſy to be had or come at upon all Occaſions; and, ſecondly, it muſt be of a fix'd unalterable Nature, or at all Times the ſame, that there may be no Variation of its Gravity in equal Bulks, in different Times or Places. Now 'tis certain ſuch a Body muſt be of the Fluid Kind, becauſe the beſt Way of finding ſpecific Gravities is by Immerſion.

4. Among Fluids there are none which promiſe the Requiſites for a Standard ſo fairly as Water. Yet here we find various Kinds, and of different Gravities, and none which are quite unexceptionable. *Rain-Water* is the moſt ſo of any, (but this is not always at hand) its ſpecific Gravity being ſo nearly always the ſame, that could it be always had, it would anſwer all our Purpoſes very well. However, *com-*

THE ſpecific Gravity of Fluids is readily determin'd by weighing one and the ſame ſolid

mon Water; by means of the Hydrometer, might be always made a Standard in the following Manner.

5. Let there be a Quantity of Salt diſſolved in Spring-Water to give it a Body or Denſity, a little greater than any Water of that Kind can be ſuppoſed to have naturally. Then let a very exact Hydrometer be ſet to float therein, and obſerve at what Diviſion of the Neck the Surface of the Water ſtands; and that will be the Point, to which the Hydrometer ought to ſink in the Water deſign'd for theſe Experiments, and which it may eaſily be made to do by the Solution of Salt therein, or a Mixture of Salt-Water therewith. And a better Method than this, for procuring a *Standard Fluid*, I am not able to think of.

6. By this Method I only propoſe to fix a Standard for very nice Enquiries; but for common Uſes, *common Water* will do, whoſe Gravity muſt be repreſented by Unity or 1, or, (in caſe of conſtructing Tables with great Accuracy) by 1,000, where three Cyphers are added to give room to expreſs the Ratios of other Gravities in larger Numbers in the Table. In doing this, we have a twofold Advantage; the firſt is, that by this Means we can expreſs the ſpecific Gravities of Bodies to a much greater Degree of Accuracy and Exactneſs. The 2d is, that the Numbers of the Table do alſo expreſs the Ounces *Averdupois* contain'd in a cubic Foot of every Sort of Matter therein ſpecified, becauſe a cubic Foot of common Water, is found by Experiment, to weigh very nicely 1000 Ounces.

7. Now an Ounce *Averdupois* weighs $437\frac{1}{2}$ Grains; and an Ounce *Troy* contains 480 Grains, therefore the *Averdupois* Ounce is to the *Troy* Ounce, as $437\frac{1}{2}$ to 480, or, as 51 to 56 nearly. The *Averdupois* Pound is to the *Troy* Pound as $437\frac{1}{2} \times 16 = 7000$, to $480 \times 12 = 5760$; that is, as 17 to 14 nearly; ſo that a cubic Foot of Water weighs $62\frac{1}{2}$ *lb.* *Averdupois*, and nearly 76 *lb.* *Troy*. And hence it is that ſuch a Table becomes ſo uſeful in the Solution of various Phyſical and Mathematical Problems, and Science of Geometry, extended by Hydroſtatic Principles.

8. From what has been ſaid, nothing can be eaſier to be underſtood, than the Method or Praxis of the Hydroſtatic Balance, for finding the ſpecific Gravities of all ſuch Solids as are heavier than Fluids, and will ſink in them; but for thoſe which are lighter, as Wood, Cork, &c. and will not

Body in them ſeverally; for ſince we ſuppoſe the Balance *in Equilibrio* with the Body ſuſpended in

be totally immerſed, a different Method muſt be taken, *viz.* the lighter is to be connected to an heavier, and ſo both together to be immerſed into the Fluid, as one compound Body, and its ſpecific Gravity thus determined; then having the ſpecific Gravity of the heavier Body, and of the compound, that of the lighter Body will be eaſily found.

9. For ſuppoſe to a Piece of Copper, weighing 18 Grains, be tied a Piece of Elm Wood, weighing 15 Grains, then will the Compound weigh 33 Grains: Again, ſuppoſe the Copper alone in Water weigh'd 16 Grains, and the Compound only 6 Grains; the Loſs of Weight in the Compound will be $27\ (= 33 - 6)$ which is the Weight of an equal Bulk of Water: If from this we ſubtract 2 (the Loſs of Weight in the Copper, and Weight of an equal Bulk of Water) the Remainder 25 will be the Weight of a Bulk of Water, equal to that of the Elm. But the Weight of the Elm was 15 Grains: The ſpecific Gravity therefore of Water is to that of Elm-Wood, as 25 to 15, or, as 1 to 0,6.

Pl. XVII. Fig. 1.

10. Another Way more ſimple to find the ſpecific Gravities of light Bodies (without tying them to heavy ones) is as follows, A B C D is a Veſſel of Water, in which is placed a ſmall Pulley E, on a Foot F; G is a light Body floating on the Surface of Water; H I is a Balance, I E G a Horſe-Hair going round the Pulley, and connecting the light Body G to the End of the Balance. Now let the Veſſel be placed in ſuch a Manner, that the Body G refuſing to go under Water, may draw the Balance H I out of an horizontal Poſition; then ſuch a Weight L, placed in the Scale K, as will draw the Body G under Water, and reſtore the horizontal Poſition, or Equilibrium of the Balance, will diſcover the ſpecific Gravity of the Body G.

11. For ſince Bodies aſcend, as well as deſcend, by the Differences of ſpecific Gravities, or relative Weight; 'tis plain the Weight L, that detains the Body under Water, muſt be equal to the Exceſs of the Weight of the Fluid, above that of the Solid under equal Bulks, and therefore the Weight L, added to that of the Body, will give the Weight of an equal Bulk of Water and of courſe the ſpecific Gravity. For Example, let the Body G be a Piece of Elm-Wood weighing 36 Grains, and the Weight L ſufficient to detain or keep it under Water, will be found equal to 24 Grains; then $36 + 24 = 60 =$ the Weight of Water, equal

the Air, the Equilibrium will be deſtroy'd when the Solid is immerſed in the Fluid, and muſt be

in Bulk to the Elm. Conſequently the ſpecific Gravity of Water is to that of Elm, as 60 to 36, that is, as 1 to 0,6; the ſame as was found before. *Note*, The Weight of the Scale K muſt be added to the Weight L, becauſe 'tis all the Force upon the Point H that keeps the Body under Water.

12. There is yet another Way, and that a very good one, for finding the ſpecific Gravities of ſuch Liquors as will not mix with each other, as Water and all Kinds of Oil; it is thus: Let ABCD be a Veſſel of Water to the Height EF, and GH a Tube dipt in Oil firſt, for the Oil to riſe a ſufficient Height therein, when the upper Orifice H is to be ſtopp'd cloſe with the Finger, and thus immerſed into the Water of the Veſſel; then the Finger being taken away, the Oil will ſubſide and ſtand at the Altitude GI in the Tube. Now ſince this Column of Oil is balanced or ſuſtain'd by a Column of Water of an equal Baſe and Altitude CF, and the Denſities of Fluids are reciprocally as the Bulks, the ſpecific Gravities (which are as the Denſities) will be as the Bulks, that is, the Altitudes reciprocally, (becauſe the Quantity of Matter is the ſame in both) therefore if GI = 100, and CF = 87, the ſpecific Gravity of the Oil will be to that of the Water, as 87 to 100, or that of the Water to that of the Oil, as 1,000 to 0,87.

Pl. XVII. Fig. 2.

13. In like Manner, if ABCD be a long Tube with a little Mercury pour'd in to a ſmall Height *bc*, and then a ſmall Tube *ae*, open at both Ends, be put into it; and laſtly, if Water be pour'd into the large Tube upon the Mercury to the Height of 14 Inches, it will by its Preſſure raiſe the Mercury to the Height *b* of one Inch in the ſmall Tube above the Surface of that in the large one; which ſhews that the ſpecific Gravity of Mercury is to that of Water nearly as 14 to 1. As will appear by the Experiment if accurately made.

14. Once more; another Way, which in ſome Caſes may do very well, to find the ſpecific Gravities of any different Liquids, is by means of a recurved Tube ADG, in which if Quickſilver be pour'd firſt to fill the Bottom Part, and then one Liquid into one Leg, and another Liquid into the other Leg, in ſuch manner that they preſs on the Quickſilver on each Side equally, which will appear by the Surface on each Side being in the ſame horizontal Line CE; then will the ſpecific Gravity of the Liquid in the Leg AC be to that in the Leg

then reſtored by Weights put into that Scale to which the Body is appended. Theſe Weights

E G, as the Altitude of the latter E F to the Altitude of the former C B.

15. By the Table of ſpecific Gravities it appears that Gold is the heavieſt Body in Nature, and that Mercury is next to it in Gravity; and conſequently Gold only will ſink in Mercury; therefore by weighing Gold in Mercury and Water, the ſpecific Gravities of theſe two Fluids may be determined by the Hydroſtatic Balance: But ſince Mercury will readily adhere to Gold, it will create us ſome Trouble in getting it off again, which muſt be done by Fire, or *Aqua Fortis*; as the latter is not always at hand, 'tis common to put the Gold into the Fire, but Care muſt be taken not to put it into a Coal Fire, becauſe the acid Spirit of the Sulphur will leſſen the Coheſion of the Particles of the Gold, and ſo render it very brittle, and apt to break, eſpecially when laid on a cold Stone to cool; the beſt way therefore to find the ſpecific Gravities of theſe Fluids is that above directed in *Art.* 13.

16. *A TABLE of SPECIFIC GRAVITIES.*

Of METALS.

Fine or pure Gold,	19,640
Gold of a Guinea of *George* II.	17,150
Gold of a Moidore,	17,140
Silver fine or pure,	11,091
Silver of a Shilling of *George* II.	10,000
Lead,	10,130
Copper,	9,000
Braſs caſt,	7,850
—— wrought,	8,000
Steel temper'd	7,704
Iron,	7,645
Tin,	7,550

17. *MINERALS, ORES,* &c.

Copper Ore,	3,775
Lead Ore,	6,800
Biſmuth,	9,700
Turbith Mineral,	8,235
Antimony from *Germany*,	4,000
———— from *Hungary*,	4,700
Speltar,	7,065

will ſeverally expreſs the Gravities of an equal Bulk of the reſpective Fluids; and conſequently,

18. *STONES, FOSSILS, &c.*

Adamant or Diamond,	3,400
A Pſeudo-Topaz,	4,270
A Pſeudo-Hyacinth,	2,631
A Pſeudo-Jaſper,	2,666
A *Bohemian* Granate,	4,360
Swediſh Granate,	3,978
Onyx-Stone,	2,510
A Cornelian,	2,568
An *Engliſh* Agate,	2,512
Turcois Stone,	2,508
Sardrachates,	3,598
A golden Marcaſite,	4,589
Rock Cryſtal,	2,659
Iſland Cryſtal,	2,720
Lapis Nephriticus,	2,894
Lapis Lazuli,	3,054
Lapis Hæmatites,	4,360
Lapis Calaminaris,	5,000
Lapis Judaicus,	2,500
Lapis Manati,	2,270
Lapis Amianthus, or Aſbeſtos from *Wales*,	2,913
—— ditto, from *Italy*,	2,360
Glaſs of the common Sort,	2,666
Flint,	2,542
Black *Italian* Marble,	2,704
White *Italian ditto*,	2,707
A fine Marble,	2,700
Another *ditto* of *Italy*,	2,718
A pellucid Pebble,	2,641
A Selenitis,	2,322
Mundick, or Gold Spar,	4,430
Kidney Stone,	3,600
Blue Stone,	2,740
Star Stone,	3,450
Hard Paving Stone,	2,460
Burford Stone,	2,049
Alabaſter,	1,875
Rag-Stone,	2,470

they may thus be compared with each other, or all of them with the Gravity of *common Water*,

Rotten Stone,	1,980
Copperas Stone,	4,300
Chalk,	2,370
Slate,	2,740
Oil-Stone,	2,380
An Hone,	2,388
China,	2,270
Piece of Brown Stone Bottle,	1,777
Piece of White Stone Mug,	2,250
Talc,	2,657
—— of *Venice*,	2,780
—— of *Jamaica*,	3,000
Armenian Bole, or Earth,	2,727
Common Sea Coal,	1,300
Magnet, or Loadſtone,	1,840
Piece of *Stonehenge*, very hard,	2,618
——— *ditto*, of a ſofter Sort,	2,500
Briſtol Stone,	2,510

19. *ANIMAL SUBSTANCES*,

Bone of an Ox,	1,656
Ivory,	1,826
The Tip of a Rhinoceros's Horn,	1,242
——— of an Ox Horn,	1,840
——— of a Stag's Horn,	1,875
Calculus humanus,	1,700
Ditto,	1,240
Ditto,	1,433
Ditto,	1,660
Oyſter Shell	2,092
Murex Shell,	2,590
A Cockle Shell,	2,520
Mother of Pearl	2,480
A Piece of hard Fiſh Skin,	1,621
A Piece of dried Fleſh of Fiſh,	1,129
The Quill Part of a Feather,	1,330

20. *VEGETABLE SUBSTANCES.*

Dry Box-Wood,	1,030
Dry Oak,	0,925
Dry Elm,	0,600

as uſual, and diſpoſed in a proper Table; making that of Water 1,000.

Dry Aſh ſappy,	0,734
Ditto more dry, about the Heart,	0,845
Dry Maple,	0,755
Dry Fir,	0,546
Dry Cedar,	0,600
Dry Walnut-tree,	0,631
Dry Yew,	0,760
Beech meanly dried,	0,854
Crab-tree meanly dried,	0,765
Lignum Vitæ,	1,327
Lignum Nephriticum,	1,200
Lignum Aloes,	1,777
Lignum Brazilicum,	1,030
Lignum Rhodium,	1,125
Lignum Aſphaltum,	1,179
Lignum Guaiacum,	1,337
Saſſafras Wood,	0,482
Red Wood,	1,031
Red Santalum Wood,	1,128
White *Ditto*,	1,041
Citrine *Ditto*,	0,809
Speckled Wood of *Virginia*,	1,313
Maſtic Wood,	0,849
Ebony,	1,177
Cork,	0,240
Good Wheat of the laſt Year,	0,757
White Oats,	0,472
Blue Peaſe,	0,795
White Peaſe very dry,	0,807
Barley of the laſt Year,	0,658
Malt made of the ſame,	0,485
Field Beans very dry,	0,807
Wheaten Meal unſifted,	0,495
Rye Meal unſifted,	0,454
Wood-Aſhes,	0,930

21. *MISCELLANEOUS SUBSTANCES.*

Amber,	1,040
Jet,	1,238
Bezoar Oriental,	1,530
——— Occidental,	1,500
Sulphur Common,	1,800

IN the ſame Manner, if divers Solids are firſt weigh'd in Air, and then afterwards immerſed

Sulphur Vivum,	2,000
Borax,	1,720
Wood petrified,	2,341
Coral Red,	2,689
—— White,	2,500
Corallachates,	2,605
Cinnabar Natural,	7,300
—— Artificial,	8,200
—— of Antimony,	6,044
The reputed Silver Ore of Whales,	7,464
The Metal thence extracted,	11,087
Ceruſe,	3,156
Tartar Common,	1,849
—— Emetic,	2,246
—— Vitrioli,	2,298
Cream of Tartar.	1,900
Camphire,	0,995
Mercury Crude,	13,593
—— Diſtilled once,	13,570
—— Sublimed 511 Times	14,110
Glaſs of Antimony,	5,280
Vitriol of *Dantzick*,	1,715
——*Engliſh*,	1,880
——White,	1,900
Sal Gemmæ,	2,143
—— *Prunellæ*,	2,148
—— *Polychreſtum*,	2,148
—— *Ammoniacum*,	1,453
—— *Mirabile Glauberi*,	2,246
Salt of Hartſhorn,	1,496
Salt of Vitriol,	1,900
Alum,	1,714
Nitre,	1,900
Gum *Arabic*,	1,375
Gum Tragacanth,	1,333
Myrrh,	1,250
Verdigreaſe,	1,714
Opium,	1,363
Litharge of Gold,	6,000
—— of Silver,	6,044
Bees-Wax Yellow,	0,960

in the ſame Fluid, as Water, for Inſtance, the *Equilibrium* will be deſtroy'd; which will be re-

Bees Wax White,	0,865
Pitch,	1,190
Tutty,	4,615
Honey,	1,450
Roſin,	1,100
Craſſamentum of the human Blood,	1,126
Serum of the human Blood,	1,030
Piece of petrified Bone,	1,895

22. *Of FLUIDS.*

Rain Water,	1,000
Diſtill'd Water,	0,993
Well or Spring Water,	0,999
River Water,	1,009
Sea Water,	1,030
Aqua Fortis,	1,300
Aqua Regia,	1,234
Oil of Vitriol,	1,700
Oil of Clove-Gilliflowers,	1,034
Oil of Amber,	0,978
Oil of Aniſeed,	0,994
Oil of Caraway-ſeed,	0,940
Oil of Linſeed,	0,932
Oil of Mint,	0,975
Oil of Olives,	0,913
Oil of Orange,	0,888
Oil of Origany,	0,940
Oil of Roſemary,	0,934
Oil of Saſſafras,	1,094
Oil of Spikenard,	0,936
Oil of Turpentine,	0,792
Spirit of Turpentine,	0,874
Spirit of Wine rectified,	0,866
Ethereal Spirit of Wine,	0,732
Spirit of Vitriol,	1,203
Spirit of Amber,	1,030
Spirit of Hartſhorn,	1,073
Spirit of Urine,	1,120
Spirit of Honey,	0,895
Spirit of Nitre,	1,315
——— *Ditto* rectified,	1,610
Spirit of Sea-Salt,	1,130

ſtored, as before, by putting in ſo much Weight as is equal to the Weight of the ſame Bulk of

Spirit of Tartar,	1,073
Tincture of Antimony,	0,866
Butter of Antimony,	2,470
Balſam of *Tolu*,	0,896
Lixivium of Salt of Tartar,	1,550
Burgundy Wine,	0,953
Canary,	1,033
Red Wine from *Pontac*,	0,993
White-Wine Vinegar,	1,011
Diſtill'd Vinegar,	1,030
Milk of Goats,	1,009
Cow's Milk,	1,030
Urine,	1,030

23. Since all Bodies are ſubject to expand with Heat, and be condenſed with Cold, it muſt follow, that the ſpecific Gravities of Bodies cannot be preciſely the ſame both in Summer and Winter. This was firſt obſerved in Experiments by M. *Homberg*, and after him by M. *Eiſenſchmid*, who found the abſolute Weight of a cubic Inch of ſeveral Sorts of Bodies in Summer and Winter was as in the Table below.

	Summer.			Winter.		
	oz.	*dr.*	*gr.*	*oz.*	*dr.*	*gr.*
Brandy,	0	4	32	0	4	42
Diſtill'd Water,	0	5	8	0	5	11
Spring Water,	0	5	11	0	5	14
River Water,	0	5	10	0	5	13
Spirit of Nitre,	0	6	24	0	6	44
Spirit of Sea-Salt,	0	5	49	0	5	55
Spirit of Vitriol,	0	5	33	0	5	38
Oil of Vitriol,	0	7	59	0	7	71
Milk,	0	5	20	0	5	25
Mercury,	7	1	66	7	2	14
Vinegar,	0	5	15	0	5	21
Ditto Diſtill'd,	0	5	11	0	5	15

24. All this may be ſtill render'd more apparent by taking the ſpecific Gravities of theſe Liquors, (1.) when they are very cold; (2.) when they are pretty warm; and (3.) when they are very hot; for in theſe ſeveral Caſes the Subject will diſcover a much leſs Denſity or Weight in equal Bulks, either by the Common or Hydroſtatic Balance.

Water: The Gravity, therefore, of every Solid is thus compared with Water, and conſequently

25. And as in this, and many other Caſes, it is required to be very exact in weighing Bodies, even beyond what is attainable by the niceſt Mechaniſm of the Inſtrument itſelf, I ſhall here give the Reader an Account of an Improvement of the Common Balance in this Reſpect, from the late learned *S'Graveſande*; and it will be the more pertinent in this Place, as it depends on an hydroſtatic Principle. This Inſtrument ſerves equally for Exactneſs in common as in hydroſtatical Matters.

26. The Figure of the Machine repreſents the Balance in its hydroſtatic Uſe. I ſhall firſt deſcribe the Machine; then ſhew the new-contrived Artifice for Exactneſs; and, laſtly, give an Inſtance of its univerſal Uſe. VCG is the Stand or Pillar fixed in the Table. From the Top at A hangs, by two ſilken Strings, the horizontal Piece or Bar BB; from which is ſuſpended, by a Ring at *i*, the fine Beam of a Balance *l*, which is kept from deſcending too low on either Side by the gentle ſpringing Piece *txyz*, fixed on the Supporter M. The Harneſs is annulated at *o*, to ſhew diſtinctly the perpendicular Poſition of the *Examen* by the ſmall pointed Index fixed above it. Pl. XVIII

27. The Strings by which the Balance is ſuſpended paſſing over two Pulleys, one on each Side the Piece at A, go down to the Bottom on the other Side, and are hung over the Hook at *v*; which Hook, by means of a Screw P, is moveable about 1¼ Inch backwards and forwards, and therefore the Balance may be raiſed or depreſſed as much. But if a greater Elevation or Depreſſion be required, the Sliding-Piece S, which carries the Screw P, is readily moved to any Part of the ſquare Braſs Rod VK, and fixed by means of a Screw.

28. The Motion of the Balance being thus provided for, the reſt of the *Apparatus* is as follows: HH is a ſmall Table fixed upon a Piece D, under the Scales *d* and *e*, and is moveable up and down in a long Slit in the Pillar above C, and faſten'd at any Part with a Screw behind. At the Point in the Middle of the Bottom of each Scale is hung by a fine Hook a Braſs Wire *ad*, *ac*. Theſe paſs through two Holes *m*, *m*, in the Table; and to the Wire *ad* is ſuſpended a curious cylindric Wire *rs*, perforated at each End for that Purpoſe. This Wire *rs* is covered with Paper graduated by equal Diviſions, and is about five Inches long.

with each other; as in the Table below.

THE *Application of Hydroſtatics* to the ſeveral

29. In the Corner of the Table at E is fixed a Braſs Tube, in which a round Wire *bl* is ſo adapted as to move neither too hard nor too freely by its flat Head I. Upon the lower Part of this moves another Tube Q, which has Friction enough to cauſe it to remain in any Poſition required; to this is fixed an Index T, moving horizontally when the Wire *bl* is turn'd about, and therefore may be eaſily ſet to the graduated Wire *r.s*.

30. To the lower End of the Wire *rs* hangs a Weight L, and to that a Wire *pn* with a ſmall Braſs Ball *g*, about ¼ of an Inch in Diameter. On the other Side, to the Wire *ac* hangs a large Glaſs Bubble R by a Horſe-hair. Let us at preſent ſuppoſe the Weight L taken away, and the Wire *pn* ſuſpended from S; and on the other Side let the Bubble R be taken away, and the Weight F ſuſpended in its Room at *c*. This Weight F we ſuppoſe to be ſuch as will keep *in Equilibrio* with the ſeveral Parts appended to the other Scale, at the ſame Time that the middle Point of the Wire *pn* is in the Surface of the Water in the Veſſel N.

31. The Wire *pn* is to be of ſuch a Size, that the Length of one Inch ſhall weigh four Grains. Hence it is evident, ſince Braſs is eight times heavier than Water, (ſee *Art.* 16.) that for every Inch the Wire ſinks in the Water, it will become half a Grain lighter, and half a Grain heavier for every Inch it riſes out of the Water: Conſequently, by ſinking two Inches below the middle Point, or riſing two Inches above it, the Wire will become one Grain lighter or heavier.

32. And therefore, if when the middle Point is at the Surface of the Water *in Equilibrio*, the Index T be ſet to the middle Point a of the graduated Wire *rs*, and the Diſtance on each Side a*r* and a*s* contain 100 equal Parts; then, when in weighing Bodies the Weight is deſired to the *hundredth Part* of a Grain, it may eaſily be had by proceeding in the following Manner. Let the Body to be weigh'd be placed in the Scale *d*, and put the Weights in the Scale *e*; and let theſe be ſo determined, that one Grain more ſhall be too much, and one Grain leſs too little. Then the Balance being gently moved up or down by the Screw P, till the Equilibrium be nicely ſhewn at *o*; and then if the Index T be at the middle Point a of the Wire *rs*, it ſhews the Weights put into the Scale *e* are juſt equal to the Weight of the Body.

Uſes of Life will be evident from the following Inſtances; having firſt premiſed, *that a Cubic*

33. But if the Index T ſtand at any Part between a and *r*, it ſhews the Number of Grains in the Scale *e* were more than equal to the Weight of the Body in the Scale *d*, becauſe the Wire *pn* is now made lighter by ſinking below the middle Point. Thus, ſuppoſe the Weights put into the Scale *e* were 1095 Grains, and the Index T cuts the 36th Diviſion above a, it ſhews that 36 hundredth Parts of a Grain are to be added, or that the Weight of the Body is 1095,36 Grains.

34. On the other hand, had the Index ſtood at 36, the Diviſion below a, it would have ſhewn the Weights in the Scale *e* were more than equal to the Weight of the Body by 36 Hundredths of a Grain; and that there the Weight of the Body was 1094,64 Grains. By this Method we find the abſolute Weight of the Body; the relative Weight is found by weighing it hydroſtatically in Water as follows.

35. Inſtead of putting the Body in the Scale *d*, as before, let it be appended with the Weight F at the Hook *c*, by a Horſe-hair as at R, ſuppoſing the Veſſel of Water O were away; then the Equilibrium being made, the Index T ſtanding between a and *r*, at the 36th Diviſion, ſhews the Weight of the Body 1095,36 Grains. As it thus hangs, let it be immerſed in the Water of the Veſſel O, and it will become lighter by much; the Scale *e* will deſcend till the Beam of the Balance reſts on the Supporter *x*. Then ſuppoſe 100 Grains put into the Scale *d* reſtored the Equilibrium preciſely, ſo that the Index T did again point to the 36th Diviſion above a: It is plain the Weight of an equal Bulk of Water would in this Caſe be exactly 100 Grains.

36. But if the 100 Grains in the Scale *d* cauſe it to preponderate a little, then by turning the Screw P the Balance may be raiſed, till the Wire *pn* becoming heavier reſtores the Equilibre. Let now the Index T cut the 6th Diviſion above a; then 36 — 6 = 30, which ſhews that the Wire *pn* is now $\frac{3}{10}$ of a Grain heavier than before; therefore the Weight of the Water is only 99,7 Grains: Whence its Gravity to that of the Body is as 99,7 to 1095,36; as required.

37. After a like Manner may this Balance be applied to find the ſpecific Gravities of Fluids; which will not be difficult to thoſe who apprehend what has been already ſaid. In Practice, it will be neceſſary to uſe great Precaution in every Particular; the Wire *pn* ſhould be oil'd, and then wiped as

Foot of common Water weighs very exactly 1000 *Ounces Averdupois, or* 62 *Pounds and an half*; which may be reduced to *Troy* Weight, by conſidering, that the *Averdupois* Pound is to the *Troy* Pound as 17 to 14, and the *Averdupois* Ounce to the *Troy* Ounce as 51 to 56.

HENCE, *to find the Quantity of Preſſure againſt the Sluice or Bank that pens the Water*, we have this Rule: *Multiply the Area of the Sluice under Water by the Depth of the Centre of Gravity in Feet, and that Product again by* $62\frac{1}{2}$; *the Product will be the Number of Pounds required. Example:* Admit the Length of the Sluice be 20 Feet, the Depth of Water 5; then will the Area under Water be 100 ſquare Feet; which multiplied by $2\frac{1}{2}$, the Depth of the Centre of Gravity, gives 250 cubic Feet; which again multiplied by $62\frac{1}{2}$, gives 15625 *lb.* equal to 7 Tons nearly (LXIV.)

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clean as poſſible; enough will remain to prevent the Water adhering thereto: Alſo the Balance ought to be raiſed very gently, and when come to an Equilibrium ſhould be gently agitated, to ſee if it will come ſo again.

(LXIV) That the Area under Water multiplied by the Depth of the Centre of Gravity gives the Number of ſolid Feet of Water preſſing againſt the Sluice, is evident from *Annot*. LV. *Art*. 13, 14, 15, &c. And this Product is to be multiplied by $62\frac{1}{2}$, becauſe ſo many Pounds is the Weight of one ſolid or cubic Foot of Water. See *Annot*. LXII. *Art*. 7.

2. This Example gives the Numbers which I took from a Pen that was made acroſs a River; and though it may ſeem wonderful that all the Water in the River ſhould affect the Pen with no more Force of Preſſure, yet this will be found conſiſtent with the Laws of the Action of Fluids, when that is underſtood which we have deliver'd in *Annot*. LVI. For the Water of the River does no otherwiſe influence the Pen, than as it ſuſtains a Plane of fluid Particles to a certain Height

AGAIN: *Since the Weight of Bodies is always as the ſpecific Gravity in equal Bulks*, it follows, that the Numbers in the Table do alſo expreſs the Number of *Averdupois* Ounces contain'd in a cubic Foot of each reſpective Sort of Matter therein mention'd. Therefore, *if the Magnitude of any Body be multiplied by the ſpecific Gravity, the Product will be its abſolute Weight.* Thus, ſuppoſe I would know what Weight of Lead will cover a Church whoſe Area is 30000 Feet, and the Thickneſs of the Lead $\frac{1}{100}$ of a Foot: Then *per* Rule, $(30000 \times \frac{1}{100} =)\ 300 \times 11325 =$ 339750 Ounces, or $97\frac{4}{5}$ Tons; the Weight required.

ANOTHER uſeful Problem is, to *find the Magnitude of any Thing, when the Weight is known*; which is done *by dividing the Weight by the ſpecific Gravity in the Table, the Quotient is the Magnitude ſought:* For Inſtance, What is the Magnitude of ſeveral Fragments of *Coral* whoſe Weight is 7 Ounces? Divide 7 by the ſpecific Gravity 2690, the Quotient is $\frac{26}{10000}$ of a cubic Foot: then $\frac{26}{10000} \times 1728 = 4\frac{1}{2}$ cubic Inches, very nearly the Magnitude required.

ALSO, *by knowing the Magnitude and Weight*,

in immediate Contact with the Pen. This ſingle Plane of Particles it is that gives all the Preſſure on the Pen; and they would do the ſame, could they be otherwiſe ſupported againſt it, if all the reſt of the Water in the River were taken away. Thus the Waters of the Sea or Ocean lying againſt any Dam or Bank will preſs it with no greater Force than the leaſt Quantity of Water ſtanding to the ſame Height, over the ſame Extent of the Dam.

we can find the ſpecific Gravity, by dividing the Weight by the Magnitude in cubic Feet. Thus ſuppoſe a Piece of Marble contain 4 cubic Feet, and weighs 603 *lb.* or 10800 Ounces; then $\frac{10800}{4} =$ 2700, the ſpecific Gravity required, as *per* Table.

HAVING given the ſpecific Gravity of Gold to Silver as 19 to 11, and ſuppoſe any Compound thereof, as King *Hiero*'s Crown, whoſe ſpecific Gravity is 16; to determine the Proportion and Weight of the Gold and Silver employ'd in making it, ſay, *As the Difference of the ſpecific Gravities of the Compound and the lighter Ingredient*, viz. 5, *is to the Difference of the ſpecific Gravities of the heavier Ingredient and the Compound*, viz. 3, *ſo is the Bulk of Gold to that of Silver made uſe of.* That is, if the whole Crown were divided in 8 Parts, the Gold would conſiſt of 5, and the Silver of 3: Then the Magnitudes 5 and 3, multiplied by the ſpecific Gravities 19 and 11 ſeverally, will give the Numbers 95 and 33, which expreſs the *Proportion of the Weight of the Gold to that of the Silver* (LXV.)

(LXV) 1. The Rules here given for ſolving the three firſt Problems are only thoſe Theorems expreſs'd in Words which you find in *Annot.* LVII, *Art.* 9, 10, 11. But that which concerns the fourth, relating to King *Hiero*'s Crown, I ſhall here demonſtrate as follows.

2. Let A and B be any two Sorts of Matter, and a and b denote their ſpecific Gravities; the Compound made of thoſe two Bodies will be $A + B$; and let c be the ſpecific Gravity thereof. Then ſince the abſolute Weight of a Body is equal to its Bulk multiplied by its ſpecific Gravity (See *Annot.* LVII. *Art.* 10.) we ſhall have $Aa =$ Weight of the Body A, and $Bb =$ Weight of the Body B, and $\overline{A + B} \times c =$ Weight of the Compound. But $Aa + Bb = \overline{A + B} \times c =$

SINCE Bodies of different ſpecific Gravities, equiponderating each other in Air, upon being immerſed into Water, will have the *Equilibrium*

$Ac + Bc$; whence $Aa - Ac = Bc - bB$; conſequently $A : B :: c - b : a - c$, which is the Rule above laid down where A is the Quantity of Gold, and B that of Silver, and $A + B$ the Compoſition of *Hiero*'s Crown.

3. When King *Hiero* (ſuſpecting the Workmen had alloy'd the Gold with more Silver than was neceſſary) ſent to *Archimedes* to examine into, and detect the Fraud, if there were any; this great Mathematician was long at a loſs to think of any Method of doing it; till one Day getting into a Tub full of cold Water to bathe himſelf, he obſerv'd, that as he enter'd the Tub the Water ran out, and he immediately ſaw it muſt follow, that if the Tub were full, the Water which ran out upon his Immerſion, muſt be equal in Bulk to his Body.

4. Hence the Philoſopher began thus to reaſon: If I immerſe the Crown in a Veſſel full of Water, it will protrude as much as is equal to its Bulk. If after this I immerſe the ſame Weight of pure Gold, and pure Silver, I ſhall get Bulks of Water equal to each; conſequently having the Bulks of Gold, Silver, and Crown of equal Weight, I have the ſpecific Gravities, which muſt be as the Bulks inverſely. Then I proceed to find the Ratio, or Proportion of Gold to the Silver in the Crown as follows.

5. Suppoſe A M L B be a Veſſel fill'd with Water to the Height D C, and that the Maſs of Gold equal in Weight to the Crown, being immerſed into the Water, raiſes the Surface thereof to F; and after that, the Maſs of Silver of the ſame Weight immerſed raiſes the Surface to G; then if the Height of the Veſſel above C be divided into equal Parts, and $LDF = 11$, and $DG = 19$; it is plain the Bulks of the Gold and Silver will be as D F to D G, and the ſpecific Gravities as D G to D F. Laſtly, if the Crown be immerſed, it will raiſe the Surface to ſome Height between F and G, ſuppoſe to H, ſo that $DH = 16$. Whence the Proportion of the Bulks of the Gold and Silver in the Crown may be determined. Pl. XVII. Fig. 5.

6. For ſince the Difference of ſpecific Gravities of the Gold and Silver is $DG - DF = FG = 8$; if the Bulk of the Crown be divided in 8 equal Parts, it is evident, that as many of thoſe Parts as conſiſt of pure Gold, ſo many of thoſe

immediately deſtroy'd by the greater Reſiſtance of the Fluid; and conſequently the greater Loſs of Weight in the lighteſt and moſt bulky Body;

Diviſions in the Side of the Veſſel will the Surface of the Fluid ſtand below the Point G, becauſe if all the 8 Parts of the Crown were Gold, the Crown would cauſe the Water to ſtand 8 of thoſe Diviſions below G. On the other Hand, as many of the 8 Parts of the Crown as are Silver, ſo many of the Diviſions will it cauſe the Surface of the Water to riſe above F, becauſe, if all the 8 Parts of the Crown were Silver, it would raiſe the Water 8 of thoſe Diviſions above F.

7. Therefore, becauſe the Crown, when immerſed, raiſes the Water to the Height D H, and H is 3 Diviſions below G, it ſhews that 3 of the 8 Parts of the Crown are Gold; and conſequently the other 5 Parts are Silver, as H is 5 of the Diviſions above F. Hence the Bulk of Gold in the Crown is to that of the Silver as 3 to 5, as before determined. That *Archimedes* took ſome Method like this, is certain; and it is ſaid, he offer'd to *Jupiter* a Hecatomb of Oxen for inſpiring him with the Thought.

8. To the Problems in the Lecture, I ſhall add the following. Let there be two Bodies A and B, one heavier, the other lighter than Water; let their ſpecific Gravities be a and b, and let the Weight of A be given, and denoted by W; and let it be required to find what Weight (w) of the lighter Body B, muſt be connected with the heavier, that ſo the Compound ſhall have the ſame ſpecific Gravity with Water. Now becauſe $A = \frac{W}{a}$ and $B = \frac{w}{b}$, therefore $A + B = \frac{W}{a} + \frac{w}{b} = \frac{Wb + wa}{ab}$ = Bulk of Water, which multiplied by its Specific Gravity (c) gives the Weight of Water $\frac{Wbc + wac}{ab} = W + w$ = the Weight of both the Bodies. Whence we have $Wbc + wac = Wab + wab$; and ſo $Wbc - Wab = wab - wac$; conſequently $\frac{Wbc - Wab}{ab - ac} = w$; or if $c = 1$, then we have $\frac{Wb - Wab}{ab - a} = \frac{b - ab}{ab - a} \times W = w$ = Weight of the Body B required.

9. For Example; ſuppoſe the ſpecific Gravity of a Man's Body, Water, and Cork be as 10, 9, $2\frac{1}{4}$; then $a = 10$,

therefore it follows, that in *buying Gold*, which is *heavier than Braſs*, we ſhould chuſe the *lighteſt Air*, *i. e.* when the Mercury in the Barometer

$c = 9$, and $b = 2\frac{1}{4}$; let $W = 150$ *lb.* = Weight of the Man's Body; then will $\frac{Wab - Wbc}{ab - ac} = w = 5$; therefore if ſuch a Man were to take 5 *lb.* of Cork with him into the Water, he would be of the ſame ſpecific Gravity of the Water, and ever ſo little more would make it impoſſible for him to ſink; whence the Art of learning to ſwim might by this Problem be greatly facilitated.

10. *To find the Weight of a Globe of Water one Inch in Diameter.* We ſhall here chuſe *Troy* Weight, where one cubic Foot weighs 76 *lb*; and therefore one cubic Inch weighs $\frac{76}{12 \times 12 \times 12}$ of a Pound, or $\frac{76 \times 12 \times 20 \times 24}{12 \times 12 \times 12}$ Grains, that is, $\frac{76 \times 40}{12}$ or $\frac{760}{3}$ Grains. But in Geometry we prove the Cube is to its inſcribed Sphere as 1 to 0,524; therefore as $1 : 0{,}524 :: \frac{760}{3} : 132$ Grains = the Weight of a Globe of Water one Inch Diameter.

11. *To find the Diameter of a Globe by weighing it hydroſtatically.* Let it firſt be weigh'd in Air, and then in Water, and the Difference of thoſe Weights will be the Weight of an equal Bulk or Globe of Water, which call W; now the Weights of homogeneous Bodies are as their Bulks, which (in Spheres) are as the Cubes of their Diameters. But the Weight of a Globe of Water 1 Inch in Diameter is 132 Grains (by *Art.* 10.) Therefore we ſay as $132 : W :: 1^3 : D^3$; Whence $\frac{W}{132} = D^3$, conſequently $\sqrt[3]{\frac{W}{132}} = D =$ Diameter of the Globe required.

12. *To find the ſolid Content in cubic Inches of any irregular Body.* Let W = Difference of its Weight in Water and Air = Weight of an equal Bulk of Water. Then as the Weight of a cubic Inch of Water $\frac{760}{3} : W :: 1 : \frac{3}{760} \times W =$ ſolid Content of the Body required.

13. *To find the Proportion of Magnitude between any two Bodies propoſed.* Let each be weighed in Air and Water, and the Loſſes of Weight they each ſuſtain will be the Weights

ſtands loweſt; but in *buying precious Stones*, *Pearls*, &c. which are *lighter than Braſs*, the beſt Time to do it in is *when the Air is heavieſt* and *moſt buoyant*, *viz.* when the Quickſilver ſtands higheſt in the Barometer: But in *ſelling Gold or Jewels*, the contrary Rules are to be obſerved in regard to the Gravity of the Air (LXVI.)

of two Bulks of Water, equal to thoſe Bodies reſpectively; and conſequently will expreſs the Ratio of the Magnitudes of thoſe Bodies.

SCHOLIUM.

14. The Excellency of this Hydroſtatic Method, above all others, appears from hence, that it is univerſal, and equally adapted to all Sorts and Shapes of Bodies, which common Geometry is not. How would it puzzle a Geometrician to exhibit the juſt Dimenſions or Bulk of a Fiſh, for Inſtance? How does the aukward Figure of his Body, the Appendage of Fins and Tail elude the Principles of his Art? Whilſt the Philoſopher immediately, and without any Trouble, gives the Anſwer by his *Hydroſtatic Balance*. Again, were a Geometer aſk'd, which is biggeſt, *a Guinea or a Shilling*, and what the Difference? He would find it vain to conſult his Art, and muſt borrow his Aid and Anſwer from this moſt uſeful Science, which excels equally in Accuracy as Univerſality.

(LXVI) 1. Since a cubic Inch of Gold weighs $10,\frac{36}{100}$ Ounces *Troy*, and a cubic Inch of Air $\frac{2}{7}$ of a Grain, when the Air is of a mean Gravity; if we ſay, As $10,36 : \frac{2}{7} :: 12 : \frac{24}{72,52}$ = the Parts of a Grain which 12 Ounces or a Pound of Gold will loſe when the Air is in a mean State. Now ſince the Air, when it ſupports a Column of Mercury 28 Inches high, is lighter by $\frac{1}{15}$ Part than when it ſuſtains a Column of 30 Inches Height; therefore $\frac{24}{72,52} + \frac{1}{20} = \frac{552,52}{1450,4}$ = the Parts of a Grain which ſo much Air weighs when heavieſt, as is equal in Bulk to a Pound of Gold; and twice the Quantity, *viz.* $\frac{1105,04}{1450,4}$ = the Weight of Air equal in

ONCE more: Since *the Goodneſs of Mineral Waters, Drugs, Metals, precious Stones,* &c. *is beſt ſhewn by their ſpecific Gravity*, it will at once appear of what vaſt Importance the Hydroſtatic Balance is, and how abſolutely neceſſary in the Hands of every judicious Dealer in any ſuch kind of Commodities (LXVII.) (LXVIII.)

Bulk to two Pounds of Gold. But this is a little leſs than one Grain; let us ſuppoſe it one Grain.

2. Then two Pounds of Gold will loſe of its Weight one Grain; and ſince Gold is twice as heavy as Braſs that it is weigh'd againſt, the Braſs Weights will have twice the Bulk, and will there loſe two Grains of its abſolute Weight. If an Equilibrium be now made, when the Air is lighteſt, or leſs by one Tenth of its former Weight, the Gold will loſe one Tenth of a Grain, and the Braſs will loſe two Tenths of a Grain; conſequently the Buyer will get one Tenth of a Grain of Gold in this laſt State of the Air, more than he would have had in the former, or heavieſt State; becauſe now one Tenth of a Grain more muſt be added to the Braſs to make an Equilibrium. And this in Value is the fifth Part of a Penny, reckoning Gold at Two-pence *per* Grain; or one Part in 115200.

3. Since the ſpecific Gravity of Diamond is to that of the Braſs Weights as 1 to 3, if we make an Equilibrium between the two Pounds of Braſs and Diamond, when the Braſs loſes two Tenths of a Grain, the Diamond will loſe ſix Tenths of a Grain; conſequently four Tenths of a Grain muſt be ſubducted from the Braſs for an Equilibrium in the lighteſt State of the Air, and ſo much the Buyer will loſe of Diamond more than when the Air is heavieſt of all; which is about one Part in 28800 of the Whole. Whence it appears, that a Regard to the State of the Air is a Matter of more Curioſity than Importance, the Advantage being ſo very inconſiderable in either Caſe.

(LXVII) 1. How great the Uſefulneſs and Importance of Hydroſtatic Knowledge is to *Phyſicians, Chemiſts, Apothecaries, Jewellers, Goldſmiths,* &c. will appear by reading Mr. *Boyle*'s excellent *Medicina Hydroſtatica*; in which Book the ſkilful Author propoſes the following Uſes to be made of Hydroſtatic Knowledge, *viz.*

2. *Firſt*, To explore the Nature and Difference of Foſſils by finding their ſpecific Gravities. For ſince the moſt pure and homogeneous Kind of Stones are in Gravity to Water as about $2\frac{1}{2}$ to 1; and Tin, the lighteſt of Metals, is to Water in Gravity as about 7 to 1; if a ſtony Subſtance be found to have a greater Proportion of Gravity than that of $2\frac{1}{2}$ to 1, it muſt be probable that it has in it ſome adventitious Matter of a metalline Nature, or is at leaſt commix'd with ſome mineral Body more heavy than pure Stone, and may therefore very probably be uſefully applied to ſome medicinal Purpoſes. For Inſtance, of this Kind he mentions the *Lapis Hæmatites* or Blood-ſtone, *Lapis Lazuli*, the Loadſtone, and *Lapis Calaminaris*; all which have their Uſes in Phyſic.

3. *Secondly*, He propoſes this Method as very certain to determine whether a Body, ſuppoſed to be a Stone of the mineral Kind, be ſo indeed, Thus Coral, which, ſays he, ſome take to be a *Plant*, and others a *Lithodendron*, but moſt reckon it among precious Stones, is in Gravity to Water as 2,68 to 1, which favours the laſt Opinion. Thus a *Calculus humanus* and a *Bezoar* were found as 1,7 and 1,5 to 1, and therefore too light to be of the ſame Species with common Stone.

4. A *Third* Uſe which he propoſes is to diſcover the Reſemblance or Difference between Bodies of the ſame Denomination, and thereby to collect and aſcertain their ſeveral Degrees of Goodneſs reſpectively. Whence he argues the Neceſſity of this Sort of Knowledge to Phyſicians, Chemiſts, Apothecaries, Druggiſts; to the Goldſmith, the Merchant, the Miner, *&c.*

5. A *Fourth* Uſe is to diſcern genuine Stones from counterfeit ones, which may be of great Help to Jewellers. Here he gives Inſtances of factitious Coral and factitious Gems, and a Bezoar, which he found out that Way not to be genuine, though a great Price was ſet on the latter.

6. Hence Mercury is found to have a different Gravity, being ſometimes but $13\frac{1}{2}$, and ſometimes above 14 times heavier than Water; and hence a notable Difference may ariſe in two Weather-glaſſes at the ſame Time, and in the ſame Place, even to a whole Inch, from the different Gravity of the Mercury in one and the other. Therefore thoſe who publiſh Regiſters of the Weather ought to find out and declare to the World the ſpecific Gravity of the Quickſilver they uſe in their Barometers.

7. Theſe Uſes he enumerates over and above what we have taken notice of, of a Mechanical and Geometrical Nature: And to let us know the high Value he had for this Science, he thus expreſſes himſelf: " As little Skill as I have in Hy-

“ droſtatics, I would not be debarr'd from the Uſe of them
“ for a conſiderable Sum of Money, it having already done
“ me acceptable Service, and on far more Occaſions than I
“ myſelf firſt expected, eſpecially in the *Examen* of Metals
“ and mineral Bodies, and of ſeveral mineral Productions.”
With much more to the ſame Purport.

(LXVIII) 1. To render this Lecture on *Hydroſtatics* more compleat, I ſhall here ſubjoin what relates to the Reſiſtance of Mediums, or to the Motion of Bodies moving in a reſiſting Medium: And to determine rightly in this Caſe, I ſhall conſider every particular Circumſtance by itſelf, *cæteris paribus*, and then repreſent the Whole connected together. We ſhall ſuppoſe the Caſe of *a Globe moving in a Fluid of an uniform Denſity for a given Time.*

2. Firſt, Let the ſame Globe move with the ſame Velocity, firſt in a denſer Fluid, and afterwards in a rarer; then it is plain, the denſer the Fluid is, the more Particles the Globe will meet with, and ſtrike in a given Time; and therefore the greater will be the Re-action or Reſiſtance of the Fluid: Therefore in this Caſe *the Reſiſtance of the two Fluids will be as their Denſities.*

3. In the next Place, ſuppoſe two different Globes, of the ſame Quantity of Matter, to move in the ſame Fluid, with the ſame Velocity. As theſe Globes move, 'tis evident the Number of Particles they meet and ſtrike, and therefore by which they are reſiſted, will be as their Surfaces, or rather as their Half-Surfaces; and they are as the Squares of their Diameters: Conſequently, *the Reſiſtance which theſe Globes meet with will be as the Squares of their Diameters.*

4. Laſtly, Let the ſame Globe move in the ſame Fluid with different Velocities. Now it is eaſy to conceive, that if one Globe moves with a Velocity double to that of the other, it will ſtrike twice the Number of Particles in the ſame Time, and its Reſiſtance will be twice as great on that account. Again: The Globe which moves with a double Velocity will ſtrike each Particle with a Force twice as great as the other does with half the Velocity; the Re-action therefore of the Particles on the former will be twice as great as upon the latter. Therefore the Reſiſtance to the Globe moving with the double Velocity will be four times as great: And ſince this will be the Caſe univerſally, *the Reſiſtance to a moving Body is as the Square of its Velocity.*

5. Now putting all theſe Caſes together, the Reſiſtance to a Globe moving uniformly in an uniform Fluid will be in the compound Ratios of the Denſity of the Medium, the Square of the Diameter, and the Square of the Velocity. So that

if A and B are two Globes, whoſe Diameters are D and d, and Velocities V and v, moving in Mediums whoſe Denſities are N and n; then will the Reſiſtance of the Globe A be to that of B as $N \times D^2 \times V^2$ to $n \times d^2 \times v^2$.

6. In this Computation we have conſider'd only that Reſiſtance which ariſes from the Re-action of Matter by its *Vis Inertiæ:* But there is ſome ſmall Reſiſtance ariſing to Bodies moving in a Fluid from two other Cauſes, *viz.* one from the Friction of the Body againſt the Parts of the Fluid as it moves, the other from the Tenacity of the Particles of the Fluid, which ariſes from their Attraction of Coheſion. But both theſe are very ſmall when compared with the other, and quite inconſiderable to Bodies moving with any conſiderable Swiftneſs. The Reſiſtance peculiar to Air, ariſing from its Elaſticity, will be conſider'd in another Place.

7. In the above Caſes we have ſuppoſed the Bodies moving in the Fluid to be homogeneal, or of the ſame Matter and Denſity; but if they are heterogeneal, or of different Denſities, then will the Reſiſtance be further variable; for it will be inverſely as the Denſities, or Quantities of Matter which the Bodies contain; becauſe the greater the Quantity of Matter, the greater will be the *Momentum* of the moving Body, and the leſs the Reſiſtance of the Medium compared therewith, and ſo will be the more eaſily overcome by it. Hence, if the two Globes mention'd in *Art.* 3. have not equal Quantities of Matter, (as is there ſuppoſed) the Reſiſtance will be as their Surfaces directly, and inverſely as the Quantities of Matter; that is, as $D^2 \times \frac{1}{D^3}$ to $d^2 \times \frac{1}{d^3}$, or as $\frac{1}{D}$ to $\frac{1}{d}$, or as d to D. That is, *the Reſiſtance to Globes of unequal Diameters and Quantities of Matter will be* (cæteris paribus) *inverſely as their Diameters.*

8. Since a Body moving in a reſiſting Medium muſt every Moment have its *Momentum* or Quantity of Motion ($Q = MV$) abated or diminiſh'd by the Re-action of the Fluid; and ſince in a given Time this Loſs or Decrement of Motion is always as the Re-action, that is, as the Reſiſtance (R) of the Fluid: But the momentary Increaſe or Decreaſe of Motion is as its Fluxion $\dot{Q}$ or $\dot{V}M$. Wherefore the Reſiſtance of the Fluid for any equal Parts of Time will be $R = \dot{Q} = \dot{V}M$; or, when the Maſs of Matter M is given, we have $R = \dot{Q} = \dot{V}$.

9. But when the Moments of Time are not given, that is, not equal, the Reſiſtance R will be as the Decrement of Motion

tion $\dot{Q}$ directly, and the Moment or Fluxion of the Time ($\dot{T}$) inverſely; for twice the Reſiſtance will deſtroy the ſame Quantity of the Motion in half the Time, and three times the Reſiſtance will deſtroy it in one Third of the Time. Therefore $R = \left(\frac{\dot{V}M}{\dot{T}} =\right) \frac{\dot{Q}}{\dot{T}}$; conſequently, when the Maſs M is given, we have $R = \frac{\dot{V}}{\dot{T}}$; that is, the Reſiſtance is directly as the Fluxion or momentaneous Decrement of the Velocity, and inverſely as the Fluxion of the Time.

10. We have before ſhewn, (*Annot.* XXI. *Art.* 5.) that in uniform Motions *the Velocity is as the Space directly, and the Time inverſely*, or $V = \frac{S}{T}$; and becauſe in every Kind of Motion, whether *in Vacuo* or in any Sort of reſiſting Medium, the momentaneous Velocity may be conſider'd as equable or uniform, or proportional to the Space deſcribed each Moment of Time; and therefore $V = \frac{\dot{S}}{\dot{T}}$, and ſo $V\dot{T} = \dot{S}$, and $\dot{T} = \frac{\dot{S}}{V}$; and, taking the Fluents, we ſhall have the *Sum of all the Rectangles* $V\dot{T} = S$ = the Space deſcribed, and the Sum of all the Quotients $\frac{\dot{S}}{V} = T$ = the Time.

11. Therefore if the Curve BPC be ſo deſcribed that its Ordinates MP, *mp*, (perpendicular to the Axis AD) expound the Velocity V, and the *Abſciſſæ* (from the fix'd Point A) AM, A*m*, the Time T; the Perpendicular AB being erected in the Point A, and meeting the Curve in B; the Area ABPM will expreſs the Space S deſcribed in the Time T. Let *mp* be infinitely near to MP, then will $Mm = \dot{T}$; therefore the Fluxion of the Area ABPM will be the Areola MP*pm* $= V\dot{T} = \dot{S}$. Pl. XVII. Fig. 6.

12. In like manner, if the Abſciſs AM expounds the Space deſcribed = S, and the Ordinate MP be as the Velocity inverſely, that is, if MP be as $\frac{1}{V}$, the Area ABPM will expound the Time T in which AM is deſcribed; for the Fluxion of the Time being $\dot{T} = \frac{\dot{S}}{V} = \dot{S} \times \frac{1}{V} = Mm \times$

MP, the Sum of all theſe Fluxionary Areolas MP*pm* will be the Area ABPM expreſſing the Time.

13. Becauſe when the Maſs of Matter in the moving Body is given, we have $\mathrm{R} = \frac{\dot{V}}{\dot{T}}$ (*Art.* 9.); therefore $\mathrm{R}\dot{T} = \dot{V}$, or *the Fluxionary Decrement of Velocity is as the Reſiſtance and the Fluxion of the Time conjointly* But *the Fluxionary Increment of the Space will be as the Velocity and its Fluxionary Decrement directly, and the Reſiſtance inverſely,* that is, $\dot{S} = \frac{V \times \dot{V}}{\mathrm{R}}$; for as above $\dot{T} = \frac{\dot{V}}{\mathrm{R}}$, and $\dot{S} = V\dot{T}$ (*Art.* 10.); therefore $\dot{S} = V \times \frac{\dot{V}}{\mathrm{R}}$, or $\dot{S} = \frac{\dot{V} \times V}{\mathrm{R}}$.

14. Hence $\mathrm{R} = \frac{V \times \dot{V}}{\dot{S}}$, or *the Reſiſtance is as the Velocity and its Fluxionary Decrement directly, and the Fluxion of the Space inverſely.* Alſo $\mathrm{R}\dot{S} = V \times \dot{V}$; that is, *the Velocity multiplied by its Fluxionary Decrement is as the Reſiſtance and Fluxionary Increment of the Space conjointly.* Becauſe the Velocity has been hitherto conſider'd as decreaſing, it ought to have the Fluxion $\dot{V}$ expreſs'd negatively in the above Equations; that is, $\mathrm{R}\dot{T} = -\dot{V}$, and $\mathrm{R}\dot{S} = -\dot{V} \times V$.

15. We have alſo hitherto conſider'd the Reſiſtance R as comprehending all that retards the Motion of the Body in the Medium. But this retarding Power may ariſe from different Principles, as the Denſity of the Medium, or its particular Reſiſtance; and alſo from a centripetal Force, as that of Gravity: Thus a Stone thrown upwards is retarded by the Reſiſtance of the Air, and Power of Gravity. Let the centripetal Force be C, the Reſiſtance of the Medium *R*; then will $\mathrm{R} = \mathrm{C} + R$ when theſe Forces conſpire, and $\mathrm{R} = \mathrm{C} - R$ when they act contrarily.

16. Thus in the Motion of an aſcending Body, where $\mathrm{R} = \mathrm{C} + R$, we have $\mathrm{R}\dot{T} = \mathrm{C}\dot{T} + R\dot{T} = -\dot{V}$; and $\mathrm{C}\dot{S} + R\dot{S} = -\dot{V} \times V$. But in the Deſcent of the Body, where the centripetal Force C is greater than that of the Reſiſtance, we have $\mathrm{C} - R$; and then the above Theorems will be $\mathrm{C}\dot{T} - R\dot{T} = \dot{V}$, and $\mathrm{C}\dot{S} - R\dot{S} = \dot{V} \times V$; for now the Velocity is increaſing, and its Fluxion poſitive. But

if the central Force be leſs than the Reſiſtance, we have $R - C$; then it will be $R\dot{T} - C\dot{T} = -\dot{V}$, and $R\dot{S} - C\dot{S} = -\dot{V} \times V$.

17. If in theſe Theorems we ſuppoſe $R = o$, they will be changed into others, by which the Motions of Bodies in non-reſiſting Mediums may be determined. Thus for the Aſcent they will be $C\dot{T} = -\dot{V}$, and $C\dot{S} = -\dot{V} \times V$; and for the Deſcent $C\dot{T} = \dot{V}$, and $C\dot{S} = \dot{V} \times V$. And thus the Motion of a Body in a reſiſting Medium may be compared with the Motions of the ſame Body in a non-reſiſting Medium, or *in Vacuo*.

18. If the Reſiſtance be equal to the central Force, that is, if $R = C$, then in deſcending Bodies we have $C\dot{T} - R\dot{T} = \dot{V} = o$; conſequently, as the Fluxion of the Velocity is equal to Nothing, the Velocity can neither increaſe nor decreaſe, but is in that Caſe uniform or equable.

19. Since, till this happens, the Velocity is continually increaſing, (becauſe its Fluxion $\dot{V}$ is always as $C - R$ for any given Time) therefore when $C = R$, or $\dot{V} = o$, the Velocity of the deſcending Body is a *Maximum*; or the greateſt poſſible: But this Velocity Bodies deſcending in a Fluid, though they conſtantly approach to it, will never attain; as will be ſeen in the Sequel hereof.

20. In order to eſtimate the Velocity, the Time, and the Space deſcribed by Bodies falling in a uniform Fluid, ſuch as Water, whoſe Parts have no conſiderable Tenacity, and where the Retardation of the Body is the Effect only of the Reſiſtance it meets with from the *Vis Inertiæ* or Re-action of the Fluid; it will be neceſſary firſt to obſerve, that when any Body moves in a Fluid with a given Velocity, the Re-action of the Fluid upon the Body is the very ſame as its Action upon it would be, were the Body to be at Reſt in the Fluid moving againſt it with the ſame Velocity: Becauſe the Magnitude of the Stroke is always as the relative Velocity, which is here the ſame in both Caſes. (See *Annot.* XXIII. 8.)

Pl. XIX. Fig. 6.

21. Alſo that a circular Plane, a Cylinder, a Globe, a Spheroid, a Cone, *&c.* whoſe Breadth is the ſame, and againſt which the Fluid moves in a perpendicular Direction, are all equally acted upon by the Fluid; or, if they move in the Fluid, are equally reſiſted by it. For let PQ be the Diameter of any of thoſe Bodies placed in a Canal EFTS, through which the Water flows from the large Veſſel ABCD, into another below fill'd with ſtagnant Water, touching the Bottom of the former CD. Then ſince the Capacity of the

Canal is alike ſtraiten'd or diminiſh'd by each of thoſe Bodies, it is plain the Fluid flowing through it muſt ſuffer the ſame Retardation from all of them, and therefore muſt act alike upon each; conſequently, were the Fluid at Reſt, and theſe Bodies moving through it, they would all be equally retarded.

22. It muſt be farther obſerved, that all Reſiſtance ariſing from the Aſperity of the Surfaces of Bodies, is not here conſider'd; alſo it is ſuppoſed, that the Fluid is continuous, infinitely compreſs'd, and whoſe Parts have no Elaſticity, Tenacity, nor Friction; and then the Caſe will be very nearly the ſame with *very deep Water*.

23. Now Sir *Iſaac Newton* has ſhewn, that if the Veſſel ABCD be indefinitely large or wide, the Velocity of the Fluid in the annular Space about the circular Plane PQ will be equal to that which a Body acquires in falling through the Height GH; becauſe it would be ſo, were the Plane PQ placed in the Orifice EF, the Motion of the Fluid through the Canal being uniform, and not accelerated by reaſon of the ſtagnant Water beneath.

24. He farther ſhews, that if this Plane were placed in the Orifice EF at the Bottom of the Veſſel, as at *pq*, it would ſuſtain a Portion of the deſcending Fluid, or Cataract AEFB, like to that repreſented by *p*H*q*; which is leſs than two Thirds of a Cylinder of the ſame Baſe *pq*, and Altitude GH, but greater than one third Part thereof: And that therefore it is nearly an Arithmetical Mean between them, or equal to Half of the ſaid Cylinder.

25. This is upon Suppoſition that the Plane *pq* is exceeding ſmall; but if it be augmented till it becomes equal to the Hole EF, it will then ſuſtain the Weight of the perpendicular Column of Water over it, that is, it will ſuſtain the Weight of a Cylinder of Water of the ſame Baſe and Altitude GH: And that therefore in all other Caſes the Weight which the Plane *pq* ſuſtains is to the Weight of a Cylinder of Water, whoſe Baſe is equal to the Plane and Altitude $\frac{1}{2}$ GH, as EF^2 to $EF^2 - \frac{1}{2} pq^2$.

26. The Reſiſtance (R) therefore to the Plane *pq* moving in the Orifice EF, and conſequently of the Plane PQ moving in the Canal, will be in all Dimenſions of the Canal and Plane compared with the Weight (W) of a Column of Water whoſe Baſe is equal to the Plane, and Altitude $\frac{1}{2}$ GH, as the Area of the tranſverſe Section of the Canal to the Exceſs of this Area above $\frac{1}{2}$ the Area of the Plane; that is, R : W as $EF^2 : EF^2 - \frac{1}{2} PQ^2$.

27. Let now the Orifices of the Canal EF and ST be cloſed up, and let the Plane PQ aſcend in the Fluid compreſs'd

on every Side; and in its Aſcent let it oblige the ſuperior Fluid to deſcend by the annular Space between it and the Canal; then will the Velocity (V) of the aſcending Plane be to the Velocity (v) of the deſcending Water, as $EF^2 - PQ^2$ to PQ; that is, $V : v :: EF^2 - PQ^2 : PQ^2$; whence $V : V + v :: EF^2 - PQ^2 : EF^2$.

28. But ſince the Motion of the Water is contrary to that of the Plane, the relative Velocity, with which they act on each other, is the Sum of both, *viz.* as $V + v$. (See *Annot.* XXIII.) Now let this relative Velocity be equal to that with which the Water before deſcended in the annular Space by the Plane PQ at Reſt, (as in *Art.* 23.) *viz.* that which is acquired by falling through GH. Then will the Action of the Water upon the Plane be the ſame as before at Reſt, and conſequently we have here alſo $R : W :: EF^2 : EF^2 - \frac{1}{2}PQ^2$; and $V : (V + v =) v^2 :: EF^2 - PQ^2 : EF^2$.

29. If now we ſuppoſe the Canal to be augmented in Width *ad infinitum*, PQ will then be infinitely ſmall in reſpect of EF; whence PQ^2 and $\frac{1}{2}PQ^2$ will vaniſh in the Terms of the above Ratios, which will then become equal; and ſo $R = W$, and $V = v$. That is, *the Reſiſtance of the Plane will now become equal to the Weight of a Cylinder of Water, whoſe Baſe is equal to that of the Plane, and Altitude* $\frac{1}{2}$ GH. Alſo, *the Velocity of the Plane is equal to that which ſuch a Cylinder would acquire by falling through the Height* GH.

30. Let $A =$ Area of the Plane PQ, then will $\frac{1}{2}GH \times A =$ the Cylinder of Water, whoſe Weight W is equal to the Reſiſtance R of the Plane. But the Cylinder $\frac{1}{2}GH \times A$, moving with the ſame Velocity V of the Plane, will have the ſame Reſiſtance. The Space through which it muſt deſcend to acquire that Velocity is twice its Length, *viz.* GH; and with that Velocity, by an uniform Motion, it would paſs over a Space equal to $2\,GH = 4$ times its Length $\frac{1}{2}$ GH. *Wherefore the Reſiſtance of the Cylinder is equal to the Force (or Weight W) by which (in falling freely) a Motion equal to that of the Cylinder may be generated, (in acquiring the Velocity V) in the Time in which the ſaid Cylinder will deſcribe a Space equal to four times its Length.*

31. Then, becauſe the Motion M of the Cylinder is as its Length L, its Baſe A, its Denſity D, and Square of its Velocity V, that is, $M : L \times A \times D \times V$; therefore when (as in this Caſe) the Quantities A, D, V are given, we have $M : L$; and ſince the Time T (in which the Motion M is generated, or 4 L is deſcribed) is alſo as L, therefore both M and T will vary with L equally, and conſequently will be proportional to each other. But the Force which generates Quanti-

ties of Motion proportional to the Time is in itſelf conſtant and immutable, and is therefore equal to the Reſiſtance of the Cylinder, which alſo remains unchanged under all Degrees of Motion.

32. If the Denſity D of the Cylinder be variable, ſo will the Quantity of Motion M be likewiſe, and alſo the Force W, by which an equal Quantity of Motion in the ſame Time may be generated or deſtroyed. Therefore, let $d =$ Denſity of a Cylinder of Water, $D =$ Denſity of a Cylinder of Lead of the ſame Baſe, $w =$ Force which would generate the Motion of the Cylinder of Water in the Time it would deſcribe four times its Length, and W the Force that would generate the Motion of the Cylinder of Lead in the ſame Time: Then $d : D :: w : W$. But we have ſhewn $w = R$, the Reſiſtance of each Cylinder: Conſequently, $R : W :: d : D$; that is, *The Reſiſtance of a Cylinder of Lead is to the Force which will generate its whole Motion while it deſcribes four times its Length, as the Denſity of Water to the Denſity of Lead.*

33. Hence, ſince the Reſiſtance of a Globe and Cylinder moving in a Fluid is the ſame, and ſince a Globe is $\frac{2}{3}$ of its circumſcribing Cylinder, the Force which will generate or deſtroy the whole Motion of the Cylinder while it moves over 4 times its Length, will generate or deſtroy the whole Motion of the Globe while it moves uniformly through $\frac{2}{3}$ of 4 Diameters, or $\frac{8}{3}$ of its Diameter.

34. From what has been ſaid, if the Velocity of a Globe or Cylinder moving in Water be known, and alſo its Diameter; then the Reſiſtance it meets with from the Fluid, while it moves uniformly through, is known alſo. Thus let its Velocity $V = 16$ Feet *per* Second, and its Diameter three Inches. Then a Body falling through the Space of 4 Feet acquires that Velocity, (ſee *Annot.* LXXV. 6.) whence $GH = 4$, and $\frac{1}{2} GH = 2$ Feet, the Length of a Column of Water, whoſe Baſe being 3 Inches in Diameter will weigh about 6 *lb.* and is equal to the Reſiſtance of the Globe.

35. Hence alſo it is eaſy to find the Height from which if the Globe falls *in Vacuo*, it ſhall acquire the Velocity which ſhall be the greateſt it can poſſibly acquire by deſcending in the Fluid by the Force of its *relative Weight*. *Which Height will be to $\frac{4}{3}$ Parts of its Diameter, as the Denſity of the Globe to the Denſity of the Fluid.*

36. For, let $R =$ Reſiſtance, $D =$ Denſity of the Globe, $d =$ that of the Fluid, and $F =$ Force which will generate the Motion of the Globe in the Time it moves over $\frac{8}{3} D$, where $D =$ the Diameter of the Globe; then we have R :

$F :: d : D$ (in *Art.* 33.) whence $F = \frac{DR}{d}$. Now let $2S =$ Space which the Globe deſcribes uniformly in the Time of the Fall, and with the Velocity acquired thereby. Then will $2S : \frac{8}{3}D :: D : d$ (*per Art.* 32) $:: S : \frac{4}{3}D$. But T and t, the Times in which uniform Spaces are deſcribed, are as thoſe Spaces, *viz.* $T : t :: 2S : \frac{8}{3}D$; whence $T : t :: D : d$. Again, ſince in the Times T and t equal Quantities of Motion are produced; *viz.* the whole Motion of the Globe; for F produces that Motion in the Time t, and in the Time T it is produced by the Force or Relative Weight of the Globe, which call P. Then, ſince the greater the Time is, the leſs will be the Force acting uniformly to produce a given Effect, we have $F : P :: T : t$; hence $F : P :: D : d$; and ſo $F = \frac{PD}{d} = \frac{RD}{d}$; whence $P = R$, that is, the Reſiſtance being equal to the Weight of the Body, it can be no longer accelerated.

37. As Sir *Iſaac Newton* was the Author of this Theory, ſo he inſtituted a Series of Experiments to confirm the ſame, for which Reaſon, as well as to exemplify the foregoing Method of Computation, I ſhall here repeat ſome of the principal, made with Globes of Wax and Lead included, let fall in Water thro' the Depth of 112 Inches. His firſt Experiment was with a Globe $\frac{84}{100}$ Parts of an Inch Diameter $= D$, whoſe Weight *in Vacuo* was $156\frac{13}{38}$ Grains, and in Water 77 Grains, therefore $156\frac{13}{31} - 77 = 79\frac{13}{38}$ Grains $=$ the Weight of an equal Bulk of Water. Wherefore $d : D :: 79\frac{13}{38} : 156\frac{13}{38} : \frac{8}{3}D$ ($= 2{,}24597$ Inches) $: 2S = 4{,}4256$ Inches; and ſo $S = 2{,}2128$ Inches.

38. The Globe falling *in Vacuo*, with its whole Weight of $156\frac{13}{38}$ Grains, will in one Second of Time deſcribe $193\frac{1}{3}$ Inches. And by its relative Weight of $79\frac{13}{38}$ Grains in Water, it would in the ſame Time by falling *in Vacuo* deſcribe 95,219 Inches. For the Spaces deſcribed in the ſame Time will be as the accelerating Forces or Weights of the Bodies. (See *Annot.* XXVII. 2, 3.)

39. But the Time T, in which the Globe by deſcending *in Vacuo* with its relative Weight of $79\frac{13}{38}$ Grains, will deſcribe the Space $S = 2{,}2128$, is to the Time $t = 1$ Second, in which it deſcribed the Space 95,219, in the ſubduplicate Ratio of thoſe Spaces; that is, $T : t :: \sqrt{2{,}2128} : \sqrt{95{,}219}$, wherefore $T = \sqrt{\frac{2{,}2128}{95{,}219}} = 0'',15244 =$ the Time in which the Globe by deſcending *in Vacuo* acquires the greateſt Velocity

V with which it can deſcend in the Water.

40. In the next Place, to find the Velocity and the Space deſcribed in the Fluid in the whole Time of the Fall (F) Sir *Iſaac* has this Method; let the Number N be found for the Logarithm $0,4342944819 \frac{2F}{T}$; then is the Velocity of the Globe at the End of the Fall $= \frac{N-1}{N+1} \times V$. Again let L be the Logarithm of the Number $\frac{N+1}{N}$; then will the Space deſcribed be $\frac{2SF}{T} - 138629S + 4,60517LS$. Theſe Velocities and Spaces deſcribed are calculated and diſpoſed into a Table by our Author for any Number of Seconds to 10 T. The Demonſtration of the above Rules is too long and intricate for this Place, but may be ſeen in the Commentary on the *Principia* by *Le Seur* and *Jacquier*. This Table will be inſerted at the End of this Note.

41. By this Table you ſee how ſoon Bodies acquire their greateſt Velocity in deſcending in Water; for Inſtance, in $5T = 0'',76 = 46'''$, the Globe has acquired a Velocity which is to the greateſt as 99990920 to 100000000. And in $10T = 1'' : 30'''$ the Velocity of the Globe is to the greateſt Velocity as $99999999\frac{3}{5}$ to 100000000. Which Difference is inſenſible to the moſt critical Eye; and from thence (having fallen through 37 Inches) it deſcends with an uniform Velocity to Appearance, tho' it really for ever increaſes.

42. If the Time of Deſcent in the Water be leſs than 10 T, the Diſtance deſcended is ſhewn by the third Column; but if it be greater, as in the preſent Experiment it is 4″, then ſay, as $T : 2S :: 4'' : \frac{8S}{T} = 116,1245$ Inches, becauſe Spaces deſcribed with an uniform Velocity are as the Times. This uniform Space muſt be diminiſh'd by ſubſtracting the conſtant Number 1,38629 S. Therefore $116,1245 - 1,38629S = 113,0569$ Inches, which the Globe ſhould deſcribe in the Water in 4″ by the Theory.

43. But in the Experiment it deſcribed only 112 Inches, which Difference of 1 Inch in 112 is inconſiderable, and would have given no Diſturbance to any but Sir *Iſaac Newton*. But he being as nice in Practice as ſkilful in Theory, attempted another Set of Experiments in a deeper Veſſel, *viz.* of 15,3 Feet or 183,6 Inches. I ſhall here give the Reader a View of the Event of his ſeveral Experiments in both Veſſels in the following Table:

In the Veſſel 112 Inches deep.

Experiment	Weight in Air.	Weight in Water.	Number of Globes.	Times of their Deſcent in Experiments.					Time by the Theory.
	Grains.	Grains.		″	″	″			″ ‴
1	$76\frac{1}{3}$	$5\frac{3}{16}$	3	15	15	15			14:48
2	121	1	3	46	47	50			40
In the Veſſel 183 Inches deep.									
3	$139\frac{1}{4}$	$7\frac{1}{8}$	4	$49\frac{1}{2}$	50	51	53		50
4	$154\frac{3}{8}$	$21\frac{1}{2}$	4	$28\frac{1}{2}$	29	$29\frac{1}{2}$	30		29
5	$212\frac{3}{8}$	$79\frac{1}{2}$	5	15	$15\frac{1}{2}$	16	17	18	15
6	$293\frac{3}{8}$	$35\frac{7}{8}$	4	$29\frac{1}{2}$	30	$30\frac{1}{2}$	31		28
7	139	$6\frac{1}{2}$	4	50	$50\frac{1}{2}$	51	52		52
8	$273\frac{1}{4}$	$140\frac{3}{4}$	4	12	$12\frac{1}{2}$	12	13		$11\frac{1}{3}$
9	384	$192\frac{1}{2}$	4	$17\frac{1}{4}$	18	$18\frac{1}{2}$	19		$15\frac{5}{9}$
10	48	4	3	$44\frac{1}{2}$	45	46			$46\frac{5}{9}$
11	141	$4\frac{3}{8}$	3	63	64	65			$64\frac{1}{2}$

44. In all the Experiments of the deep Veſſel, the Times are expreſs'd in *Half Seconds*, that is, in all but the two firſt. And the ſmall Differences from the Theory, Sir *Iſaac* very eaſily accounts for from the oſcillatory Motion with which thoſe which deſcended moſt ſlowly were found to move, as in the 5th, 8th, and 9th Experiments. But in thoſe which deſcended more ſwiftly, the Times very nearly correſpond with the Theory. Theſe Experiments were made in Water; others were made in Air, to ſhew that the Phænomena agree with the Theory in an elaſtic and rare Medium, as well as in a non-elaſtic and denſe one.

45. In order to this the Denſity of Air to that of Rain Water is aſſumed as 1 to 860; and 5 Globes form'd of Hog's Bladders, blown in a wooden Mould, while green, and taken out when dry, were let fall from the Dome of St. *Paul*'s to the Pavement, from the Height of 272 Feet, and the Time meaſured by Half-Second Pendulums; the Event of all the five Globes is ſhewn in the following Table. I ſhall here ſubjoin a Calculation for one of them, *viz.* the 3d, as follows.

46. The Diameter of this Globe was $5{,}3 = D$, and it weigh'd 137,5 in Air. A Globe of Air of the ſame Diameter weighs 23 Grains, very nearly; therefore $137{,}5 + 23 = 160{,}5$ Grains, the Weight of the Globe *in Vacuo*. Where-

fore, as 23 : 160,5 :: $\frac{8}{3}$D : 2$S = 98,626$. Again, as 160,5 : 137,5 : : 193$\frac{1}{3}$: 165,628 = Inches the Bladder would fall in the Air in one Second. But with the ſame relative Weight 137$\frac{1}{2}$, it will *in Vacuo* deſcend thro' the Space $S =$ 49,313 in the Time 0″,5456 = T, and acquire the greateſt Velocity with which it can deſcend in Air. The Time of the Fall was 18$\frac{1}{4}$″, in which with that Velocity the Globe would deſcribe 278 Feet 8 Inches by an uniform Motion; for 0″,5456 : 98,626 Inches :: 18$\frac{1}{2}$″ : 278$\frac{2}{3}$ Feet. Laſtly, ſubduct 1,3863 $S = 5\frac{2}{3}$ Feet, and there remains 273 Feet, tho' in the Table, by a more accurate Calculation, it is but 272 Feet : 7 Inches, and by the Experiment it was 272 Feet.

47. The following Table ſhews, in the firſt Column, the Weight of each Globe or Bladder; the ſecond its Diameter; the third the Time in which they ſeverally deſcended thro' the Height of 272 Feet; the fourth ſhews the Spaces they ſhould have deſcended thro' by the Theory; and the laſt Column ſhews the Difference between the Theory and Experiments.

The Weight in Grains.	*Diameter in Inches.*	*Times of the Fall.*	*Spaces* per *Theory.*		*Difference.*	
			Feet.	Inch.	Feet.	Inch.
128	5,28	19″	271 —	11	0 —	1
156	5,19	17	272 —	0$\frac{1}{2}$	0 —	$\frac{1}{2}$
137$\frac{1}{2}$	5,3	18$\frac{1}{4}$	272 —	7	0 —	7
97$\frac{1}{2}$	5,26	22	277 —	4	5 —	4
99$\frac{1}{8}$	5	21	282 —	0	10 —	

48. According to this Theory, having found the Reſiſtance which a Globe meets with in deſcending in any reſiſting Medium, agreeing to its greateſt Velocity, it is eaſy then to find its Reſiſtance for any leſſer Velocity propoſed; becauſe the Reſiſtance for any given Velocities, will be as the Squares of thoſe Velocities.

49. The Globe has been hitherto ſuppoſed to begin its Motion in the reſiſting Medium from a State of Reſt; but we may ſuppoſe it alſo projected into ſuch a Medium, or to enter it with a certain Degree of Motion or Velocity; and in this Caſe we may determine the Velocity, the Reſiſtance, and the Space deſcribed in the Medium, from any given Time; by knowing the Denſity of the Globe and Medium, and the Velocity with which it enters it; as alſo what Part

of its Motion the Globe ſhall have loſt in that Time.

50. But in order to this, we muſt take to our Aſſiſtance the *Hyperbola*. Therefore let $AB =$ Time in which the Globe would loſe its whole Motion by the Reſiſtance it meets with (upon entring the Fluid,) if uniformly continued. In the Points A and B erect the Perpendiculars A D and B C; and let B C repreſent the Motion or Velocity, and alſo the Reſiſtance of the Globe at its Entrance into the Fluid; thro' the Point C, let the Hyperbola be deſcribed to the Aſymptotes A D and A B continued to any Point E; in E erect the Perpendicular E F meeting the Hyperbola in F; compleat the Parallelogram C B E G; and draw A F cutting B C in H.

Pl. XVII. Fig. 7.

51. Then if the Globe in any Time B E deſcribes *in Vacuo* the Space repreſented by the Parallelogram B C G E by its firſt Velocity B C uniformly continued, it will in a reſiſting Medium deſcribe the hyperbolic Space or Area BCFE; and its Motion or Velocity at the End of the Time B E will be repreſented by the Ordinate E F, and the Part loſt by F G. Laſtly, its Reſiſtance at the End of that Time will be B H, and the Part loſt H C.

52. For $BC^2 : EF^2 :: BC : \frac{EF^2}{BC} =$ the Reſiſtance at the End of the Time B E, as B C is that at the Beginning. And from the Nature of the Hyperbola, $BC : EF :: AE : AB$; and by ſimilar Triangles $AE : AB :: EF : BH$; wherefore $HB = \frac{EF^2}{BC}$, the Reſiſtance at the End of the Time B E.

53. Hence if the Globe in any Time T loſes its whole Motion M by the Reſiſtance R uniformly continued; the ſame Globe ſhall in a reſiſting Medium, by the Reſiſtance R decreaſing in the duplicate Ratio of the Velocity, in the Time t loſe the Part $\frac{tM}{T+t}$ of its whole Motion M; and then $M - \frac{tM}{T+t} = \frac{TM}{T+t}$ will be the Part of the Motion remaining. For let m be the reſidual Motion, then $T : t :: AB : BE$ (by *Art*, 50, 51.); and hence $T + t : T :: AE : AB$; and moreover $M : m :: CB : FE :: AE : AB$; whence $T + t : T :: M : m$; conſequently $m = \frac{TM}{T+t}$.

54. Laſtly, the Space (S) deſcribed in the reſiſting Medium in the Time t will be to the Space (2 S) deſcribed by the uniform

form Motion M, as the Logarithm of the Number $\frac{T+t}{T} \times$ 2,302585 to $\frac{t}{T}$. For let $AB = a$, $BC = b$, $BE = x$, and $AE = a + x$, and $L =$ Logarithm. Then (*per* Conics) the Area $BCEF = ab \times L\frac{a+1}{a}$, and the Rectangle $BCGE = bx$, whence $s : 2S :: ab \times L\frac{a+x}{a} : bx$; and dividing by ab, we have $s : 2S :: L\frac{a+x}{a} :: \frac{x}{a} :: L\frac{T+t}{T} : \frac{t}{T}$; but the Expreſſion $L\frac{T+t}{T}$, as it here ſtands, is an hyperbolical Logarithm, which is to the common tabular Logarithm as 2,302585 to 1. Conſequently the tabular $L\frac{T+t}{T} \times$ 2,302585 = Logarithm ſought.

55. The Spaces, Times, and Velocities of Bodies moving either by their *Vis inſita* alone, or conjointly with an accelerating Force, as Gravity; in reſiſting Mediums of various Denſity and Ratio of Reſiſtance, may be analytically inveſtigated as follows. Let the initial Velocity, or that with which it begins its Motion in the Fluid be V, the Time t, the Space deſcribed s, and the Velocity at the End of the Time v, the Denſity of the Medium d, and the Reſiſtance r as $\frac{dv^n}{a^n}$. Then (by *Art.* 14.) $r\dot{s} = -\dot{v}v$, wherefore $\frac{dv^n\dot{s}}{a^n} = -\dot{v}v$; and ſo $d\dot{s} = a^n\dot{v}v^{1-n}$.

56. Whence if the Denſity be uniform, as of Water, then $d = 1$. And if $n = 2$, then $\dot{s} = a^2\dot{v}v^{-1}, = -\frac{a^2\dot{v}}{v}$. Now s is the Fluent of the Fluxion $\dot{s}$, and the Fluent of $-\frac{a^2\dot{v}}{v}$ is $Q - a^2Lv$, where Q is ſome conſtant Quantity, to find the Value of which, ſuppoſe $s = Q - a^2Lv = o$, then $Q = a^2Lv$, but when $s = o$, $v = V$; therefore $Q = a^2LV$. Wherefore $s = a^2LV - a^2Lv = a^2L\frac{V}{v}$.

57. The Time t is found by the *Formula* in *Art.* 10. where we had $\dot{t} = \frac{\dot{s}}{v}$, but becauſe when d is given, $\dot{s} = a^n\dot{v}v^{1-n}$

(*Art.* 55.) therefore $\dot{t} = \frac{\dot{s}}{v} = \frac{a^n \dot{v} v^{1-n}}{v} = a_n \dot{v} v^{-n}$.

Whence taking the Fluents we have $t = \frac{Q - a^n v^{1-n}}{1-n}$; and putting $t = o$, we have $v = V$, wherefore $Q = a^n V^{1-n}$; conſequently, ſubſtituting for Q its Value, we have $t = \frac{a^n V^{1-n} - a^n v^{1-n}}{1-n}$. Hence, if $n = 2$, $t = \frac{a^2 V - a^2 v}{Vv}$ $= a^2 \times \frac{V - v}{Vv}$.

58. We had above $\dot{s} = a^n \dot{v} v^{1-n}$; and taking the Fluents on both Sides, we have $s = \frac{Q - a^n v^{2-n}}{2-n} = \frac{a^n V^{2-n} - a^n v^{2-n}}{2-n}$. Let $2 - n = m$; then $ms = a^n V^m - a^n v^m$; hence $a^n V^m - ms = a^n v^m$; therefore $v = \frac{\overline{a^n V^m - ms}^{\frac{1}{m}}}{a^{\frac{n}{m}}}$. Which Theorem therefore gives the Velocity when the Space s is known.

59. In theſe Theorems we ſee the Agreement with the former Methods of determining theſe Quantities. Thus the Space by Theorem in *Art.* 56. is $s = a^2 \mathrm{L} \frac{V}{v} = 2{,}302585$ $\mathrm{L} \frac{T + t}{T}$ (in *Art.* 54.) $=$ BCFE, the Hyperbolic Space in the Figure. Whence, in this Caſe, $a^2 = 2{,}302585. \times \frac{8}{3} D$.

60. To determine the Space s which the Globe ſhall have paſs'd by its *Vis inſita* when it has loſt half its Velocity or Motion, that is, when $V : v :: 2 : 1$; we have $\mathrm{L} \frac{V}{v}$, or the Logarithm of $\frac{2}{1} = 0{,}301030$, which multiplied by 2,302585 gives 0,6931; and this again, multiplied by $\frac{8}{3} D = 2{,}6\, D$, gives $1{,}84832\, D = s$, which is not quite 2 Diameters.

61. The Time t being always as $a^2 \times \frac{V - v}{Vv}$, or (becauſe a^2 is conſtant) as $\frac{V - v}{Vv}$; therefore when $v = \frac{1}{2} V$, we have $\frac{V - v}{Vv} = \frac{1}{2} = t$, which ſhews the Time in which the Globe

loſes half its Motion is half the whole Time; that is, BE $=$ AB (in the Figure) the Time in which by an uniform Velocity BC it would deſcribe $\frac{8}{3}$ Parts of its Diameter.

62. If the Medium be ſimilar, or of an uniform Denſity, where $d = 1$; and if the Reſiſtance be in the ſimple Ratio of the Velocity, then $n = 1$; and by the Theorems above, $s = aV - av$, and $t = a\,\mathrm{L}\,\frac{V}{v}$, and $v = V - \frac{s}{a}$.

63. If in the foregoing Equations the Velocity $v = o$, then $s = \frac{a^n V^{2-n}}{2-n}$, when n is leſs than 2. But when $n = 2$, $s = a^2\,\mathrm{L}\,\frac{V}{o} =$ Infinite; if n be greater than 2, then alſo $s = \frac{a^n V^{n-2} - a^n v^{n-2}}{\overline{n-2} \times V^{n-2} \times v^{n-2}} = \frac{a^n}{\overline{n-2} \times o} =$ Infinite. That is, there is but one Caſe in which the Globe moving in the Fluid can loſe its Velocity, or come to a State of Reſt in deſcribing a finite Space, and conſequently in a finite Time, *viz.* when n is leſs than 2. In all other Caſes the Motion will never be totally deſtroyed. And when $n = 1$, then, though the Space be finite, *viz.* $s = aV$, yet the Time of deſcribing it will be infinite, *viz.* $t = a\,\mathrm{L}\,\frac{V}{o} =$ Infinite.

64. After a like Manner we may raiſe Theorems for the Spaces, Times, and Velocities of a Body deſcending by the Force of Gravity in any reſiſting Medium. For ſuppoſe the Medium of an uniform Denſity, and the Reſiſtance in any multiplied Ratio of the Velocity, as $r = \frac{v^n}{a^{n-1}}$; then for a deſcending Body we have (by *Art.* 16.) $c\dot{s} - r\dot{s} = v\dot{v}$, and therefore $c\dot{s} - \frac{v^n \dot{s}}{a^{n-1}} = v\dot{v}$; whence $\dot{s} = \frac{v\dot{v}a^{n-1}}{\overline{ca^{n-1} - v^n}}$.

65. Alſo becauſe (by *Art.* 10.) $\dot{t} = \frac{\dot{s}}{v}$, therefore we have $\dot{t} = \frac{\dot{v}a^{n-1}}{ca^{n-1} - v^n}$. And in like manner the Theorems for aſcending Bodies are $\dot{s} = \frac{-v\dot{v}a^{n-1}}{ca^{n-1} + v^n}$, and $\dot{t} = \frac{-\dot{v}a^{n-1}}{ca^{n-1} + v^n}$.

66. If the Reſiſtance be as the Velocity, then $n = 1$; and in the deſcending Body $\dot{s} = \frac{v\dot{v}}{c-v} = -\dot{v} + \frac{c\dot{v}}{c-v}$; the Fluents of which are $s = Q - v - c\ \mathrm{L}.\ \overline{c-v}$, but ſince when $s = o$ it is $v = \mathrm{V}$, therefore then $Q = \mathrm{V} + c\ \mathrm{L}.\ \overline{c-\mathrm{V}}$; and ſo $s = \mathrm{V} - v + c\ \mathrm{L}.\frac{c-\mathrm{V}}{c-v}$. Alſo the Time is had from the Equation $\dot{t} = \frac{\dot{v}}{c-v}$; of which the Fluents are $t = Q - \mathrm{L}\ \overline{c-v} = \mathrm{L} = \frac{c-\mathrm{V}}{c-v}$. And for aſcending Bodies $s = \mathrm{V} - v + c\,\mathrm{L}\frac{c+\mathrm{V}}{c+v}$; and $t = \mathrm{L}\frac{c+\mathrm{V}}{c+v}$.

67. If the Reſiſtance be as the Square of the Velocity, then $n = 2$, and $r = \frac{v^2}{a}$. Let $V =$ the greateſt Velocity the Body can acquire by deſcending in the Fluid, and becauſe then the Reſiſtance is equal to its Gravity or Force of its Weight (by *Art.* 36.) we have $c = \frac{V^2}{a}$, (for in that Caſe $v = V$,) whence $rc = V^2$. Let $\mathrm{S} =$ Space deſcribed *in Vacuo* to acquire the Velocity V; then becauſe (by *Art.* 17.) in that Caſe $c\dot{\mathrm{S}} = V\dot{V}$, therefore $c\mathrm{S} = \frac{1}{2}VV$, and ſo $2c\mathrm{S} = VV = ac$, conſequently $a = 2\mathrm{S}$.

68. Therefore ſince (*Art.* 64.) $\dot{s} = \frac{av\dot{v}}{ac-v^2} = \frac{2\mathrm{S}v\dot{v}}{V^2-v^2}$ and putting $Vv - vv = xx$, and taking the Fluxions $v\dot{v} = -x\dot{x}$, and therefore $\dot{s} = -\frac{2\mathrm{S}x\dot{x}}{xx} = -\frac{2\mathrm{S}\dot{x}}{x}$; and taking the Fluents, $s = Q - 2\mathrm{S}\,\mathrm{L}.\,x = Q - \mathrm{S}\,\mathrm{L}.\,x^2 = Q - \mathrm{S}\,\mathrm{L}.\ \overline{V^2 - v^2}$. Wherefore when $s = o$, we have $v = \mathrm{V}$, the initial Velocity, then $Q = \mathrm{S}\,\mathrm{L}.\ VV - \mathrm{V}^2$. And conſequently $s = \mathrm{S}\,\mathrm{L}.\ \frac{V^2 - \mathrm{V}^2}{V^2 - v^2}$.

69. Let $\mathrm{L}.\,b = 1$; then $s\,\mathrm{L}.\,b = \mathrm{S}\,\mathrm{L}.\frac{V^2-\mathrm{V}^2}{V^2-v^2}$; $\frac{s}{\mathrm{S}}\mathrm{L}.b = \mathrm{L}\,b^{\frac{s}{\mathrm{S}}} = \mathrm{L}\frac{V^4-\mathrm{V}^2}{V^2-v^2}$, therefore $b^{\frac{s}{\mathrm{S}}} = \frac{V^2-\mathrm{V}^2}{V^2-v^2}$,

whence we get $v^2 = \frac{V^2 b^{\frac{s}{\mathrm{S}}} + \mathrm{V}^2 - V^2}{b^{\frac{s}{\mathrm{S}}}}$. Wherefore the Velocity v is from thence eaſily found.

70. The Times t it obtain'd by the Equation $\dot{t} = \frac{a\dot{v}}{ac - vv}$ (in *Art.* 65.) $= \frac{2\mathrm{S}\dot{v}}{V^2 - v^2} = \frac{\frac{\mathrm{S}}{V}\dot{v}}{V + \dot{v}} + \frac{\frac{\mathrm{S}}{V}\dot{v}}{V - v}$; and taking the Fluents we have $t = \mathrm{Q} + \frac{\mathrm{S}}{V}\mathrm{L}.\, V + v - \frac{\mathrm{S}}{V}\mathrm{L}.\, V - v = \mathrm{Q} + \frac{\mathrm{S}}{V}\mathrm{L}.\frac{V + v}{V - v}$; and putting $t = o$, and ſo $v = \mathrm{V}$, we ſhall find $\mathrm{Q} = -\frac{\mathrm{S}}{V}\mathrm{L}.\frac{V + \mathrm{V}}{V - \mathrm{V}}$; and therefore at length we have $t = \frac{\mathrm{S}}{V}\mathrm{L}\,\frac{V + v \times V - \mathrm{V}}{V - v \times V + \mathrm{V}}$.

71. If the Body deſcends in the Fluid from a State of Reſt, then $\mathrm{V} = o$; and ſo the Space $s = \mathrm{S\,L}.\,\frac{V^2}{V^2 - v^2}$; the Time $t = \frac{\mathrm{S}}{V}\mathrm{L}.\,\frac{V + v}{V - v}$; and the Velocity $v = \sqrt{\frac{V^2 b^{\frac{s}{\mathrm{S}}} - V^2}{b^{\frac{s}{\mathrm{S}}}}}$.

And in like Manner are found the Equations for the Spaces, Times, and Velocities of Bodies aſcending in a reſiſting Medium, which are the ſame with theſe having only the Signs of V and v changed.

72. Thus I have given a Specimen of the ſeveral Ways that have been uſed to repreſent and compute the Motion of Bodies moving in reſiſting Mediums of any Denſity, and according to any Law of Reſiſtance. The Theorems in the latter Articles expreſs moſt of the Caſes in *Prop.* I, II, III, VI, VIII, XI, of Sir *Iſaac Newton*'s ſecond Book of *Principia*; I could have inſerted many more, but have already extended this Note to a great Length; and ſhall refer the Reader to the admirable Commentary on our Author by Meſſrs. *Le Seur* and *Jacquier*, from whence I have collected moſt of theſe Articles.

73. I ſhall conclude with inſerting the Table refer'd to in the foregoing Computations, which is as follows.

Time of the Fall.	*Velocities acquired in the Fluid.*	*Spaces described by falling in the Fluid.*	*Spaces described with the greateſt Velocity.*	*Spaces deſcribed by falling* in Vacuo.
0,001 T	$99999\frac{29}{30}$	0,000001 S	0,002 S	0,000001 S
0,01 T	999967	0,0001 S	0,02 S	0,0001 S
0,1 T	9966799	0,009983 S	0,2 S	0,01 S
0,2 T	19737532	0,039736 S	0,4 S	0,04 S
0,3 T	29131261	0,088681 S	0,6 S	0,09 S
0,4 T	37994896	0,155907 S	0,8 S	0,16 S
0,5 T	46211716	0,240229 S	1,0 S	0,25 S
0,6 T	53704957	0,340270 S	1,2 S	0,36 S
0,7 T	60436778	0,454540 S	1,4 S	0,49 S
0,8 T	66403677	0,581507 S	1,6 S	0,64 S
0,9 T	71629787	0,719660 S	1,8 S	0,81 S
1 T	76159416	0,867561 S	2 S	1 S
2 T	96402758	2,650005 S	4 S	4 S
3 T	99505475	4,618657 S	6 S	9 S
4 T	99932930	6,614376 S	8 S	16 S
5 T	99990920	8,613796 S	10 S	25 S
6 T	99998771	10,613718 S	12 S	36 S
7 T	99999834	12,613707 S	14 S	49 S
8 T	9999980	14,613706 S	16 S	63 S
9 T	9999997	16,613705 S	18 S	81 S
10 T	$9999999\frac{3}{5}$	18,613705 S	20 S	100 S

LECTURE

LECTURE V.

HYDRAULICS.

Of the Equilibrium *and* Motion *of* Fluids *in general. Of the* Rise *of* Fluids *in* Reservoirs *and* Pipes *of* Conduit. *Of the* Running *of* Rivers. *Of the* Origin *of* Fountains *and* Springs. *Of* Perennial, Intermitting *and* Reciprocating Springs. *The Nature of the* Recurved Syphon *and* Tantalus-Cup. *The* Theory *of* Jet d'eau's *their* Adjutages, Pipes, *&c.* Theorems *of the* Velocities, Momenta, Distances, *&c. Of* Spouting Fluids. *Various* Calculations *relating to the Motion of Fluids thro' Pipes applied to the* Animal Oeconomy, Cataracts, *&c. Of the Properties and Uſe of the* Crane *or* Syphon. *The whole* Theory *of* Pump-Work *at large. The Nature of the* Common Pump *explain'd. The Structure of a* New Pump *without Friction. The Nature and Uſe of a* Mercurial-Pump, *working with* Quicksilver *inſtead of a* Piston, *explain'd.* Archimedes's Screw *explain'd. The Nature and Structure of* Newsham's Water-Engine, *of* Newcomen's Fire-Engine, Savery's Fire-Engine, *and* Paine's Improvement *of* Fire-Engines *explain'd. Of the* Æolipile. *The* Theory *of the* Tides *at large explain'd.*

BY HYDRAULICS we are to underſtand the Science of the Motion of Fluids, and the Conſtruction of all Kinds of Inſtruments and Machines relating thereto; alſo the Nature of Springs, the Theory of the Tides, &c. are uſually explain'd under this Head.

THE *Motion of Fluids*, viz. their *Deſcent* or *Riſe* below or above the common Surface or Level of the Source or Fountain is cauſed either (1.) By the natural Gravity or Preſſure of the Fluid contain'd in the Reſervoir or Fountain; or (2.) By the Preſſure or Weight of the Air on the Surface of the Fluid in the Reſervoir, when it is at the ſame time either taken off, or diminiſhed, on ſome Part in Aqueducts, or Pipes of Conduit. (3.) By the Spring or elaſtic Power of compreſſed or condenſed Air, as in the common Water-Engine. (4.) By the Force or Preſſure of Piſtons, as in all Kinds of forcing Pumps, &c. (5.) By the Power of Attraction, as in the Caſe of Tides, &c. Of all which in Order.

I. THE moſt natural Motion of Fluids is that ariſing from the Force of their own Gravity, by which thoſe Parts which ſtand higheſt preſs upon others below them, till by that means they riſe to the ſame horizontal Level. Thus Water in a Fountain ABCD by its Preſſure raiſes that in the Aqueduct FGH to the ſame Height IKLM, in every Direction or Poſition of the Duct GH or GN; unleſs the Orifice of the ſaid Duct be below that Level, in which Caſe the Water will continually flow from the ſame. The Reaſon hereof is evident from the Principles of *Hydroſtatics*, where it was ſhewn, *that the Preſſure* Pl. XVI. Fig. 5.

ſure of Fluids was in Proportion to the Altitude only, and not according to the Quantity thereof; and therefore the Effect or Riſe of the Fluid in the Duct muſt be equal thereto. Or thus; *The Velocity of the Fluid in the Duct at* L, *is to that in the Fountain at* IK, *as the Quantity of the Fluid in the Fountain is to that in the Duct in given Altitudes*; whence (from the Principles of *Mechanics*) the *Momentum* of the Fluid in each will be equal; and conſequently an *Equilibrium* will enſue at equal Altitudes K and L in both. (LXIX)

(LXIX.) 1. Since the Particles of Fluids are to be conſider'd as ſmall ſolid Bodies, they muſt obſerve the ſame Laws of Motion, and have their *Momenta* determined in the ſame Manner, as Bodies of larger Magnitude; that is, *by the Maſſes or Quantities of Matter drawn into their Velocities*.

Pl. XVI. Fig. 5. 2. Now 'tis evident the Velocities of the Fluid in the Reſervoir A D, and in the Duct S L, muſt be as the Quantities of Matter, or of the Fluid, contain'd in equal Altitudes in each, inverſely; that is, as the Cylinder T V W L to the Cylinder ICDK. For when all the Fluid in the Cylinder ICDK is paſs'd into the Tube S H, it will fill a Space above V W in that Tube, or make a Cylinder equal to the former. Now ſince all Cylinders are as their Baſis and Altitudes jointly; if B be the Baſe, and H = the Height or Altitude of the Cylinder ICDK; and *b*, *b*, the Baſe and Altitude of the equal Cylinder in the Tube above V W; then becauſe $B \times H = b \times b$, we have $H : b :: b : B$, but the Altitudes H and *b* are the Spaces paſs'd thro' in the ſame Time, and will therefore repreſent the Velocities of the moving Fluid; but *b* is to B as the Cylinder T W to the Cylinder I D; (for Cylinders of equal Altitudes are as their Baſes) therefore $H : b :: TW : ID$. Or the Velocities are as the Quantities of the Fluid contain'd under equal Altitudes inverſely.

3. Or it may be otherwiſe eaſy to underſtand that the Water can riſe no higher in the Tube S H than in the Reſervoir A D, becauſe (ſuppoſing the Duct every where of an equal Bigneſs, and therefore EQ = TL) 'tis plain, if we continue F E to O, and R Q to P; that then only that cy-

HENCE we have Conduits often ſupplied with Water from Springs which lie above them; and Cocks to ſupply the Inhabitants of a Town with Water by Pipes of Conduct proceeding from a Reſervoir in a Situation above the higheſt Part of the Town (LXX).

lindric Body of the Fluid in the Reſervoir which ſtands over the Orifice of the Duct E Q, that is, only the Cylinder of Water OEQP can affect or preſs upon the Water in the Duct; for all the Fluid (beſides this Cylinder) ſtands over, and therefore preſſes upon the Bottom of the Reſervoir, as C E and Q D, and ſo by equal Re-action has all its Force deſtroy'd. The Water therefore will deſcend through the Orifice E Q, till by its Preſſure it has raiſed a Quantity in the other Leg S H equal to the Cylinders O Q + Q F, and therefore to an equal Height; and thus the Preſſure at F R and S G being equal, there muſt be an Equilibrium.

4. That it ought to riſe to the ſame horizontal Level M Z in the inclined Tube S N, whether that Tube be leſs, having an elliptic Baſe whoſe longeſt Diameter is S X; or whether it be equal to the Tube S H, as having the ſame Baſe; or, laſtly, if it be greater, as having a circular Baſe on the Diameter S G; I ſay, that in every Caſe it ought to riſe to the ſame Level Z M is evident, becauſe there can be no Equilibrium till the Preſſure of the Fluid in each Tube, or on each Side the Baſe S X, or S G, be equal; that ſo, being contrary, they may deſtroy each other. But there can never be an equal Preſſure on each Side the common Baſe, unleſs there be an equal perpendicular Altitude of the preſſing Fluid on each Side, becauſe (as we have ſhewn *Annot.* LIV.) the Preſſure of Fluids on any given Area or Baſe is ever proportional to the Altitude of the Fluid, and that only.

5. Hence it appears, that an Inſtrument like this in the Figure may ſerve very well to take a Level for a ſmall Diſtance, or to draw a Line truly horizontal; for the two Points I and M will always be in a true horizontal Level or Line. But for this Purpoſe the Tube S G ſhould be at leaſt $\frac{1}{4}$ of an Inch Diameter, to avoid the riſing of the Fluid by Attraction; it ſhould alſo move very nicely by a Joint fixed on to the Part F S.

(LXX) 1. Let A C D B be a Reſervoir with a Duct or Conduit-Pipe F K L, and at L turn'd off from the Perpendi- Pl. XVII. Fig. 8.

HENCE alſo the Deſcent of Water in Rivers, Streams, and Canals, from Springs and Sources

cular to give the Water by a Stop-Cock into a Ciſtern for the Uſe of a Family. It is plain the Water will iſſue out from L with the ſame Force as it would be made to riſe to the Height L O, were the Tube or Pipe continued upright to O; for on one Side the common Baſe K, the Preſſure (when the Cock is ſhut) is proportional to the Altitude E K, but on the other it is only L K; the Difference therefore, E I = L O, is the Force of Preſſure by which the Water is protruded from the Cock at L. In the ſame manner it is ſhewn, that the Force by which it is made to ſpout from the Cocks at M and N is proportional to the Altitudes H E and G E, or M O and N O, of the Fluid in the Reſervoir above them.

2. The *Running of Rivers* is upon the ſame Principle as the Deſcent of Bodies on inclined Planes; for Water no more than a Solid can move on an horizontal Plane, the Re-action of ſuch a Plane being equal and contrary to Gravity entirely deſtroys it, and leaves the Body at reſt; here we ſpeak of a Plane of ſmall Extent, and ſuch as coincides with the curved Surface of the Earth. But if we conſider a large Extent or long Courſe of Water, then we ſhall find that ſuch Water can never be at reſt, but when the Bottom of the Channel coincides every where with the curv'd Surface of the Earth.

Pl. XVII. Fig. 9. 3. Let A D F be the curved Surface of the Earth, C its Centre, C D, C E two right Lines drawn from thence, and E G a Tangent to the Earth in the Point D. Then 'tis plain if B D were a Channel of Water, the Water could not run, or move, becauſe they are every where at an equal Diſtance from the Center C, and therefore equally affected by Gravity. But if there be any Place above the Surface of the Earth as E, where Water can be found, 'tis evident that Water can deſcend in a Channel to any Part of the Earth's Surface between B and D, becauſe every Point in the Line E D is nearer to the Centre of the Earth, and therefore below the Point or Place E; and its Velocity will be ſo much the greater as it tends to a Point nearer B, and ſloweſt of all, when it moves in the Direction of the Tangent E D.

4. Hence it appears that the Source E of all Rivers and Streams muſt be more than a Semi-diameter of the Earth C B diſtant from the Centre C. And ſince all great Rivers run to the Sea or Ocean where they diſembogue their Waters at the Point D, the Line D C is a Semidiameter, and = 4000 Miles nearly. Alſo the Courſe of all long Rivers being in the Di-

above the common Surface of the Earth; and the Breaking out of Springs at the Bottoms and

rection of the Tangent at the Point D, if they were repreſented by the Tangent-Line E B, then the Height of the Source E above the common Surface of the Earth at B would be eaſily found. Thus, ſuppoſe E D were the River *Niger* in *Africa*, whoſe Source is more than 3000 Miles from the Sea; but put $ED = 3000$, and ſince $CD = 4000$, we ſhall have $CE = 5000$, and $CE - CB = 1000 = BE =$ the Height of the Source. But ſince we know of no Mountains above three or four Miles high, it is plain the River *Niger*, and all ſuch long Rivers, are ſo far from moving in a Tangent, that their Courſe muſt be very nearly of the ſame Curvature with the Earth's Surface, and inſenſibly diſtant from it.

5. Since Bodies move on Planes ever ſo little inclined, except ſo far as they are prevented by Friction, and ſince the Friction of the Particles of Water among themſelves is inconſiderable, it follows that the Water ſituated on a Plane ever ſo little inclined will commence a Motion; and if the Plane be conſiderably inclined, and the Quantity of Water great, its Velocity will be proportional, and its *Momentum* ſuch as will ſoon begin to wear away the Earth, and create itſelf a Courſe or Channel to glide in. In Rivers that are made, it is uſual to allow the Fall of one Foot in 300.

6. If we allow the ſame Declivity to Rivers which make their own Way, then we find their Height at their Source above the common Surface of the Sea, as in Example of the *Niger* thus: As $300 : 1 :: 5280 : \frac{5280}{300} =$ the Height at one Mile, or 5280 Feet. Then again ſay, As $1 : \frac{5280}{300} :: 3000 : \frac{5280 \times 3000}{300} = 5280 \times 10 = 10$ Miles. From whence it is evident, that the Continents and Iſlands ought to be much above the Surface of the Sea, to give a neceſſary Deſcent and Courſe to the Waters through them.

7. Let ABCD be the Section of a Reſervoir, and BCIK the Section of a Canal of Water ſupplied from thence, and ABN the horizontal Line. Now ſince the Particles of Water are govern'd by the common Laws of Gravity, the Velocity of a Particle at any Part of the Bottom of the Canal, as F or H, will be the ſame as it would acquire by falling

Pl. XVII. Fig. 10.

on the Side of Hills, from Ciſterns and Reſervoirs in the internal Parts above them, which re-

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through the perpendicular Altitude OF or LH, (by *Annotat.* XXVII. *Art.* 9) that is, as $\sqrt{OF}$ to $\sqrt{LH}$ (by *Annotation* XXVI. *Art.* 4.). Hence the Velocity of the Stream is accelerated.

8. For the ſame Reaſon the Velocity of a Particle at the Bottom of the Stream H is to the Velocity of a Particle at the Top G, as $\sqrt{LH}$ to $\sqrt{MG}$; conſequently the Stream moves with a greater Celerity at Bottom than at Top.

9. The Quantity of Water which paſſes through the Section of the Stream HG is the ſame that paſſes through the Section of the Reſervoir BC in the ſame Time. The ſame may be ſaid of any other Section FE; therefore the Quantity of Water paſſing by any two Sections of the Stream FE and GH in the ſame Time is the ſame.

10. Since there runs the ſame Quantity of Water by GH as by FE in the ſame Time; and ſince the Velocity at GH is greater than at FE; and laſtly, ſince the Breadth of the Canal is ſuppoſed to be every where the ſame; therefore it follows, that the Depth GH muſt be leſs than the Depth FE, and ſo the Depth of the Stream muſt continually decreaſe as it runs.

11. As the Stream proceeds, the Depth HG decreaſing, the Lines MG and LH will approach nearer to an Equality, and therefore the different Velocities of the Water at Top and Bottom will approach much faſter to an Equality, as being proportionate to the Square Roots of thoſe Lines.

12. This Approach to an Equality is much farther promoted by the upper Parts being continually accelerated by the lower, and the lower Parts retarded continually by the ſlower Motion of the Waters above, incumbent and preſſing upon them.

13. Since the Difference of the deſcending Velocities, or the relative Velocities, are greateſt near the Beginning or Head of the Stream, the Waters will there fall or deſcend with the greateſt Impetuoſity, or cauſe the loudeſt Noiſe.

14. In the Courſe of Rivers, the accelerated Velocity is quickly reduced to an equable or uniform Velocity, by the Reſiſtance it meets with from the Bottom and Sides of the Channel, which Reſiſtance will be as the Squares of the Velocities, and therefore ſoon become ſo great as to equal the accelerating

ceive their Waters from *Rain*, *Dew*, *condenſed Vapours*, *melted Snow*, &c. diſtilling or percolated through the Pores or Crevices and Chaſms of the upper Part of the Earth (LXXI.)

Force, and be communicated to the middle Parts of the Stream; cauſing the Whole to move uniformly.

15. Hence in Rivers the Motion of the Water is ſloweſt at the Sides and Bottom of the Chanel; becauſe there the Reſiſtance begins, and is communicated to all other Parts.

16. In different Parts of the ſame River the uniform Velocity is greateſt where the Bottom of the Channel has the greateſt Inclination or Declivity, becauſe the relative Gravity of the moving Particles is here greateſt.

17. In thoſe Parts of the River where the Velocity of the Stream is leaſt, the Depth of the Water is the greateſt, and; *vice verſa*: Becauſe equal Quantities paſs thro' unequal Sections of the River in the ſame Time. Hence alſo it follows, that the *Momentum* of running Water muſt be every where the ſame; or a given Quantity.

(LXXI.) 1. Various have been the Theories, or rather Hypotheſes, relating to the Origin of Fountains, many of which have ſavour'd ſo little of Philoſophy, that they ſcarce deſerve to be mention'd or confuted. Thoſe who pretend to derive the Waters of Springs from the Waters of the Sea by ſubterranean Ducts, ſeem either wholly ignorant of the hydroſtatic Laws of Fluids, or reſolved to maintain a Theory by meer Hypotheſis.

2. For let ECAF be a Part of the Earth's Surface cover'd with Water at A; ſuppoſe ABC a ſubterranean Duct, by which the Waters deſcend to the internal Parts of the Earth towards B; if B be the loweſt Point, or that neareſt the Centre of the Earth, the Water will be forced to riſe from B to C by the Preſſure of the Water in the Part AB. But when the Water is arrived to C it will there ſtop, and an Equilibrium will neceſſarily enſue (from what we have ſhewn in *Annotat.* LXX.) Conſequently no Water can riſe above C to the Top of the Mountain D by the Force of Preſſure; and we can ſhew no other Power in Nature to drive it thither; and therefore ought to ſuppoſe none. Pl. XVII. Fig. 11.

3. They who advance the Capillary Hypotheſis, or ſuppoſe the Waters riſe from the Depths of the Sea thro' the porous Parts of the Earth, as it riſes in Capillary Tubes, or

By the Preſſure of the Fluid it may eaſily be convey'd over Vallies and Hills by bended Pipes

Tubes of Sand or Aſhes, ſeem not to conſider one principal Property of this Kind of Tube, or this Sort of Attraction; for tho' the Water riſe to the Top of the Tube or Sand, yet will it riſe no higher, becauſe it is by the Attraction of the Parts above that the Fluid riſes, and where that is wanting it can riſe no farther. Therefore, tho' the Waters of the Sea may be drawn into the Subſtance of the Earth by Attraction, yet can it never be raiſed by this means into a Ciſtern or Cavity to become the Source of Fountains.

4. The true Principles which ſupply the Waters of Fountains or Springs, are undoubtedly *melted Snow*, *Rain-Water*, and *condenſed Vapours*. Several have attempted to ſolve the *Phænomenon* by Snow or Rain only; but others making an Eſtimate of the Quantity of Rain and Snow, that falls in the Space of a Year, to ſee if it would be equal to that which the Rivers diſcharge annually into the Sea, found that it was much ſhort of that Quantity.

5. But that which was moſt extraordinary was, that they found by their Experiments, that the Rain and Snow which fell in one Year would not produce more than $\frac{2}{3}$ of what was raiſed in Vapour; for by Experiment it was found, that the Rain and Snow that fell in a cylindric Veſſel, raiſed a Column of Water about 19 Inches high; whereas the Water raiſed in Vapour was yearly about 32 Inches Altitude. This great Deficiency of 13 Inches plainly indicated another Way, by which the Waters circulated from and to the Sea.

6. This (among many other Things) was left to the Diſcovery of that ſagacious Naturaliſt Dr. *Halley*; who being on the Tops of the Mountains in the Iſle of St. *Helena* (in South Lat. 16°) making his Obſervations for a Catalogue of the Southern Stars, (about 800 Yards above the Level of the Sea,) found that the Quantity of Vapour, which there fell in Dew, was ſo great, as very much impeded his Obſervations, by covering his Glaſſes over in 6 or 7 Minutes, even when the Sky was clear.

7. Upon this he was induced to determine by Experiment, the Quantity of Vapours raiſed from the Surface of the Sea, as far as it ariſes from Heat; for other Cauſes, as Wind, and the Warmth of common Air, contribute to exhale the Water, as the ſame Gentleman found by making the Experiment in a cloſe Room, where 8 Inches Altitude of Water

or Syphons, from Ponds or Reſervoirs in a higher Situation; which the Ancients being ignorant of,

was exhaled in one Year; and 'tis well known from common Obſervation, that a ſtrong Wind will carry off a great Quantity of Water in a few Hours time, as in the late *January* Storm ſeveral ſhallow Ponds had all their Water blown away by the Wind in one Day. Alſo, we find by Experience, that one windy Day will make the Ways dryer after Rain, than 3 or 4 Days Sun-ſhine.

8. But the Doctor's Deſign was to ſee what Quantity was evaporated by the Heat of the Sun only. For this Purpoſe he inſtituted the Experiment in the following Manner: He took a Veſſel of Water, and made it ſalt to the ſame Degree with Sea-Water, by means of the Hydrometer. In this he placed a Thermometer, and by a Pan of Coals he brought the Water to the ſame Degree of Heat with that of the Air in the hotteſt Summer.

9. This done, he affix'd the Veſſel of Water, with the Thermometer in it, to the End of a Pair of Scales, and nicely counterpoiſed it with Weights in the other; then at the End of two Hours he found by the Alteration made in the Weight of the Veſſel, that about the 60th Part of an Inch of the Depth of the Water was gone off in Vapour, and therefore in 12 Hours, one Tenth of an Inch would have been evaporated. *Note*, 12 Hours only are here allow'd for the Time of Exhalation, that being the Length of the Day at a Mean; and in the Night, as much, if not more Water, returns by Dew than is exhaled.

10. Upon this Suppoſition, every 10 ſquare Inches of the Surface of the Water yields in Vapour, *per Diem*, a cubic Inch of Water, which weighs 253⅓ Grains *Troy*; therefore every ſquare Foot will yield ½ a Pint Wine Meaſure; every Space of 4 Feet ſquare, a Gallon; and every ſquare Mile 6914 Tons. A Degree ſquare (reckoning 69 *Engliſh* Miles to a Degree) will produce 33 Million of Tons; and if the *Mediterranean* be 40 Degrees long, and 4 broad (the narrow Parts compenſating for the broader,) which is the leaſt that can be ſuppoſed, then will there be in its Surface 160 ſquare Degrees, which will evaporate *per Diem*, 5280 Millions of Tons in the Summer Time.

11. The *Mediterranean* receives Water from the following nine great Rivers, *viz.* The *Iberus*, the *Rhone*, the *Tyber*, the *Po*, the *Danube*, the *Neiſter*, the *Boryſthenes*, the *Tanais*,

were at vaſt Pains and Expence very often to effect. Hence the running of Water through

and the *Nile*; all the reſt being of no great Note, and their Water inconſiderable. Each of theſe nine Rivers (together with all the ſmaller) are ſuppoſed to bring as much Water to the Sea, as is equal to ten Times the Water of the *Thames*, at leaſt.

11. In order to eſtimate the Quantity of Water, which paſſes daily thro' the *Thames*, the Doctor aſſumes the Breadth of the River at *Kingſton* Bridge (where the Flood ſeldom reaches) to be 100 Yards, and the Depth 3; ſo that the Section of the Channel is 300 ſquare Yards, and allowing the Velocity of the Water to be at the Rate of 2 Miles *per* Hour, there will run in 24 Hours, the Length of 48 Miles, or 84480 Yards; therefore $84480 \times 300 = 25344000$ cubic Yards, which make 20300000 Tons which the River *Thames* yields *per Diem*.

12. Now each of the above nine Rivers being ſuppoſed to bring ten times as much as the *Thames*, will yield 203000000, or 203 Millions of Tons, and therefore all the nine will produce 1827 Millions of Tons; which is but little more than $\frac{1}{3}$ of the Quantity (5280 Millions of Tons) evaporated every Day from the Sea. The $\frac{2}{3}$ nearly of this prodigious Quantity of Water, the Doctor allows to Rains which fall again into the Seas; and to the Rain imbibed by the low Parts of the Earth, and ſpent in Vegetation in general.

13. The Quantity of Water exhauſted from the *Mediterranean*, being ſo much greater than what is return'd by the Rivers, is the Occaſion of the Water's ſetting in from the Ocean, to ſupply the Deficiency, by a continual Influx or Stream, which before this Diſcovery was quite unaccountable. Alſo, hence it appears, why the *Caſpian* Sea, tho' it receives the Waters of many large Rivers, is yet never liable to overflow; becauſe the Waters, brought in by the Rivers, are exhauſted by the Sun, Wind, &c. Yea, the Difficulty is on the other Hand, ſince the Exhalation is ſo much greater than the Supply by the Rivers, why the Sea does not appear diminiſh'd, or its Waters continually waſting? which Thing has never been obſerved.

14. The prodigious Quantity of Vapours raiſ'd by the Sun's Heat, and otherwiſe, being carried by the Winds over the low Lands to the very Ridges of Mountains, as the *Pyrenean*, the *Alps*, the *Apennine*, the *Carpathian*, in *Europe*; the *Taurus*

the *bended Syphon* accounts for the odd Phænomena of *intermitting and reciprocating Springs*;

Caucaſus, *Imaus* and others in *Aſia*; *Atlas*, the *Montes Lunæ*, or Mountains of the Moon, with other unknown Ridges in *Africa*; I ſay, the Vapours being compelled by the Stream of Air to mount up with it to the Top of thoſe Mountains, where the Air becoming too light to ſuſtain them, and condenſed by Cold, they precipitate in Water, and gleet down by the Crannies of the Stone; and Part of the Vapour entering into the Caverns of the Hills, the Waters thereof gather as in an *Alembic*, into the Baſons of Stone it finds, which being once fill'd, all the Overplus of Water that comes thither, runs over by the loweſt Places, and breaking out by the Sides of the Hills, forms ſingle Springs.

16. Many of theſe Springs running down by the Vallies, between the Ridges of the Hills, and coming to unite, form little Rivulets, or Brooks; many of theſe again meeting in one common Valley, and gaining the plain Ground, being grown leſs rapid, become a River; and many of theſe being united in one common Channel, make ſuch enormous Streams as the *Rhine*, the *Rhone*, the *Danube*, &c. And it may almoſt paſs for a Rule, that the Magnitude of a River, or the Quantity of Water it diſcharges, is proportional to the Length and Heights of the Ridges from whence the Fountains ariſe.

17. This beautiful Account of the Origin of Springs and Rivers has been received with univerſal Applauſe and Satisfaction in the learned World. I ſhall now add a Word or two concerning the different Sorts of Springs which we find in divers Parts of the Country. But one Thing I ſhall firſt remark, and that is, it has been aſſerted, and often taken for granted, that *there are Springs of Water upon the very Tops, Summits, or higheſt Parts of Mountains*; which Poſition is contrary to the Hydraulic Law of Fluids, by which they riſe to the Level of the Fluid in the Reſervoir, but no higher; and not only that, but it is undoubtedly falſe in Fact: For I have myſelf examin'd moſt of the remarkable Hills in *Suſſex*, *Surry*, *Hampſhire*, *Buckinghamſhire*, *Berkſhire*, *Glouceſterſhire*, *Wiltſhire*, *Somerſetſhire*, and other Counties, but could never diſcover any ſuch Thing; tho' I often found them on the Sides, not far below the Tops, but low enough to account for their being there. And Mr. *Caſwell* carefully examin'd the Tops of the Mountains in *Wales* (at the Deſire of a Friend) and could find no Spring there. I ſhall therefore conclude

and alſo in that Inſtrument call'd the *Tantalus-Cup*. The Riſe of Water in Wells alſo

there never was, nor can be, any ſuch Thing in Nature.

17. The ſeveral Sorts of Springs obſerv'd are (1.) *common Springs*, which either run continually, and then they are call'd *Perennial Springs*; or elſe run only for a Time, or at certain Times of the Year, and then they are call'd *Temporary Springs*. (2.) *Intermitting Springs*, or ſuch as flow and then ſtop, and flow and ſtop again, by regular Alternations or Intermiſſions. (3.) *Reciprocating Springs*, whoſe Waters riſe and fall, or flow and ebb, by regular Intervals, or Reciprocations of the Surface. All which are produced after the following Manner.

Pl. XIX. Fig. 1. 18. Let ABCDE repreſent the Declivity of a Hill, whoſe Section, from Top to Bottom perpendicularly, is ſhewn in the Figure; in which let FGH be a Cavern or Baſon near the Top which collects the Water gleeting thro' the Crannies, and has a Drain or Duct leading from H to the Side of the Hill at B. 'Tis evident when the Water riſes to the Drain H, it will deſcend thro' it to B, where it will break out in Form of a Fountain or Spring, and will continue thus, ſo long as the Reſervoir is ſupplied with Water above the Level FH, and after that it will become dry. Thus we ſee a Spring may be generated near the Top of the higheſt Hills or Mountains. Of this we have a notable Inſtance near the Top of *Landſdown-Hill* by *Bath*.

19. There is one odd *Phænomenon* of ſuch Springs that is ſometimes obſerv'd, and that is, *that they run in dry Weather, and are dry in wet Weather*; to account for which, we need only obſerve, that while the Weather continues wet, the Waters are gathering into the Baſon or Reſervoir, till there is a ſufficient Quantity to run over and make the Spring play, by that Time the Weather is alter'd and become dry; during the dry Weather, the Remains of the laſt Rains (for theſe Springs proceed chiefly from Rain Water) are continually feeding the Reſervoir, and by that means ſupplying the Spring, but by the Time theſe are all ſpent, the Weather again alters to Wet, and the Spring ceaſes of courſe, till it meets a freſh Supply from the preceding Rains, and ſo on.

20. The INTERMITTING SPRINGS may be thus accounted for: Let IKL be a Cavity in the Mountain, to which at I there is a feeding Stream that brings the Water from other Parts, and at K, on the lower Part, there goes a Duct K*k*C, which is of a curved or crooked Form, and conveys the Water to

is owing to the Weight of that in the ſubterranean Ciſterns from whence they are ſup-

the Side of the Hill at C, where it breaks out in a Spring. Here 'tis evident, that as the Water riſes in the Cavern, it will riſe in the Duct at the ſame Time, till the Surface of the Water IL be level with the higheſt Part *k* of the Duct, after which the Water will deſcend from *k* to C, which Point C, being lower than the Orifice of the Duct K, will exhauſt the Cavern of all its Water, and then the Spring will ſtop till the feeding Stream I repleniſhes the Ciſtern to the ſame Level IK, and then the Fountain will play again; this ſuppoſes the Duct KC to carry off more Water than the other at I brings to the Ciſtern; and then every Thing follows from the Nature of the *bended Syphon* as explain'd in the next *Annotation*.

21. The RECIPROCATING SPRINGS are occaſion'd after this Manner. Let MNO be a Cavity ſupplied by a feeding Stream PM, and alſo by a Syphon K*k*O, which brings Water from another Cavity above, as IKL; the Duct ND carries the Water to the Side of the Hill at D, and there makes a conſtant Spring, by Virtue of a conſtant Supply of Water, by the Drain PM. The Water at D will alſo flow and ebb alternately, for when the Syphon K*k*O works, the Surface of the Water MO will be ſuddenly raiſed, and preſs upon the Water at N with a greater Force, by which Means it will iſſue out at D with a greater Velocity, and raiſe the Surface if confin'd; but when the Syphon intermits, or ceaſes, the *Momentum* of the Water at D is not ſo great as before, and then the Spring will ſink or decreaſe.

22. In like Manner we account for the riſing of Water in Wells. Thus ſuppoſe a Well be ſunk at the Foot of the Hill at E, to ſuch a Depth EV, as will bring the Diggers to an Eruption of a Spring at V, whoſe Water is brought by the Duct RV (or many of them) from a Cavity QRS in the Hill, (or otherwiſe from a Pond, a River, the Sea, &c.) 'tis evident the Water in the Well will riſe from the Bottom V, to an Altitude VT, where the Surface of the Water at T is upon a Level with that in the Reſervoir QS, and thus conſtitutes a Well.

23. If thoſe Reſervoirs of Water in the Body of Mountains be ſituated where Mineral Ores abound, or the Ducts or feeding Streams run thro' Mineral Earth, 'tis eaſy to conceive the Particles of Metal will mix with, and be abſorb'd

plied by proper Canals or Springs (LXXII.)

FROM the Preſſure of the ſuperior Fluid we

by the Water, which being ſaturated therewith, becomes a *Mineral Spring or Well*. If Salt, Sulphur, Lime-ſtone, &c. abounds in the Strata, thro' which the Water paſſes, it will then be ſaline, ſulphureous, Lime-Water, &c. If Sulphur and Iron ſhould both abound in the Parts of the Hill, whence the Waters come, the Waters will partake of the Warmth or Heat, which is occaſion'd by the Mixture of two ſuch Subſtances in the Earth, where they are found. Now, tho' every Thing may not happen preciſely in the Manner, as here repreſented, yet that it is in ſome Way analogous to it, I believe no Perſon will doubt, who has been at the *Peak* in *Derbyſhire*, or at *Wookey-Hole* in *Somerſetſhire*, and ſeen the wonderful Caverns, Receptacles and Streams of Water, which Nature has there furniſh'd in the Bowels of the Mountains.

SCHOLIUM.

24. I ſee but one Objection which can be made to the foregoing noble Theory of Dr. *Halley*, and that is, that a Body of Water heated to the ſame Degree with the hotteſt Summer Air will certainly evaporate more from a given Space in a given Time than the deep Waters of the Sea, whoſe Surface only is affected by the Heat of the Summer Air; and therefore in all probability the Doctor's Quantity of Water raiſed in Vapour was much too great. To which I anſwer (1.) The Rivers are ſuppoſed to return much more Waters to the Sea than they really do, for no one can imagine there can be 90 Times the Water of the *Thames* brought to the *Mediterranean* by all its Rivers. (2.) The Doctor accounts for Springs by the Quantity of Vapour exhaled by the Sun alone, whereas if we abate near half this Quantity, and take in what is raiſed by the Wind and Agitations of the Sea, together with the large Supplies from Rain, Melted Snow, &c. we ſhall undoubtedly find ſufficient Matter for the Sources of all the Rivers and Springs. (3.) As the Vapour is exhaled from the Surface of the Water only, it will be proportional to the Heat in the Surface only, which muſt be nearly the ſame for the moſt Part of the Day with the contiguous Air, and therefore the Caſe is not ſo widely different from the Doctor's Experiment as might be at firſt imagined.

(LXXII) The Nature of the *bended Tube or Syphon* (of which we have made ſo much Uſe in the preceding *Annota-*

have the Invention of a *Natural Fountain*, which playing from the ſmall Orifice of an Adjutage

Pl. XIX, Fig. 2.

tion) will eaſily appear by conſidering it as follows. ABCD is a Veſſel, into which is inſerted at the Bottom a bended Pipe BGHIK to repreſent the Syphon. Now if Water be pour'd into the Veſſel, it will as it riſes in the Veſſel riſe alſo into the Pipe or Duct, till the Water in the Veſſel be upon a Level FE with the upper Part G of the curved Tube; what more is pour'd in after this will run over the Bend at G down the lower Part GH; and from the loweſt Point H it will, by the ſame Principle as before, be forced into the riſing Part HI, where at the Bend I it will again run over, and deſcend thro' the Part IK, and at K it will run out, as being lower than the Level of the Bottom of the Veſſel CL.

2. That the Pipe muſt neceſſarily be full to the ſame Altitude every where as the Water ſtands in the Veſſel, by the Preſſure of that in the Veſſel, is evident from what we have ſhewn in *Annotation* LXX. And ſince it can riſe no higher in the Veſſel than in the Pipe, and in the Pipe it can riſe no higher than the bended Parts G or I, therefore it can riſe no higher in the Veſſel than thoſe bended Parts of the Pipe. Again, becauſe the Preſſure of the Air upon the Water at FE in the Veſſel, and upon the Orifice at K is equal, therefore if the Orifice K were in the ſame Level with the Bottom of the Veſſel CL, the Water, having the ſame Altitude EB and ML, wou'd not run out at the Orifice L, but remain *in Equilibrio* with that in the Veſſel.

3. But, laſtly, ſince the Orifice K is below the Level, the Water at K will have a greater Altitude MK, and therefore a greater Preſſure, or *Momentum*, and ſo will prevail againſt the equal Preſſure of the Air, and run out; and will continue to run out, till all the Water in the Veſſel be exhauſted, becauſe the Preſſure of the Air upon the Surface FE will force the Water into the Pipe to prevent a *Vacuum*, which muſt otherwiſe happen in ſome Part theteof, which is abſurd or contrary to Nature.

4. For ſuppoſe the Water were not to enter at B as faſt as it runs out at K; there muſt happen a Separation of the Water in ſome Part of the Pipe, as at I for Inſtance; then the Water running out at K will cauſe the ſeparated Part to deſcend from I towards L, and ſince nothing can get into the Space between the ſeparated Surfaces of Water at I and L, all that Space muſt be a pure *Vacuum*, and therefore the Water muſt

at the Bottom, will ſpout very beautifully nearly to the Height of the Water from the Foun-

be at reſt in the Pipe between B and I; tho' the Preſſure of the whole Atmoſphere acts upon the Part at B, and nothing at all at the Part I to reſiſt or counteract it, which is contrary to the Nature or Definition of a Fluid, as has been abundantly ſhewn in *Annot*. LIII, and all the ſubſequent ones.

5. This bended Syphon is uſually call'd a TANTALUS, from the Semblance of the Experiment made with a Man in a Glaſs to the Caſe of *Tantalus* in the Fable, who, according to the Poets, was ſentenced for his horrid and unnatural Crime of killing, boiling, and ſerving up his own Son at a Feaſt of the Gods, to ſtand in a River of Hell up to his Chin in Water, which yet he is not permitted to ſip to quench his eternal Thirſt. Now this *Tantalus* is the Man in the Glaſs, with a Syphon conceal'd in his Body, beginning in the Bottom of one of his Feet, and aſcending to the upper Part of his Breaſt; there it makes a Turn, and deſcends through the other Leg on which he ſtands, and from thence down through the Foot of the Glaſs, where it runs out, and cauſes the Water to ſubſide in the Glaſs, as ſoon as it riſes to the Height of the Syphon, or to the Chin of the Man; and thus aptly expreſſes the Fable.

6. This Syphon may very eaſily be conceal'd alſo in the Handle of a common Pint Pot or Cup, and then it is call'd the TANTALUS CUP: For the Top of the Handle being a little below the Top or Rim of the Cup, and the loweſt Part of the Handle or Syphon below the Bottom or Orifice by which it communicates with the Cup, will cauſe the Cup to have the Property of emptying itſelf when fill'd to the Brim; as is eaſy to underſtand from what has been ſaid above.

SCHOLIUM.

7. The Nature of this Syphon, or the hydraulic Law of Fluids, on which it depends, was not underſtood by the Ancients; for though we read ſurpriſing Accounts of the *Roman* Aquæducts and Pipes, by which they convey'd Water to the Diſtance ſometimes of five or ſix Miles, yet we find no mention made of any other than horizontal and deſcending Pipes or Ducts, and never of any riſing or aſcending. If the Water was to be convey'd where a Valley interpoſed, they with great Expence and Labour conſtructed a Water-Courſe over the ſame, ſupporting it with vaſt Piles and Arches ſometimes

tain; but it cannot quite reach that Altitude, by reaſon of the Air's Reſiſtance (LXXIII.)

from the Depth of near an 100 Feet. Whereas a ſingle Pipe laid over the Vale would have done the Buſineſs moſt effectually. If a Mountain interpoſed, we find they would ſometimes carry their Aquæduct through the very Body of it by Dint of Labour, which ſurely could not have been their Choice, had they known that a Pipe laid over the Top coming from a Reſervoir of a proper Height would have anſwer'd the ſame End. Such Reſervoirs indeed ſuppoſe the Uſe of Pumps to raiſe the Water into them; and Pumps they had of various Kinds, but could not apply them for want of knowing the Theory. They who would acquaint themſelves with the Magnificence of the *Romans* in regard to their Water-Works and Aquæducts may conſult *Frontinus* and others, who have wrote on this Subject.

(LXXIII) 1. The Theory of Natural Fountains, or *Jet d'Eau's*, depends on the following Principles. It has been ſhewn, that Water coming from a Reſervoir ABCD, through the Pipe EFGH, will riſe from the loweſt Part G to the ſame Altitude H, in the Part GH, as is upon a Level with the Surface of the Water AB in the Reſervoir: And alſo that it thus roſe from the Point G by a Force of Preſſure proportional to the Altitude of the Water in the Reſervoir, which is equal to the Altitude GH. *Annot.* LXIX. Pl. XIX. Fig. 3.

2. Now I preſume 'tis very evident that the Tube GH itſelf can contribute nothing towards the Water's riſing in it; but on the contrary, it rather impedes the Aſcent by the Friction it occaſions to the Particles which move againſt the internal Surface thereof. Therefore if the Part GH be taken away, the Water would riſe to the ſame Height H, excepting ſo far as it is obſtructed by other concurring Incidents; for in all Fountains the Height GI is ſomewhat leſs than GH, the Height in the Tube, for the following Reaſons.

3. *Firſt*, The Air's Reſiſtance is an Obſtruction to the Jet, and diminiſhes its Height; and ſince we know that the Reſiſtance of Fluids is proportional to the Squares of the Velocity, (*Annot.* LXVIII. *Art.* 4.) and the Deficiency of Height HI is proportional to the Reſiſtance; therefore a Jet that plays with a double Velocity will have that Deficiency four Times as great; and with three Times the Velocity, nine Times as great; and ſo on.

4. The ſecond Impediment is the Friction againſt the Sides

FROM this Preſſure of Fluids, as the Cauſe, we can eſtimate the various *Momenta* or Forces of ſeveral Jets ſpouting through the Sides or Bot-

of the Hole, and the Adjutage at G; and ſince this is in proportion to the Quantity of Surface in the Hole, it will be greater in a ſmall Hole than a large one, in reſpect of the Body of the Jet, or ſpouting Water; becauſe the Jet will increaſe in Magnitude with the Square of the Diameter of the Hole, whereas the Reſiſtance will increaſe only with the Diameter ſimply. Or a Hole of twice the Diameter emits four Times the Water, and gives but twice the Reſiſtance. This makes a ſmall Jet riſe to a leſs Height than a large one, from the ſame Reſervoir.

5. A third Impediment ariſes from hence, that ſince all the Particles ſet out from G with an equal Velocity, and this Velocity is continually diminiſhing, it follows that the Velocity of the inferior Parts is greater than the Velocity of the Parts above them, and therefore muſt in ſome Degree ſtrike againſt them; by this Impulſe (ſince Fluids move every way) the Particles will be urged ſideways, and the Column of the Jet become wider, and conſequently ſhorter than it would otherwiſe be.

6. The fourth Cauſe why Jets do not riſe to the Height of the Reſervoirs, is becauſe the Water upon the Top of the Jet does not immediately run off, but ſpreading into a Head, lies with its Weight upon the aſcending Water below, and hinders it from riſing ſo high as it would do. This will appear by inclining the Jet a little, that the upper Water may not bear upon the riſing Stream, for the Jet will then play higher, but be leſs beautiful.

7. If the Hole of the Adjutage G be leſs than $\frac{1}{4}$ of an Inch in Diameter, the Force of the Attraction of Coheſion will extend itſelf thro' the Body of the Jet at the Hole, and greatly obſtruct its Aſcent; hence the Riſe of Jets in Models, for Experiments cannot be expected ſo high as others, becauſe of the very ſmall Holes made uſe of in them. Whence all Jets of larger Fountains (ſuch as are in Gentlemens Gardens) ought to exceed $\frac{1}{4}$ of an Inch in Diameter, and that in Proportion to the Height of the Reſervoir, as will be ſhewn hereafter.

8. If the Conduct-Pipe EFG be not of a proper Size to ſupply Water, as faſt as it can be expended at the Adjutage G, the Jet will likewiſe be check'd, and will not riſe to the full Height. To aſcertain the Proportion of the Conduct-Pipe to the Bore of the Adjutage, is the Buſineſs of Ex-

toms of Veſſels at different Depths below the Surface: For the *Momenta* of Fluids, as well as ſolid Bodies, is *compounded of the Quan-*

periments; and what has reſulted from thence, in regard to this Matter, will be ſhewn in a Table by and by.

9. And one Thing more is neceſſary to be known, that the Jet may play the higheſt poſſible, *viz.* that the Part of the Conduct-Pipe at the Adjutage does not turn up at a Right Angle, but with a gentle eaſy Curve; that is, not as at G, but as at L, where the Jet plays to a greater Height at K. The upright Part at G directly reſiſts the Water coming from F, where the Curve at L cauſes the Impulſe of the Water againſt it to be very oblique; and therefore a leſs Part of its *Momentum* will be deſtroy'd, and conſequently the greater remaining Force will throw the Jet the higher.

10. From what has been ſaid in *Art.* 4. it appears (*cæteris paribus*) the Hole in the Adjutage ought to be made in a thin Plate of Braſs, and not thro' the Bore of a Tube of any Length, becauſe of the Quantity of Surface in ſuch an Adjutage-Piece, which muſt greatly retard the Jet, and diminiſh the Height. This is a Fault which moſt People have run into for want of an experimental Knowledge of this Science. For the ſake of Beauty they ſcrew on ſuch a conical Piece as repreſented at M, of an Inch in length, inſtead of a Plate of Braſs, whoſe thickneſs need not exceed the 40th Part of an Inch for very large Jets, and whoſe Friction will therefore be 40 Times leſs.

11. If the Jet be play'd from G, the Velocity with which it ſets out from the Adjutage, will be as $\sqrt{FQ}$, and throw it to its full Height G C at once; but if the Pipe deſcends below F G to H I, and thence returns upwards again to the ſame Level F K, and the Jet be play'd from K, the Velocity with which it ſets out from K at firſt will not be the ſame as the Velocity from G, that is, it will not be as $\sqrt{FQ}$, or $\sqrt{QH-FH}$, but it will be as $\sqrt{QH}-\sqrt{FH}$, which is leſs, and therefore the Height at firſt will be only K L, from whence by Degrees it will riſe to M upon the ſame Level with C, ſo that its laſt Height K M will be equal to the Height G C of the Jet from G. Pl. XIX. Fig. 4.

12. The Truth of the laſt Article is thus demonſtrated. The Water in the Legs Q H and I K are to be conſider'd as two Non-Elaſtic Bodies whoſe *Momenta* (being as the Preſ-

tity of Matter and Velocity; but in ſpouting Fluids, *the Quantity of the Fluid iſſuing through the ſame Hole in the ſame Time is always as the*

ſure) are as the Altitudes QH and IK; and ſince both the Velocities and Quantities of Matter moved in a given Time, are as the Square Roots of the Altitude (as will be ſhewn in the next Annotation) therefore we have $QH = \sqrt{QH} \times \sqrt{QH}$, and $IK = \sqrt{IK} \times \sqrt{IK}$, and ſince the Difference of the *Momenta*, divided by the Sum of the Bodies, gives the Velocity after the Stroke (*Annot.* XXIII. *Art.* 4.) which we will call x, therefore $\frac{\sqrt{QH} \times \sqrt{QH} - \sqrt{IK} \times \sqrt{IK}}{\sqrt{QH} + \sqrt{IK}} = x = \sqrt{QH} - \sqrt{IK}$, (becauſe $\sqrt{QH} \times \sqrt{QH} - \sqrt{IK} \times \sqrt{IK} = \overline{\sqrt{QH} + \sqrt{IK}} \times \overline{\sqrt{QH} - \sqrt{IK}}$; which is the Velocity of the Water at firſt ſetting out from K.

13. But ſince Fluids act not by a ſingle Impulſe, as Solids do, but conſtantly; the Velocity of the Water at K will not be equable at firſt, but accelerated for ſome Time, that is, till all the Water in the Leg IK ſhall be brought to move with the ſame Velocity with that in the Leg QH when it comes to be uniform: For as the Velocity in the Leg IK is accelerated, ſo by its Re-action does it retard the Motion or diminiſh the Velocity of the Water in the Leg QH; but as it cannot totally deſtroy the Motion of QH, that which remains muſt be equable or uniform; and what that is will eaſily appear if we only conſider, that that Part of the Motion which is deſtroy'd in QH by the Re-action of the Leg IK muſt be equal to its own *Momentum*, which is as IK; (becauſe Action and Re-action are equal) therefore the remaining *Momentum* of QH will be $QH - IK = QH - FH = QF$, wherefore the laſt Velocity will be as $\sqrt{QF}$, and therefore equal to that at G, and ſo carry the Water as high; that is, the Jet will riſe from L to M, as high as from G.

14. To give an Example: Suppoſe $QH = 25$ Feet, $FH = IK = 9$; then $QF = (QH - HF =)\ 16$; therefore the firſt Velocity will be to the laſt Velocity as $\sqrt{QH} - \sqrt{IK} : \sqrt{QH - IK}\ (= \sqrt{QF}) :: \sqrt{25} - \sqrt{9} = 5 - 3 = 2 : \sqrt{25 - 9} = \sqrt{16} = 4$. Conſequently the

Celerity of its Motion, as is eaſy to conceive: Whence the *Momenta of ſpouting Fluids are proportional to the Squares of the Velocities, or Quantities of Matter iſſuing out in a given Time:* But ſince the *Momenta* are the Effect of Preſſure, it is evident, *the Velocity, or Quantity of ſpouting*

the Height of the Jet K L is to the Height at laſt K M, as 2 to 4. And thus you carry on the Calculation for any Length of the Leg I K whatſoever.

15. I ſhall here ſubjoin a Table of the Height to which Jets will riſe in Feet and decimal Parts from Reſervoirs of an Height of five Feet to 150 Feet.

Ref.	*Jet.*	*Ref.*	*Jet.*	*Ref.*	*Jet.*	*Ref.*	*Jet.*	*Ref.*	*Jet.*	*Ref.*	*Jet.*
5	4,91	23	21,46	41	36,55	59	50,49	77	63,54	95	75,83
6	5,88	24	22,33	42	37,35	60	51,24	78	64,24	96	76,49
7	6,84	25	23,20	43	38,14	61	51,99	79	64,94	97	77,15
8	7,8	26	24,06	44	38,93	62	52,73	80	65,64	98	77,81
9	8,74	27	24,92	45	39,75	63	53,47	81	66,33	99	78,47
10	9,68	28	25,78	46	40,53	64	54,20	82	67,02	100	79,12
11	10,62	29	26,63	47	41,31	65	54,93	83	67,71	110	85,58
12	11,55	30	27,48	48	42,09	66	55,66	84	68,40	120	91,86
13	12,48	31	28,32	49	42,87	67	56,39	85	69,08	130	97,99
14	13,40	32	29,16	50	43,65	68	57,12	86	69,76	140	103,97
15	14,31	33	30.	51	44,42	69	57,84	87	70,47	150	107,87
16	15,22	34	30,83	52	45,19	70	58,56	88	71,14		
17	16,13	35	31,63	53	45,96	71	59,28	89	71,81		
18	17,03	36	32,47	54	46,72	72	60.	90	72,48		
19	17,93	37	33,29	55	47,48	73	60,71	91	73,15		
20	18,82	38	34,11	56	48,24	74	61,42	92	73,82		
21	29,70	39	34,93	57	48,99	75	62,13	93	74,49		
22	20,58	40	35,74	58	49,74	76	62,84	94	75,16		

16. By various Experiments that have been made by Mr. *Mariotte*, Dr. *Deſaguliers*, and others, it has been found, that if the Reſervoir be five Feet high, a Conduct-Pipe 1 $\frac{3}{4}$ Inch Diameter will admit a Hole in the Adjutage from $\frac{1}{4}$ of an Inch to $\frac{3}{8}$ of an Inch; and ſo on as in the following Table:

Fluids is ever proportional to the Square Root of the Preſſure or Altitude of the Fluid (LXXIV.)

Height of Reſervoir.	*Diameter of the Adjutage.*	*Diameter of the Pipes of Conduct.*
5 Feet	$\frac{1}{8}$ to $\frac{3}{4}$	1 $\frac{3}{4}$ Inch.
10	$\frac{1}{4}$ to $\frac{1}{2}$ Inch	2 Inches.
15	$\frac{1}{2}$ an Inch	2 $\frac{1}{4}$ Inches.
20	$\frac{1}{2}$ an Inch	2 $\frac{1}{2}$ Inches.
25	$\frac{1}{2}$ an Inch	2 $\frac{3}{4}$ Inches.
30	$\frac{1}{2}$ to $\frac{3}{4}$ of an Inch	3 Inches, or 3 $\frac{1}{2}$.
40	$\frac{3}{4}$ of an Inch	4 $\frac{1}{2}$ Inches.
50	$\frac{3}{4}$ of an Inch	5 Inches.
60	1 Inch	5 $\frac{3}{4}$ Inches.
80	1 $\frac{1}{4}$ Inch	6 $\frac{1}{2}$ Inches.
100	1 $\frac{1}{4}$ or 1 $\frac{1}{2}$ Inch	7 or 8 Inches.

17. Here the Jet is ſuppoſed to be within 100 or 150 Feet of the Reſervoir; but if the Conduct-Pipe much exceeds this Length, it muſt be of a larger Diameter than what is here aſſign'd. Thus for Jets from $\frac{3}{4}$ of an Inch to thoſe of an Inch, and from Reſervoirs from 40 to 90 Feet height, if the Diſtance be from 150 Yards to $\frac{1}{4}$ of a Mile, the Diameter of the Pipe ſhould be of 6 Inches; from $\frac{1}{4}$ of a Mile to two Miles, it muſt be of 7 Inches; and from 2 Miles to 5, it muſt be of 8 Inches Diameter for the ſame Jets.

18. If it be required to keep any Number of Jets playing, whoſe Adjutages are given in Diameter, by one common Conduct-Pipe, we muſt find the Diameter of an Adjutage equal to all the given ones. Thus if there be four Adjutages of $\frac{3}{4}$ of an Inch Diameter each, then the Square of $\frac{3}{4}$ is $\frac{9}{16}$, which multiplied by the Number of Adjutages 4, makes $\frac{36}{16}$; the Square Root of which is $\frac{6}{4} = 1\frac{1}{2} =$ the Diameter of the Adjutage equal to all the four ſmall ones. A Pipe of Conduct of 10 Inches Diameter will ſupply all the Jets, as being a little more than ſix times as great as the Diameter of the one large Adjutage now found. After this Manner the Dimenſions of a Conduct-Pipe may be found for any other Number of Adjutages.

Pl. XVI. Fig. 6. (LXXIV) To repreſent theſe Propoſitions in Symbols may render them more perſpicuous, and, perhaps, more elegant. Therefore let Water ſpout from the Side of a Veſſel at two different Depths B and E from the Surface of A; the *Momen-*

FROM hence it appears, *that the Velocity of a Fluid ſpouting at any Depth below the Surface, is equal to the Velocity a heavy Body would acquire by falling from the ſame Height*; becauſe the Velocity, as was obſerved, is always as the Square Root of the Space deſcended through. If therefore on the Altitude of the Fluid A K, as a Diameter, we deſcribe the Semicircle A F K; and from any Point therein, as I, we draw the Perpendicular I H, that ſhall be proportional to the Diſtance to which the Fluid will ſpout from an Adjutage at H: For the *Velocity* will be as the

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ium of the ſpouting Fluid will be as the Force which protrudes it, and this Force is every where proportional to the Altitude of the Fluid, therefore the *Momentum* of the Water at B and E will be as A B, and A E, which let be denoted by m and M. Then let v and V be the Velocities with which the Water ſpouts; and q and Q the Quantities of Water iſſuing out in the ſame Time from the Adjutages B and E, then we ſhall have $m = vq$, and $M = VQ$. And therefore $\frac{m}{q} = v$, and $\frac{M}{Q} = V$.

2. Now the Quantity of Water ſpouting from any Hole in a given Time muſt neceſſarily be as the Velocity with which it flows; for the greater the Velocity, the greater will be the Number of Particles which come to the ſame Point in the ſame Time, or the greater will be the Quantity of Water which paſſes by the Orifice of the Adjutage, as is very eaſy to conceive; therefore $q : Q :: v : V :: \frac{m}{q} : \frac{M}{Q}$; conſequently $\frac{M}{Q} \times q = \frac{m}{q} \times Q$; or $Mqq = mQQ$; wherefore $m : M :: qq : QQ$ ($:: vv : VV$, as is evident from the Proportionality of the Roots.) Therefore alſo $q : Q :: v : V :: \sqrt{m} : \sqrt{M} :: \sqrt{AB} : \sqrt{AE}$.

3. Hence if $AB : AE :: 1 : 4$, the Velocity at E will double the Velocity at B, and the Quantity of Water which flows from E will be twice as great as that which flows from B

Square Root of A H, and the *Time* as the Square Root of H K; whence the Product of theſe two will expreſs both the Space paſs'd over by the projected Body, and alſo the Line H I, as is well known to Geometricians.

HENCE it follows, that *a Fluid will ſpout from a Hole or Adjutage* D *in the Centre of the Semicircle, or middle Point of the Altitude* A K, *to the greateſt horizontal Diſtance* K M *poſſible*; becauſe the Perpendicular F D is the greateſt that can be drawn to the Diameter A K. Alſo 'tis evident, that *from two Holes* B *and* H, *equally di-*

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in one Minute, or any given Time. If A E be equal to 9 A B, the Velocity and Quantity of Water will be three times greater at E than at B, and ſo on.

4. Hence 'tis very evident the Velocity of a Spouting Fluid, and that of an heavy Body, which it acquires by falling from the Surface of the Water A to the Adjutage B or E (*in Vacuo*) will be both the ſame, as being both equal to $\sqrt{AB}$ or $\sqrt{AE}$. See *Annot.* XXII. *Art.* 4.

5. Hence alſo, if A B D E be a Veſſel of Water whoſe Surface is A B, then a Parabola D C E deſcribed on the Bottom D E and Height of the Water O C, will expreſs by its Ordinates *a c*, *d f*, *g i*, *k m*, E D, the Velocities with which the Water will ſpout from the Veſſel thro' Holes G, I, L, N, D, made in the Side upon a Level with thoſe Ordinates reſpectively; becauſe theſe Ordinates are as the Square Roots of the Abſciſſæ C*b*, C*c*, C*h*, *&c.* that is, as the Square Roots of the Altitude of the Fluid above the ſaid Holes. Thus if $Cl = 4Cb$, then $ac : km :: \sqrt{Cb} : \sqrt{Cl} : 1 : 2 ::$ Velocity of the Water ſpouting from G to the Velocity of that ſpouting from N. And ſo in every other Place.

Pl. XIX. Fig. 5.

6. Hence, if A B the Axis of a Parabola be equal to $\frac{1}{2}$ BC its greateſt Semi-ordinate, the Point B will be the *Focus*, and B C the *Semi-parameter* of the Parabola. Now ſince Water, or any heavy Body, falling from the Height A B, or Water ſpouting at the Depth A B below the Surface, will acquire at B a Velocity that in an uniform Motion will carry it over a Space equal to 2 A B or B C; and ſince in one Second of

Pl. XVIII. Fig. 1.

ſtant above and below the middle Altitude D, *the Jets of Water will be made to the ſame horizontal Diſtance* KN; becauſe the Perpendiculars to theſe two Points, *viz.* CB and IH, are equal.

MOREOVER, the horizontal Diſtance KM, to which the Water ſpouts from D the Centre, is equal to the Diameter or Altitude AK, or twice DF. For ſince the Velocity of the Jet at Q is equal to that acquired by falling through the Height AD, or DK, it will, as being uniform, carry the Fluid in an horizontal Direction over twice the Space DK or DF in the ſame Time:

Time a Body falls through 16 Feet; therefore if we divide AB into 16 equal Parts, and BC into 32, and Ordinates be drawn through every Point of Diviſion in the Axis AB, and Perpendiculars to the ſame Point BC, then there will be exhibited at one View the *uniform Velocities* acquired by the ſeveral Falls, or Jets of Water, from all Heights to 16 Feet.

7. Theſe Velocities are expreſſed by the reſpective Ordinates, and their Meaſure in Feet ſhewn by the Perpendiculars in the Scale BC on the Baſe. Thus, for Example, a Fall of 4 Feet gives an uniform Velocity of 16 Feet *per* Second; a Fall of 9 Feet gives 24; and ſo of the reſt. Alſo the Time of the Falls, being proportional to the Ordinates, might have been expreſſed by another Scale of 60 Thirds (equal to one Second) in the Baſe BC; but to avoid Confuſion, I have given the Scale of Time upon the Curve of the Parabola: So that any Queſtion relating to the *Time*, *Velocity*, or *Height* of a Fall or Jet of Water leſs than 16 Feet, may be readily ſolved by Inſpection, and that to an Exactneſs ſufficient for the niceſt Caſe in Practice.

8. After the ſame Manner may a Parabola be conſtructed for all Heights to an 100 Feet, but that I leave to the Reader, that which I have given being nearly ſufficient for all common Occaſions. Such a Scale has been greatly wanting in Books of this Sort, and ſuperſedes the Neceſſity of thoſe large Tables which we find in ſome voluminous Treatiſes of *Hydroſtatics*; as thoſe of M. *Belidor*, &c. Note alſo, the ſeveral Ordinates of this Parabola expreſs the Lengths of a Cylinder or

And therefore, ſince the Diſtances of Jets from D and B are as D F to B C, and the Diſtance of the Jet from D is equal to twice D F, the Diſtance of the Jet from B will be alſo equal to twice B C, or $KN = 2\,BC$ (LXXV.)

Column of Water iſſuing from an Adjutage at any of thoſe Depths in one Second of Time, as being proportional to the Velocity at each reſpective Place.

Pl. XVI. Fig. 6.

(LXXV.) 1. The Velocity of the Fluid ſpouting from B we have ſhewn to be as $\sqrt{AB}$, and the Time it takes in going to the horizontal Line will be as $\sqrt{BK}$, becauſe that would expreſs the Time if it were to deſcend to K in the Perpendicular BK, by *Annot.* XXVI. 4. And though it goes not in the Perpendicular, but is projected from B with an Impetus in an horizontal Direction, it will not by that means be any longer arriving to the horizontal Line, but only go in a different Tract, as is evident from what has been ſaid in *Annotation* XXIV. Therefore the Time of its coming to the Ground at N will be as $\sqrt{BK}$, that is, as the Square Root of the Altitude of the Spout BK.

2. Now the Spaces which ſpouting Fluids paſs over in an horizontal Direction muſt be conſider'd as uniform, becauſe it proceeds from a ſingle Impulſe; for a Particle of Water, after it has made its Exit from the Orifice of the Adjutage, is no more affected by the Preſſure or Impulſe of the confined Water, and conſequently its Velocity in the Direction BC will be uniform. But in all uniform Motion the Spaces deſcribed are as the Rectangle under the Times and Celerities, or as $\sqrt{AB} \times \sqrt{BK}$; but this Rectangle is alſo as the Line BC.

Pl. XX. Fig. 1.

3. For let there be drawn AC and CK, 'tis evident the Triangle ACK is ſimilar to the Triangle BCK, becauſe the Angles ACK and CBK are right, and the Angle CKB is common to both; therefore $AC : CK :: BC : BK$. In the ſame Manner we ſhew the Triangle ACK is ſimilar to the Triangle ACB; therefore alſo $AC : CK :: AB : BC$. Conſequently, $AB : BC :: BC : BK$; therefore $AB \times BK = BC^2$. And ſo $BC = \sqrt{AB \times BK} = \sqrt{AB} \times \sqrt{BK}$.

4. Now becauſe the Space the Fluid deſcribes according to the Horizon, and the Line BC, are both proportional to the

I SHALL only obſerve farther in regard to ſpouting Water, that the Jet will go farther through the bare Hole in the Veſſel at P, Q, or

ſame Thing, (*i. e.* to $\sqrt{AB} \times \sqrt{BK}$) therefore they muſt alſo be proportional to each other; and ſo the horizontal Diſtance of any Jet, as B, will be in Proportion to the Line BC; and the Jet from D will be proportional to the Line DF, and ſo on.

5. If now D be taken in the Centre of the Circle AFK, the Velocity at D would (as being uniform) carry it through twice the Space $AD = 2DF = DP$ in the horizontal Direction or Line DP, as is evident from *Annot.* XXVI. 7. becauſe the Particles of a Fluid obſerve the ſame Laws of Motion with larger Bodies. Conſequently, though the Fluid goes not in the Direction or Line DP, it will go to the ſame Diſtance from the Perpendicular DK on the Horizon; becauſe $DK = AD$, or the perpendicular Spaces, and conſequently the Times of the Fall, are equal. Whence, becauſe the horizontal Diſtances deſcribed by Jets from D and B are as DF and BC, and the horizontal Diſtance of the Jet from D is $2DF = DP = KM$, therefore the horizontal Diſtance of the Jet from B will be equal to $2BC = KN$.

6. From what has been hitherto ſaid, 'tis eaſy to obſerve that the Motion of a ſpouting Fluid is every way ſimilar to that of a projected Solid. The Path of the Fluid is a Parabola, becauſe it is impell'd by two Forces, one horizontal, the other of Gravity in the Perpendicular, in the ſame Manner as was ſhewn of the projected Solid. The Impetus of the Jet B, D, H, E, is the Height of the Reſervoir AB, AD, AH, AE; the greateſt horizontal Random KM is that from the Jet D, directed to the middle Point F of the Semicircle; and any two Jets equally diſtant, as B, H, go to the ſame Diſtance KN on the Horizon. If the Adjutage be horizontal, the Jet will be a Semi-parabola; if oblique, it will be a whole Parabola. All which Particulars were obſerved to be the Properties of the Motion of ſolid Projectiles; a Jet of Water being every way the ſame as a Diſcharge of ſmall Shot.

7. But to apply this more particularly: Let ABCQL be a Fountain and Conduct-Pipe with an Adjutage moveable on a Joint, to be ſet in any Poſition; as perpendicular, as at I; oblique, as at K; or horizontal, as at L. In the firſt Caſe a Jet will play upright to D; the Height of which Jet ID is Pl. XX. Fig. 2.

R, than through the *Adjutage* PB inſerted in thoſe Holes, and farther ſtill in Proportion as the Adjutage is ſhorter and the Bore larger. On

the *Impetus*, or the Meaſure of the Force of the Machine to project the *Fluid*. If the Adjutage be placed obliquely, as at K, the Jet of Water will be the Parabola KFH; but if placed horizontally, as at L, it will form the Semi-parabola LN. Now in this latter Caſe it will be LM : MN :: MN : $\frac{MN^2}{LM}$ = *Latus Rectum* of the Parabola LN when compleated.

8. But it has been ſhewn, thar the *Impetus* ID, or Force by which the Parabola is deſcribed, is equal to *one Fourth Part* of the *Latus Rectum*, (*Annot.* XXXIII. 12.) therefore it will always be $ID = \frac{MN^2}{4LM}$; conſequently, 4 LM : MN :: MN : ID. So that if we ſee a Jet of Water at any Time, and would know the Height of the Reſervoir whence it comes, we can eaſily find it by the Proportion above, becauſe the three firſt Terms are always given by Meaſurement.

9. By this means alſo we can eſtimate the Force by which any Fluid is projected, where no Reſervoir or Preſſure of Fluids is concern'd, but the Action of Solids; as the Force by which the Urine is protruded; the Force of Reſilition in the Blood ſpouting from Veins and Arteries in Bleeding; the Force by which the Chyle aſcends from the *Receptaculum* by the Thoracic Duct to the Left Subclavian Vein, or by which the Blood aſcends from the loweſt Parts of the Body to the Heart. In ſhort, nothing in the Animal Oeconomy can be well underſtood but by an Acquaintance with the Hydraulic Laws of Fluids: For an Animal Body is the moſt curious and compounded Hydraulic Machine in Nature.

10. A ſignal Example of the Uſe of *Hydraulics* in this Reſpect I ſhall produce from the admirable Experiment of Dr. *Hales* in his *Hæmoſtatics*. The Doctor cauſed a Mare to be tied down alive, and laid open the Left Crural Artery, in which he inſerted a Braſs Tube, and on that fix'd a Glaſs Tube; and then cutting the Ligature on the Artery, the Blood roſe in the Tube in an Inſtant about 4 Feet, and continued to riſe till it attain'd the Height of 8 Feet and 3 Inches. In a

the contrary, if a Pipe be inſerted perpendicularly into the Bottom of a Veſſel, it will diſcharge itſelf ſooner through it, than through the bare

Gelding he tried the ſame Experiment, and the Blood roſe to the Height of 9 Feet 8 Inches: This was in the ſame Artery as before. In another Mare he fix'd a Pipe and Tube into an Inciſion of the Left Jugular Vein, and the Blood roſe to 4½ Feet; and then he fix'd it in the Left Carotid Artery, where the Blood roſe to 9 Feet and 6 Inches. In an Artery of a Doe it roſe to 4 Feet 2 Inches; in a Dog, to 6 Feet 8 Inches; in a Sheep, to 6 Feet 5½ Inches; and he conjectures that it will very probably riſe in a Man to the Height of 7 Feet and 6 Inches.

11. Now to eſtimate the Force with which the Left Ventricle of the Heart (of the firſt Mare) propell'd the Blood, the Doctor found by means of injected Wax, that the Capacity of the Ventricle was 10 Cubic Inches, and the Surface on the Inſide was equal to 26 Square Inches; alſo the Area of the greateſt Section of the Ventricle from *Apex* to Baſe was 6,83 Square Inches: Conſequently the whole Preſſure upon the internal Surface of the Ventricle was equal to $26 \times 114 = 2964$ Cubic Inches. (See *Annot.* LV. 10, 11.) Alſo the Weight or Preſſure downwards was equal to $6{,}83 \times 114 = 778{,}63$ Cubic Inches, that is, to the Height multiplied by the Area of the greateſt Section of the Ventricle.

12. Now a Cubic Inch of Blood weighs 267,7 Grains; wherefore $2964 \times 267{,}7 = 113{,}22$ Pounds *Averdupois*, allowing 7008 Grains to a Pound; which therefore is the Force of Preſſure upon the Surface of the Ventricle when dilated. Again, $6{,}83 \times 114 = 29{,}7$ Pounds = the Force of Weight, or the *Momentum* of the Blood which the muſcular Fibres of the Ventricle muſt reſiſt in ſupporting ſuch an Altitude of Blood, on ſuch an Area of Baſe.

13. The Velocity of the Blood entering the *Aorta* is thus computed: The Ventricle each Syſtole throws out 10 Cubic Inches of Blood, and the Area of a tranſverſe Section of the *Aorta* at the Heart is 1,036; therefore $\frac{10}{1{,}036} = 9{,}64$ Inches = the Length of the Cylinder of Blood form'd in paſſing through the *Aorta*'s Orifice at each Syſtole of the Ventricle. The Ventricle contracted, or the Pulſe in the Horſe beat 36

Hole; and ſtill quicker as the Pipe is longer and larger (LXXVI.)

times in a Minute, or 2160 times in an Hour; but 9,64 × 2160 = 20819,4 Inches, or 1734,9 Feet, would paſs in an Hour, if the Time of the Contraction of the Ventricle were equal to the Time between two Pulſes: But it is but a third Part of that Time; therefore three times 1734,9 Feet, or 5204,7 Feet, will be the Velocity *per* Hour, or one Mile nearly, or 86,7 Feet *per* Minute.

14. But this Velocity is abated by the Diviſions and Subdiviſions of the Artery into numberleſs Branches, ſo that in the fine Capillaries it muſt be nearly deſtroy'd, did not other Powers take place to continue and augment it; ſuch as the Re-action of the elaſtic Coats of the Arteries, the Attraction of Coheſion, and perhaps an electrical Virtue ariſing from conſtant Attrition, and extended through all the Tubular Compages.

(LXXVI.) 1. Let AB be a Veſſel fill'd with Water, it will run out at the Bottom through a Hole with a Velocity at firſt proportional to the Height AC, which will conſtantly decreaſe with the Altitude till the Veſſel be emptied. If another Pl. XX. Fig 3. Veſſel NK, equal to the former, has a Tube inſerted at the Bottom, as LM, the Water will begin to run out at M with a Velocity proportional to IM; the Veſſel NK will therefore be exhauſted ſooner than the Veſſel AB. If another Veſſel EH be of an equal Diameter with the Veſſel NK or AB, and equal in Length to the Veſſel and its Pipe, the Surface of Water in each will begin to deſcend with equal Velocities; but that in EH will ſoon become greater than that in NK, becauſe of an equal Altitude of the Water, and a freer Exit at the Bottom, as having no narrow Tube to retard it.

2. The Theory of Water running through a Hole EF in Fig. 4. the Bottom of a Veſſel ABDC is of a curious Conſideration, and may be thus explain'd. AB is the Surface of the Water, or Diameter of the Orifice of the Veſſel; and ſince the Water muſt move from the Surface to the Hole EF, it muſt move in a Body of a particular Figure AMEFNB, which is call'd the *Cataract* of deſcending Water. Now the Nature of the Curve AME or BNF is thus determined.

3. The *Momentum* of moving Water is every where the ſame, (*Annot.* LXX. 17.) *viz.* at AB, MN, or EF; alſo the Velocity at MN or EF is as $\sqrt{HK}$ or $\sqrt{HG}$, (*Annota-*

II. THE ſecond Cauſe of the Riſe or Motion of Fluids is the *Preſſure of the Air* on the Surface of that in the Fountain or Reſervoir. Thus

tion LXXIV.) If now we take M N as an Ordinate to the Curve AME, and put MR $= y$, and HR $= x$; then ſince the Quantity of Water flowing by M N in a given Time is as the Area of the Section or Square of its Diameter, *viz.* MN^2, which is as yy, and alſo becauſe the Velocity is as $\sqrt{HR} = \sqrt{x}$, therefore the *Momentum* of the Water will be $yy \times \sqrt{x}$. And becauſe this is every where the ſame in the moving Cataract, the Quantity $y^2 \times \sqrt{x}$ is given or conſtant; and ſo $y^2 \sqrt{x} = 1$, and $\sqrt{x} = \frac{1}{yy}$, whence $x = \frac{1}{y^4}$; which ſhews the Curve A M E, is an *Hyperboloid* of the fourth Order, whoſe Aſymptotes are X I parallel to the Horizon, and I G perpendicular thereto.

4. To find the Quantity or Magnitude of the Cataract we proceed thus; let *mn* be drawn infinitely near M N, then will $Rr = \dot{x}$, and becauſe $x = \frac{1}{y^4}$, we have $\dot{x} = -\frac{4x\dot{y}}{y} = -\frac{4\dot{y}}{y^5} = Rr$. The Area of the Circle upon the Diameter M N is pyy, but $pyy \times \dot{x} = pyy \times -\frac{4\dot{y}}{y^5} = -\frac{4p\dot{y}}{y^3} = -4py^{-3}\dot{y} \doteq 2py^{-2} = \frac{2p}{yy} = C =$ the Cataract: But becauſe $x = \frac{1}{y_4} = \frac{1}{y^2} \times \frac{1}{y^2}$; therefore $xyy = \frac{1}{yy}$, conſequently $2xpyy = \frac{2p}{yy} =$ Cataract; but $2x = 2IR$, and pyy is the Area of the Circle M N. Therefore the whole Cataract from any Section to the Aſymptote XX, is equal to a Cylinder whoſe Baſe is the Section of the Cataract, and its Length twice the Height of the Cataract.

5. Now 'tis very evident from what has been ſaid of the Nature of the Curve Y A M E, that as ſoon as the Water begins to run out at E F, there will be at that Inſtant a Velocity in all Parts of the Cataract even in thoſe of the Surface A B. For let $A =$ Area of the Circle upon A B, or Orifice of the Veſſel, and $a =$ Area of the Hole E F; $V =$ Velocity of the Water in paſſing thro' the Hole, and $v =$ the Velocity at the Surface A B. Then becauſe it is $Av = aV$; we have

if a SYPHON or *Crane* be immerſed with the ſhorter Leg in Water, and the Air ſuck'd out of the Inſtrument, the Fluid will aſcend into the va-

$V : v :: A : a$; therefore whatever Proportion a has to A, the ſame Proportion will there be between v and V. So that till A becomes infinite, or till A B coincides with X X, there will always be ſome Degree of Velocity in the Surface of the Water. But when A is infinitely greater than a, v will be infinitely leſs than V, that is nothing at all.

6. Since the Aſymptote XIX (or the infinite Surface of the Water) is the higheſt Part of the infinite Cataract, and where the Velocity commences, it is evident the Velocity will be in all Parts below as at A B, E F, as $\sqrt{IH}$ and $\sqrt{IG}$; conſequently the Velocity of the Water is different from that of Bodies falling from the Height of the Fluid, for of ſuch a Body the Velocity at EF would be as $\sqrt{HG}$, and at A B it would be nothing.

7. Let $GH = H =$ the Height of the Water in the Veſſel, and $IH = z$, then will $H + z = IG$. And ſo $V : v :: A : a :: \sqrt{IG} : \sqrt{IH} :: \sqrt{H+z} : \sqrt{z}$; or $V^2 : v^2 :: A^2 : a^2 :: H + z : z$; whence, by Diviſion of Proportion, we have $H + z : H :: V^2 : V^2 - v^2 :: A^2 : A^2 - a^2$; conſequently $H + z = IG = \frac{V^2 H}{V^2 - v^2} = \frac{A^2 H}{A^2 - a^2}$.

8. According to Sir *Iſaac Newton* Bodies fall *in Vacuo* at the Rate of $193\frac{1}{3}$ Inches, or $16\frac{1}{9}$ Feet, the firſt Second, which Velocity becoming uniform will carry a Body over twice the Space in one Second, *viz.* $32\frac{2}{9}$ Feet. But uniform Velocities are as the Spaces deſcribed by them in the ſame Time; and the laſt acquired Velocities of falling Bodies are as the Square Roots of the Spaces deſcended through; therefore $32\frac{2}{9} : V :: \sqrt{16\frac{1}{9}} : \sqrt{IG}$ or $\sqrt{\frac{A^2 H}{A^2 - a^2}}$. Wherefore $\frac{32\frac{2}{9}\sqrt{IG}}{\sqrt{16\frac{1}{4}}} = V = 8{,}02773\sqrt{IG}$, or $8{,}02773\sqrt{\frac{A^2 H}{A^2 - a^2}}$ Feet, or $96{,}33276\sqrt{\frac{A^2 H}{A^2 - a^2}}$ Inches *per* Second.

9. When A is very great in reſpect of a, then a^2 may be rejected as inconſiderable; and ſo the Expreſſion will be reduced to $8{,}02773\sqrt{H}$ Feet, or $96{,}33276\sqrt{H}$ Inches, for

cuous Space by the Preſſure of the Air on the Water in the Veſſel, and fill the whole Cavity of the Tube; and becauſe there is a greater Co-

the Velocity of the effluent Water *per* Second. But (as Sir *Iſaac Newton* found, and is eaſy to obſerve) this Velocity is not ſo great in paſſing through the Hole in Experiments, as the Theory gives it: The Reaſon is, becauſe the Vein of effluent Water contracts itſelf, and grows narrower to the Diſtance of about a Diameter of the Hole below it; where, by meaſuring, Sir *Iſaac* found the Diameter of the Vein to that of the Hole as 21 to 25; whence the Area of a Section of the Vein there will be to the Area of the Hole as 441 to 625, that is, as 1 to $\sqrt{2}$; conſequently, the Velocity of the Vein is to the Area of the Hole as $\sqrt{2}$ to 1 in that Part, becauſe of equal *Momenta* in both Caſes.

10. Hence, if $IG = 48$ Inches, and the Diameter of the Hole 1 Inch; then $48 + 1 = 49$ Inches will be the Height from the Place where the Velocity is greateſt. Now the natural Velocities acquired by Water deſcending freely, from the Heights of 48 and 49 Inches, are as 69 to 70, and therefore nearly equal: Conſequently, the great Inequality of 1 to $\sqrt{2}$, or the Diminution of the Velocity in the Hole, muſt be owing to the Retardation occaſion'd by the lateral Water reſiſting in ſome meaſure its perpendicular Deſcent.

11. Hence the Velocity of the Water in the Hole is very nearly equal to that which would be acquired in deſcending through $\frac{1}{2}IG$; becauſe the Velocity acquired by deſcending through $\frac{1}{2}IG$ is to that acquired by deſcending through IG as 1 to $\sqrt{2}$. Whence the Velocity in the Hole is to be diminiſhed in the Ratio of 1 to $\sqrt{2} = 1{,}414$. Thus, as $1{,}414 : 1 :: 8{,}02773 : 5{,}6773$; and as $1{,}414 : 1 :: 96{,}33276 : 68{,}1278$; ſo that the true Velocity of the Water in the Hole is $5{,}6773\sqrt{H}$ Feet, or $68{,}1278\sqrt{H}$ Inches *per* Second.

12. We obtain an Expreſſion of the Velocity alſo in other Terms, thus: The Quantity (Q) of Water diſcharged is greater in proportion to the Area of the Hole (a), the Velocity (V), and the Time (T) which it ſpends. Therefore $Q = TVa$; and ſo $V = \frac{Q}{Ta} = 68{,}1278\sqrt{H}$; and conſequently $Q = 68{,}1278\,Ta\sqrt{H}$. Or, becauſe $a = 0{,}785398\,dd$, therefore $Q = 53{,}507476\,ddT\sqrt{H}$ Cubic Inches of Water.

lumn or Body of Water in the longeſt Leg, it will preponderate and deſcend through it; and

And becauſe a Cubic Inch of Water weighs $253\frac{1}{3}$ Grains; the Weight of the Water diſcharged will be $W = 13555{,}32\,ddT\sqrt{H}$.

13. Suppoſe a Veſſel 10 Inches ſquare, 4 Feet or 48 Inches high, and having an Hole at the Bottom 1 Inch ſquare. Then will $A = 100$, $a = 1$, and $H = 48$; alſo $A^2 = 10000$, $a^2 = 1$, and ſo $\frac{A^2}{A^2 - a^2} = \frac{10000}{9999} = 1{,}0001$, which may be rejected as inconſiderable. Wherefore $68{,}1278\sqrt{H} = 472$ Inches *(ferè)* the Velocity of the iſſuing Water *per* Second. $Q = 100 \times 48 = 4800$ Cubic Inches: Whence $\frac{4800}{472} = \frac{Q}{V} = T = 10$ Seconds, the Time in which ſuch a Veſſel would diſcharge a Bulk of Water equal to its own Bulk, if kept continually full all the while.

14. Now becauſe, when a is given, $Q = TV$; and when Q is given, $TV = 1$, and $T = \frac{1}{V}$, and $V = \frac{1}{T}$; it is plain all theſe Caſes are the ſame with thoſe of Bodies moving uniformly, as in *Annot.* XXII. And ſince V is a given Quantity when it increaſes or decreaſes in a given Ratio, therefore the above Theorems will be equally adapted to the uniformly accelerated and retarded Motion of Bodies, whether ſolid or fluid. From whence we can eſtimate the Time in which the abovemention'd Veſſel full of Water will be diſcharged by its own Gravity only.

15. Thus 'tis evident, that Column of Water that ſtands over the Hole will deſcend alone through it in $\frac{1}{2}$ a Second, becauſe Bodies fall at the Rate of 4 Feet in $\frac{1}{2}$ a Second, and that is the Height of the Column. If the Veſſel contains any Number of equal Columns beſides, the Time of their Evacuation will be proportional; that is, Q will always be as T, becauſe V in all Caſes (uniformly decreaſing with the $(\sqrt{H})$ Square Roots of the Altitudes) is a given Quantity; and becauſe there are 100 ſuch Columns in the Veſſel, it will require 100 Half-Seconds, or 50 Seconds, for the Time of the Diſcharge. This would be the Time, were the Water to ſet out with the Velocity of the Deſcent from the Height of 4 Feet: But ſince the initial Velocity is leſs than that in the Proportion of 1 to $\sqrt{2}$, (*Art.* 9.) and ſince the Times are

will keep flowing out, till the Veſſel is exhauſted to the Orifice of the ſhorter Leg (LXXVII.)

THE Effect of the COMMON PUMP is from the ſame Principle: For the Bucket or Sucker, raiſing a Column of Water above it in the Barrel, makes a vacuous Space, or greatly attenuates the Air in the Barrel and Pipe below it; whence the Water muſt neceſſarily riſe in the Pipe, by the ſuperior Preſſure of the Air on the Surface of that in the Well. The Water aſcending in the Pipe, paſſes through the Valve in the Bottom of the Barrel, which ſhutting upon the Depreſſion of the Sucker, the Water is thereby forced through the Valve of the Sucker into the

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inverſely as the Velocities when Q or the Quantity of Water is given, (*Art.* 14.) therefore as $1 : \sqrt{2} = 1{,}414 :: 50'' : 70{,}''7$ = the true Time of the Diſcharge.

(LXXVII) 1. It has been already ſhewn, that when a *Syphon* is firſt fill'd with, and then immerſed in Water, it is the Preſſure of the Air that cauſes it to continue full, and that it runs out of the longer Leg by the Preponderancy of Water in that Leg. (*Annot.* LXXII.) I ſhall only add here, that when the Air is drawn out by the Mouth-Piece, the Preſſure of the Air on the Surface of Water in the Veſſel cauſes it to riſe into and fill the Syphon; to evince which, we need only put a Syphon, while working, under the Receiver of an Air-Pump; for upon exhauſting all the Air, the Syphon will ceaſe running.

2. Or if the Syphon be equicrural, or has both the Legs of an equal Length, the Water will not run through it, becauſe the Force of Water againſt the Air at each Orifice is equal. The ſame Thing will be effected with a Syphon of unequal Legs, though the ſhorter Leg be immerſed in Water, if the Syphon be ſo inclined that the Orifice of both Legs be on the ſame horizontal Level. I need not after this give the Reaſon why the Water will not run out of the ſhorter Leg, if the longeſt be placed in the Water.

Ciſtern of the Pump, and diſcharged through the Spout. All which is clearly ſhewn by the Model of a Pump in Glaſs (LXXVIII.)

(LXXVIII) 1. I ſhall here give the Theory of *Pump-Work*, and the Structure of ſuch Pumps as are found to be moſt uſeful for the various Purpoſes of Life. The Theory depends in a great meaſure upon thoſe Properties of the inverted Syphon, or Fountain, deſcribed in *Annot.* LXXIII. *Art.* 11, 12, 13, 14. Let ABCDEFG repreſent the inverted Syphon there deſcribed: Only here the Part ABCD repreſents a Column of Air, and DG the lower Part of the Pipe of a Pump immerſed in the Water of the Well HI. Let P be the Piſton of the Pump at E in its loweſt Situation, and at F in its higheſt. Now as both theſe Parts communicate with the Water, one by preſſing on it, the other by opening into it, they may be look'd upon as communicating with one another.

2. Now ABC, the Column of Air, would by its Weight or Preſſure force up a Column of Water into the Pipe DG to the Height of 33 Feet were the Air exhauſted from the ſaid Pipe, and continued to that Height; (the Weight of a Column of Air being equal to the Weight of ſuch a Column of Water of the ſame Baſe, as will be ſhewn hereafter.) If now the Piſton P be thus thruſt down to E, meeting the Water there, and from thence it be raiſed to F with an uniform Motion, the Water will riſe from E to follow the Piſton with a variable Velocity; the leaſt of which is as $\sqrt{AC} - \sqrt{DE}$, and the greateſt as $\sqrt{AC - DE} = \sqrt{AB}$, (by *Annotation* XXIV. 12.)

3. If in lifting up the Piſton the Velocity of Water be leſs than that of the Piſton, it will not be able to follow it, but will leave a void Space between them, which will increaſe more and more as the Velocity of Water becomes leſs than that of the Piſton. The Conſequence of this will be, that a Part of the Stroke of the Piſton will be loſt; and not only that, but the Piſton, when the Water leaves it, will riſe very hard, as having a Weight of Water upon it, and the Air of greater Denſity above than below; whence the Equilibrium, which ought to be in Pumps, is deſtroy'd, and the Balance againſt the Workman. Now as this can happen where the Diameter of the Sucking-Pipe is equal to that of the Pump-Barrel, much ſooner will it happen when the Pipe is leſs than the Barrel, becauſe the Water riſing through a leſs Paſſage will

III. IV. But the moſt conſiderable of all Water-Engines is that call'd *Newſham*'s En-

be longer in filling the Pump-Barrel, and therefore quit the Piſton, and leave the greater void Space between.

4. On the contrary, if the leaſt Velocity of Water riſing in the Pipe be greater than that of the Piſton, there will be no void Space; and the Pump-Barrel may be made in proportion as much wider than the Pipe as the Velocity of Water is greater than that of the Piſton. Now that this may be the Caſe, we ſhall ſhew by Calculation what Diameters the Barrel and Pipe ought to have compared with the Velocity of the Water and Piſton. Let A repreſent the leaſt Altitude of the Atmoſphere $AC = 31$ Feet of Water; $B = DF$ the higheſt Elevation of the Piſton above the Surface of the Water H I, which let be 16 Feet. And let the greateſt Velocity of the Piſton which can well be given to a Pump be that of four Feet in a Second $= v$; and $V =$ the leaſt Velocity of Water that riſes in the Pipe; $D =$ the Diameter of the Barrel; and $d =$ Diameter of the Pipe.

Pl. XX. Fig. 5.

5. Now here we have $\sqrt{A} - \sqrt{B} = V =$ the leaſt Velocity of Water; and the Fall which will produce that Velocity is the Square of that Expreſſion, *viz.* $A + B - 2\sqrt{AB}$, that is, $31 + 16 - 2\sqrt{31 \times 16} = 2$ Feet 6 Inches, the Height of the Fall required. Whereas by the common Way of taking the Square of $\sqrt{A - B}$, *viz.* $A - B$ for the Height, we have 15 for the Fall, which extraordinary Error muſt be of very bad Conſequence in Practice.

6. Here the Velocity $\sqrt{A} - \sqrt{B} = \sqrt{31} - \sqrt{16} = 5{,}6 - 4 = 1{,}6$ *per* Second. The Velocity of the Water at the Bottom of the Pipe D is as $\sqrt{A} = 5{,}6$; that alſo muſt be the Velocity of the Piſton at D, that the Water may follow it; whence the Piſton moving with the ſame Velocity at F, where the Velocity is but 1,6, we have $5{,}6 - 1{,}6 = 4$ Feet of void Space, therefore 4 Parts in 5,6 of ſuch a Stroke would be loſt or ineffectual. We may here obſerve by the Way, that ſince the Velocity of the Water at D is 5,6, and the greateſt Velocity which can be given to the Piſton (without damaging the Machinery) is but 4; therefore a Piſton working at the lower End of the Pipe or Barrel will always have Water more than enough to prevent any void Space or Loſs of Labour.

7. But ſince we find ſo great a Void in the Pipe at F of the ſame Diameter with the Piſton, 'tis evident if we contract all

gine, from the great Improvements which Mr. *Newſham* made in it: the Nature and Effect of

the Part below F into a ſmall Pipe as *F D*, and let the Part F G remain as it was for the Barrel for the Piſton to play in, as at *FG*, that then the Water will riſe into the Barrel *F G* with a greater Velocity than before, in proportion as the Pipe is leſs; conſequently *if the Bore of the Pipe F D be to that of the Barrel F G, as the Velocity of the Piſton* P, *or the Water in the Barrel, is to the Velocity of the Water in the Pipe*, there will always be a Sufficiency of Water to prevent a *Vacuum* in the Barrel. Which Rule in Symbols is thus expreſs'd; $DD : dd :: V : v$; whence $D^2 v = d^2 V$ for a general Canon; any three of which Quantities being given, the fourth may be found.

8. Thus, for Example, ſuppoſe D, *v*, and V were given to find *d*, we have $d = \frac{D\sqrt{v}}{\sqrt{V}}$. Let the Diameter of the Barrel D = 6 Inches. And ſuppoſe the Piſton gives 20 Strokes in a Minute, each a two Foot Stroke, ſpending as much Time in its Aſcent as Deſcent, then will the Motion of the Piſton be 80 Feet *per* Minute, or $1\frac{1}{3}$ *per* Second; whence $v = 1\frac{1}{3}$. Laſtly, to obtain the Value of U, we muſt fix on the Length of the Pipe *FD*, which let be 16 Feet, then the higheſt Elevation of the Piſton will be 18 Feet (if it comes to the Bottom of the Barrel, as it ought to do;) wherefore an Height of Water of 18 Feet in the Pump *GD* acts againſt the Weight or Height 31 Feet in the Leg AC. Now the Velocity V of the Water in the Pipe *F D* being uniform, or conſtantly the ſame, we muſt find what Difference of uniform Velocities will be generated by Falls from 31 and 18 Feet Heights. Thus $\sqrt{16\frac{1}{9}} : 32 :: \sqrt{31} : 43$ nearly; and $\sqrt{16\frac{1}{4}} : 32 :: \sqrt{18} : 32\frac{3}{4}$; whence the Difference of theſe uniform Velocities will be $43 - 32\frac{3}{4} = 10\frac{1}{4} =$ 10 Feet 3 Inches *per* Second; therefore $V = 10\frac{1}{4}$. Wherefore $\frac{D\sqrt{v}}{\sqrt{V}} = d = 2\frac{1}{10}$ Inches for the Diameter of the Pipe *F D*. Pl. XX. Fig 3.

17. The Reaſon why we make no Uſe of the Expreſſion $\sqrt{AC} - \sqrt{DF}$ in this Caſe, is becauſe this gives only the Difference of the inſtantaneous Velocities, or the leaſt Velocity with which the Water at F can begin its Motion upwards, whereas we here want to find what the conſtant and uniform Motion of the Water will be, or how much it will ſupply every Second uniformly, which is done by the Method above.

which will be eaſily underſtood from a perpendicular Section of a Model, with ſome Variation.

For ſince a Fall of 16 $\frac{1}{9}$ gives an uniform Velocity of 32 Feet *per* Second, a Fall of 31 Feet will give 43; thus a Fall of 18 Feet will give 32 $\frac{3}{4}$, and their Difference muſt be that of the Water at F.

18. If we know the Velocity of the Piſton v, the Diameter of the Barrel D, and the Diameter of the Pipe, d, we ſhall find the Velocity of the Water in the Pipe $V = \frac{vDD}{dd}$; or thus in Words, *Multiply the Square of the Diameter of the Barrel by the Velocity of the Piſton; divide the Product by the Square of the Diameter of the Pipe, the Quotient will be the Velocity ſought from the Water in the Pipe.* This Velocity when found muſt be taken from 43, the Remainder $43 - V$ will be the uniform Velocity produced by a Fall from the higheſt Situation of the Piſton to the loweſt Surface of the Water in the Well, and which is found by ſaying, as $32 : \sqrt{16\frac{1}{9}} :: 43 - V : \sqrt{DF}$; whence DF the higheſt Elevation of the Piſton will be known.

19. When we know D, V, and d, we find v the Velocity of the Piſton by this Theorem $\frac{Vdd}{DD} = v$; that is, in Words, *Multiply the Square of the Diameter of the Pipe by the Velocity of the Water in it, and divide the Product by the Square of the Diameter of the Barrel, the Quotient is the Velocity of the Piſton required.*

20. Having given the Velocity of the Water in the Pipe V, the Diameter of the Pipe d the Velocity of the Piſton v, to find the Diameter of the Barrel D, we have this Theorem $D = \sqrt{\frac{Vdd}{v}}$; that is, *Multiply the Square of the Diameter of the Pipe by the Velocity of its Water, and divide that Product by the Velocity of the Piſton; the Square Root of the Quotient is the Diameter of the Barrel ſought.*

21. If a Piſton works in a Barrel or Pipe AD of the ſame Diameter from Top to Bottom; then we can find the Height to which the Water will riſe at every Stroke of the Piſton playing at any Diſtance above the Surface of the Water BD. Thus, let E be the loweſt Part to which the Piſton comes, and let $ED = 12$ Feet Diſtance from the Water, and ſuppoſe its Length of Stroke $EF = 4$ Feet. Then when the Piſton is Pl. XX. Fig. 6.

which I have made in the Structure thereof. The Motion of the Water in this Engine is effected

at E, the lower Part of the Pipe ED is fill'd with Air of the ſame Denſity or Spring with the common Air, which call S; when the Piſton is rais'd from E to F, ſo that $FD = 16$, two Effects will follow; one is, the Air will now expand itſelf into a larger Space beneath the Piſton, and have its Spring weaken'd; the other is, that ſince the Spring of the Air in the Pipe is leſſen'd it will preſs leſs on the Surface of the Water in the Pipe than the external Air preſſes on that without, the Water will riſe ſo far into the Pipe, till its Weight, together with the diminiſh'd Spring of the Air be equal to the Preſſure of the external Air.

22. Suppoſe the Water riſes to c; call its Height $CD = x$; then the Space into which the Air is expanded will be $FC = 16 - x$. Let its Spring thus diminiſh'd be call'd s; then the Spring of the Air being inverſely as the Spaces which it takes up, we have $S : s :: 16 - x : 12$; therefore $\frac{12S}{16 - x} = s$. The Weight of a Column of Water 32 Feet high is equal to the Preſſure of the Atmoſphere, that is, equal to S; whence $32 : S :: x : \frac{Sx}{32}$ = the Weight of the Water x. But this added to the weaken'd Spring of the Air is equal to S; that is, $\frac{12S}{16 - x} + \frac{Sx}{32} = S$; and, dividing by S, we have $\frac{12}{16 - x} + \frac{x}{32} = 1$; or $384 + 16x - x^2 = 512 - 32x$; or $x^2 - 48x = -128$, and by adding on each Side the Square of 24 (to compleat the Square) we have $x^2 - 48x + 576 = 576 - 128 = 448$, and extracting the Root, $x - 24 = \pm\sqrt{448} = \pm 21{,}166$. Wherefore $x = 2{,}834 = DC$, the Height of the Water required.

Pl. XIX. Fig. 5.

23. This Height taken from $12 = ED$; leaves $EC = 9{,}166$, and calling the whole Height to which the Water will riſe by the firſt and ſecond Stroke together x; we ſhall have this ſecond x determin'd by the Equation $\frac{9{,}166}{16 - x} - \frac{x}{32} = 1$, inveſtigated and reſolved as before. For ſince the Water CD cannot recede (by reaſon of a Valve below) when the Piſton a ſecond Time comes to E, the Valve in the Piſton will be open'd, and the Part EC again fill'd with common Air; whence $S : s :: 16 - x : 9{,}166$, &c. as before. The E-

by

quation reſolved gives $x = 5{,}098$, from which taking the firſt $x = 2{,}834$, there remains 2,264, for the Height it riſes the ſecond Stroke. In the ſame Manner we calculate that the Water riſes

Firſt Stroke	2,834	2,834
Second	2,264	5,098
Third	2,025	7,123
Fourth	2,043	9,166
Fifth	2,413	11,579
Sixth	3,620	15,199

24. But to ſave the Trouble of a Calculation every Time we would find x for the various Heights of the Piſton, let us raiſe a general Theorem for all Caſes; thus let $a =$ the greateſt Elevation of the Piſton, $b =$ leaſt, $a - b = p =$ the Length of the Stroke, and $b =$ Height of a Column of Water equal to the Weight of the Atmoſphere; then the foregoing Theorem will become $\frac{b}{a-x} + \frac{x}{b} = 1$, and ſo $ax - x^2 + bb = ab - xb$; or $ax + bx - xx = ab - bb = pb$; putting $\frac{a+b}{2} = s$, we have $2sx - xx = pb$; and changing the Signs we have $xx - 2sx = -pb$; and compleating the Square, $xx - 2sx + ss = ss - pb$; and ſo $x - s = \pm\sqrt{ss - pb}$; and finally $x = \pm\sqrt{ss - pb} + s$.

25. Now this general Theorem finds not only x, but any of the other Quantities, having the reſt given. Thus ſuppoſe the higheſt Elevation of the Piſton be 16 Feet $= a$, $b = 32$; and I would raiſe the Water the firſt Stroke 10 Feet: Quære p the Length of the Stroke? Here $x = 10$, and $s = \frac{a+b}{2} = 24$; then above we had $2sx - xx = pb$, whence $p = \frac{2sx - xx}{b} = 11{,}9$, or near 12 Feet for the Length of the Stroke required.

26. Since after every Stroke or Aſcent of the Piſton, the Air in the Barrel is in a dilated State; and ſince when the Piſton deſcends the Water by its Tendency downwards carries down the Valve at D, and by this Means cuts off the Communication of the Air at D, and ſupports the Column of rais'd

Pl.XXIII. Fig. 1. following Manner, *viz.* When the Piſton R is raiſed, a *Vacuum* would be made in the Barrel

Water, 'tis plain the Valve in the Bucket of the Piſton will be affected on its upper Part by the Weight or Spring of the common Air, and on the under Side by the Spring of the dilated Air only; and therefore upon the Deſcent of the Piſton the Valve of the Bucket will not riſe but compreſs the internal dilated Air into a Space gradually leſs till its Spring by that Means comes to be equal to that of the external Air, and then as the Piſton deſcends it will ſuffer no farther Compreſſion, but force up the Valve, and make its Eſcape.

27. Thus for Example, by the firſt Stroke (in *Art.* 23.) the Water roſe 2,834, which ſubſtracted from 16, leaves 13,166 for the Space into which 12 Feet of Air was dilated. Now its Spring being weaker than that of the external Air, upon the Deſcent of the Piſton, it will be compreſs'd into the former Space of 12 Feet before it can recover its former Spring, the Piſton will then have deſcended through 1,166; and ſince the Stroke is 4 Feet, and the Air will now be no farther compreſs'd, therefore $4 - 1{,}166 = 2{,}834$ will be the Quantity of Air which will make its Eſcape through the Valve of the Piſton; which in this Caſe is equal to the Column of Water raiſed.

28. The Piſton being now again at E, upon its Aſcent, it will permit the Column of Air E C $= 9{,}166$ to expand itſelf into a Space $= 16 - 5{,}098 = 10{,}902$; and upon deſcending it will compreſs the Air again into 9,166 Feet, before it can eſcape. To do this the Piſton muſt deſcend through the Space $= 1{,}736$; and therefore $4 - 1{,}736 = 2{:}264$ is the Quantity of Air that will be exhauſted the 2d Stroke; and the Column of remaining Air E C will now be $9{,}166 - 2{,}264 = 6{,}902$ Feet. And in this Manner are the Numbers found for every following Stroke.

29. Hence 'tis eaſy to obſerve, that whenever the Space thro' which the Piſton muſt paſs in its Deſcent to reduce the internal Air, by Compreſſion, to a Denſity or Spring equal to the external Air; I ſay, when this Space happens to be greater than the Length of the Stroke, there can no Water riſe in the Pump; becauſe in that Caſe, the Air will be no farther able to make its Eſcape thro' the Valve of the Sucker, by reaſon of the greater Preſſure of the external Air upon it.

30. When this is the Caſe of a Pump, let x be the greateſt Height to which the Water can be raiſed in it; when the high-

TV,

T V, did not the Water follow it from the inferior Canal E M (through the Valve H) which

eſt Elevation of the Piſton is a, the loweſt b, the Height of the Atmoſphere b, and $a-b=p$ the Stroke of the Piſton, as before. Then when the Piſton is down, the Space in which the Air is contain'd is $b-x$; and when up, it is expanded into $a-x$; its Preſſure is then a $\frac{b-x}{a-x}$ Part of what it was before, that is of the Atmoſphere; the Weight of the Water being the other Part, *viz.* $\frac{x}{b}$; and both together being equal to the Atmoſphere, we have $\frac{b-x}{a-x}+\frac{x}{b}=1$; therefore $bb-bx+ax-x^2=ab-bx$; or, $ax-x^2=ab-bb=pb$. And changing the Signs, $x^2-ax=-pb$; and compleating the Square, $x^2-ax+\frac{1}{4}aa=\frac{1}{4}aa-pb$; wherefore $x-\frac{1}{2}a=\sqrt{\frac{1}{4}aa-pb}$; therefore $x=\pm\sqrt{\frac{1}{4}aa-pb}+\frac{1}{2}a$.

31. Hence if the Stroke of the Piſton and its Height above the Water be given, we know the greateſt Height to which the Water can riſe, and conſequently, if ſuch a Pump can be effectual. For Example, let $a=28$, $b=25$; then $p=3$, and put $b=32$; then $x=\pm 10+14$; that is, the greateſt Height to which the Water can riſe $=14-10=4$ Feet; for the other Root, $14+10=24$, is impoſſible, becauſe all Heights between 4 and 24 will be too great.

32. Again, ſuppoſe $a=24$, $b=20$; then, $p=4$, and $b=32$; $x=\pm 4+12$, that is, $x=12-4=8$ Feet, the greateſt Height. If $a=20$, $b=16$, $p=4$; then we ſhall have $x=\pm\sqrt{\frac{1}{4}aa-pb}+\frac{1}{2}a=\sqrt{100-128}+10=\sqrt{-28}+10$. Now the ſquare Root of a negative Quantity being impoſſible, ſhews the Caſe in Queſtion has no Place in Nature, or that here is no Space between x, and the loweſt Situation of the Piſton. Nor can there ever be any when pb is greater then $\frac{1}{4}aa$, the Water riſing to the Piſton in all thoſe Caſes. When $\frac{1}{4}aa=pb$, then $\sqrt{\frac{1}{4}aa-pb}=0$; and $x=\frac{1}{2}a$; and in this Caſe, $p=\frac{\frac{1}{4}aa}{b}$, or $\frac{aa}{4b}$. Thus if $a=16$, $\frac{1}{2}a=8$, $=x$, the greateſt Height the Water will riſe to, and $p=\frac{\frac{1}{4}aa}{b}=\frac{64}{32}=2$ Feet, the Length of the Stroke.

riſes through the Glaſs Tube EF immerſed in the Water of a Veſſel by the Preſſure of the At-

33. From this Theorem we learn this general Rule concerning Pumps, *That no Pump can raiſe Water, but when the Length of the Stroke of the Piſton exceeds the Square of the higheſt Elevation of the Piſton divided by* 128. Thus if that Elevation be 16 Feet, the Square of that is 256, which divided by 128, quotes 2; therefore the leaſt Stroke of the Piſton muſt be greater than 2 Feet. If the Elevation be 20 Feet, then $20 \times 20 = 400$, and $\frac{400}{128} = 3{,}13$, which ſhews the Elevation of the Piſton muſt be in that Caſe greater than 3,13 Feet. If the Elevation be 12 Feet, then $12 \times 12 = 144$, and $\frac{144}{128} = 1{,}12$, which is leſs then the leaſt Length of the Stroke of the Piſton. And theſe Things, I think, finiſh the Theory of Pumps.

34. I ſhall now proceed to the Deſcription of two or three moſt uſeful Pumps, but firſt give an Account of that Sort of Valves which I find are exceeding good, and ought to be uſed in Pumps and all Kinds of Water Engines where Valves are neceſſary. Let ABCD be the Bucket of a Piſton, or any other Part where a Valve is required; in the Middle there is a circular but tapering Hole from Top to Bottom, in which is fitted the tapering or conical Piece EFGH, with a Piece IK to be ſcrew'd in and out of the bottom Part HI. It is to be ſcrew'd out, when the ſaid Solid EG is put into its Place or Hole in AC; and afterwards ſcrew'd in, when the Whole together appears as in the Figure LMNO. The Piece EG now becomes a Valve, or capable of permitting the Water to aſcend, and to prevent its Deſcent. Pl. XX. Fig. 7.

35. That the Water puſhing againſt the Bottom of the Valve will raiſe it upwards, is evident from the conical Form thereof, and its lying in the Hole only by its own Weight; the Length of the Key at K being ſufficient to permit ſuch a Riſe of the Valve, as will admit a Space between it and the Hole for the Water to paſs as freely as required: And, that the Valve may not be thrown quite out of the Hole, the croſs Piece I is added, of a greater Length than the Diameter of the loweſt Part of the Valve.

36. If the Valve EG and its Socket AC be of Braſs, and fitted, by grinding them with Emery firſt and Putty afterwards, with a Drill-Bow into each other, they will not only be Water-

moſphere on the Surface thereof. The Water of the Barrel TV, by the Depreſſion of the Pi-

tight, but even Air-tight; and that too if but ſlightly touch'd with fine Emery or Putty; for if they are ground to a Poliſh, the Attraction of Coheſion will take place, and prevent the Valve from riſing ſo freely as it ought to do; yea, ſometimes thoſe Surfaces have been found to cohere ſo ſtrongly, that the Force of the riſing Water could not overcome it: But all this will be prevented, and every thing ſucceed to one's Wiſh, if they are made as above directed, as I have learnt by long Experience and trying every Way.

37. The COMMON SUCKING PUMP (as it is call'd) is ſo far deſcribed in the Theory, that ſcarce any thing more can be ſaid to explain it. However, I have given the Figure of ſuch a Pump, in which A is the Ciſtern, AB the Barrel or Pipe ſtanding in the Water EF, the Piſton GD with its Bucket and Valve D, and the Valve C open for the Aſcent of the Water through it. Pl. XX. Fig. 8.

38. A FORCING PUMP of the common Sort is thus conſtructed: AB is the Barrel ſtanding in the Water of the Well or Mine at B; GC is the Piſton; and C a ſolid Piece without any Hole or Valve, becauſe no Water is to paſs through it, as in the other. This Piece ſhould be well leather'd, to fit the Barrel very nicely, that in its Motion neither Air nor Water ſhou'd paſs between. At a diſtance below, a Valve with its Diaphragm is fix'd in the Barrel, as at D. Between this and the loweſt Situation of the Piſton C there goes off a Pipe at H, in which there is fix'd a Diaphragm and Valve, as at E. Now the Piſton being drawn up from C towards A, attenuates the Air above D, by which means the Water ruſhes into the Space CD; then when the Piſton is forced down, as the Water cannot repaſs at D, it is forced to aſcend into the Pipe at H, and through its Valve E into the Ciſtern F, (which may be ſituated at any Diſtance above the Pump) from whence the Water will run off by the Spout. Fig. 9.

39. A LIFTING PUMP is a Forcing Pump of another Form or Structure. AB is a Barrel fix'd in a Frame IKLM, which alſo is fix'd immoveable, with the lower Part in the Water to be exhauſted. GEQHO is a Frame with two ſtrong Iron Rods, moveable through Holes in the upper and lower Parts of the Pump IK and LM; in the Bottom of this Frame QH is fix'd an inverted Piſton BD, with its Bucket and Valve upon the Top at D. Upon the Top of the Barrel Fig. 10.

ſton R is forced through the ſuperior Canal ON, to enter by the Valve I into the Air-Veſſel *abcd*;

there goes off a Part ER, either fix'd to the Barrel, or moveable by a Ball and Socket, (as here repreſented at F) but in either Caſe ſo very nice and tight, that no Water or Air can poſſibly get into the Barrel, which would ſpoil the Effect of the Pump. In this Part, at C, is fix'd a Valve opening upwards. Now when the Piſton-Frame is thruſt down into the Water, the Piſton D deſcends, and the Water below will ruſh up through the Valve D, and get above the Piſton; where, upon the Frame's being lifted up, the Piſton will force the Water through the Valve C up into the Ciſtern P, there to run off by the Spout. *Note*, this Sort of Pump is ſet ſo far in the Water, that the Piſton may play below the Surface of it.

40. The **MERCURIAL PUMP**, or that which works by Quickſilver, is the next which I ſhall deſcribe. It was the Invention of Mr. *Joſhua Haſkins*, and brought to its preſent Perfection by Dr. *Deſaguliers*, together with his Workman Mr. *William Vreem*. As this is one of the moſt curious of modern Inventions, I ſhall be a little the more prolix in its Deſcription; which I ſhall give from a Model that I have made to explain the Nature and Manner of working in this Pump to thoſe who attend my Lectures in Philoſophy. This Model works very well, and is here repreſented in its real Magnitude in the Figure.

Pl. XXI. Fig. 1. 41. AB is a Ciſtern placed on the Top of a Braſs Tube POQR, open at each End PR and OQ. Towards the upper Part of the Tube at F is inſerted (or ſolder'd) a curved Pipe FG, opening into the Tube at F; and in the End G is cemented a Glaſs Tube GH, fix'd below in an hollow Box LK, full of Holes for the Admiſſion of Water, into which it is immerſed. MNST is an exterior Tube, open at the Top MT, and cloſe at the Bottom NS; in which Bottom is firmly fix'd another hollow Tube VXYW, cloſe on the Top XY. This inner Tube may be a ſolid Piece of Wood, as in my Model it is. This inner Tube or Cylinder of Wood goes up into the Tube QOPR, at the ſame Time that this is inveſted by the exterior Tube MNST, as is ſeen in the Figure. In the lower Part of the Pipe at H is a Diaphragm, and a Valve I, opening upwards for the Aſcent of Water when a *Vacuum* is made. At CD in the main Tube, above the Inſertions of the Pipe, is another Diaphragm and Valve E, opening up-

and the like being done alternately by the other Barrel W X, and its Piſton S, the Air-Veſſel is

wards alſo to give Paſſage for the Water in the forcing Part of the Stroke.

42. The conſtituent Parts of the Pump being thus deſcrib'd, we are next to view the peculiar Manner of working by *Quickſilver*, which for that Purpoſe is pour'd into the exterior Tube MS, which, when applied to the Pump in its Place, will be made to riſe in two cylindric Shells; one about the Tube of the Pump outwardly; the other within, about the innermoſt Tube or Plug XQ, as repreſented in the Figure at *a*, *b*, and *c*, *d*. At the Bottom there is but one cylindric Shell, becauſe the middle Tube does not reach the Bottom, leaving the Space VOWQ.

43. From this Account 'tis eaſy to obſerve, that the Part COQD anſwers to the Barrel of the Common Pump, FGH to the Pipe, the inner Tube XQ to the Piſton, and the Quickſilver at Bottom to the Leathers of the common Piſton; for it prevents all Communication of the external Air and internal Part of the Barrel, where the *Vacuum* is to be made. Whence 'tis evident, that upon letting down the outer Tube MS, it carries down the inner Tube XQ at the ſame Time, and makes a larger Space in the Barrel and Pipe, in which the Air will be expanded or become rarer, and its Spring thereby weaken'd. In conſequence of this, the Water will riſe through the Valve at I into the Pipe, and alſo the Mercury will riſe in the inner Shell by the Preſſure of the Air on the outer Shell, till the Equilibrium be reſtored: And the Height of the Water raiſed will be nearly 14 times as great as that of the Mercury.

44. When the Tube SM is raiſed again, the Air will be compreſs'd within the Barrel; and, its Spring increaſing, it will act upon the Water, the Mercury of the inner Shell, and the Valve E; the Water above the Valve I it cannot move, becauſe of the Valve being ſhut below; its whole Force is therefore ſpent on the Mercury, and Valve E; it will act on the inner Shell of Mercury, and drive it down to the Level of that in the outer Shell, as at *a b*, *c d*; and then the Preſſure will be every where equal, *viz.* on the inner and outer Shell, and on the Valve E; as the Tube after this continues moving up, the Air will be farther compreſs'd, and its Springs made greater than that of the outward Air, which therefore it will overcome, and ſo thruſt up the Valve E, till ſo much

by this means continually filling with Water, which greatly compreſſes the Air above the Sur-

has eſcaped as leaves the Remainder of an equal Tenour, or *in Equilibrio* with the Atmoſphere. The two Shells of Mercury will be upon a Level all the while the Air is going thro' the Valve, becauſe the Preſſure is not greater within than without.

45. By repeating the Operation a ſecond Time, the Air will be farther rarified, and the Water will again riſe in the Pipe; and thus on till the Pipe and Barrel be full as in the common Pump; and hence 'tis evident all thoſe Theorems raiſed in the preceding Articles, are equally applicable here as there. This Pump made at large for Uſe, ſhould have the following Dimenſions: The Length of the outer Tube $MN = 30$ Inches; of the inner Tube $XO = 31$ Inches; the Diameter of the inner Tube XY or $QO = 6$ Inches; the Thickneſs of the outer Tube $= \frac{1}{10}$ of an Inch, of the middle one $\frac{8}{100}$ of an Inch, and of the inner one, $\frac{13}{100}$ of an Inch, and the Bottom of the Tube ZO to come within an Inch of the Bottom NS of the outer Tube.

46. Theſe Dimenſions afford ſufficient Strength if the Tubes or Barrels are made of Copper, or caſt Iron, and of ſuch a Diameter, that the Diſtance between each may be $\frac{1}{2}$ the Tenth of an Inch; and this ſhould be nicely effected by having the Tubes truly turn'd in a Lathe. A tranſverſe Section of a Part of the Circumference of thoſe Tubes, their Thickneſſes, and Spaces between, you ſee repreſented in the *Fig.* 2. where AB is the outer, CD the middle, and EF the inner Tube. The Spaces between are ſo narrow, becauſe otherwiſe too great a Quantity of Quickſilver would be neceſſary, and yet of no Uſe; becauſe Fluids preſs only according to the Altitude, and not the Quantity.

47. If now every Part be fitted for Work, and Mercury pour'd into the Height of 24 Inches, as ſhewn by *a*NS*d*; and the Barrel and Pipe fill'd with Water, ſo that the whole Pump be full, and *in Equilibrio* with the Atmoſphere; if the outer Tube MS be moved down thro' the Space of 14 Inches, at the Commencement of the Motion, the *Equilibrium* in the Pump is deſtroy'd by the greater Space which would enſue upon the Deſcent of the inner Tube XQ, and which cannot be ſuffer'd, becauſe of the Preſſure of the Air on the Water at H, and on the Mercury in the outer Shell at *a*, *d*. And becauſe theſe Preſſures of the Air outwardly at

face of the Water in the Veſſel, and thereby proportionably augments its Spring; which at length

H and *a* are equal, but there is not an equal Preſſure inwardly on the Valve I, and the inner Shell of Mercury *b*, *c*; the Valve being preſs'd with all the Water in the Pipe above it, and which is proportional to the Altitude C*g*; and the Surface of the Mercury of the inner Shell at *b*, *c*, being preſs'd only with the Altitude of Water C*b*, 'tis plain the Water will not ruſh in at I till the Preſſure on *b*, *c*, becomes equal to it, and that will be when the Height of the inner Shell at *b* is greater than that of the outer Shell at *a*, by near a $\frac{1}{14}$ Part of the Difference of the Altitudes *bg*; and then the Preſſure being equal at *b* and I, upon the Motion of the Tube X Q downwards, the Water will be forced up thro' the Valve at I, and the Height of the Mercury in the inner Shell will always exceed that in the outer Shell, in ſuch Manner, that the Exceſs will be about $\frac{1}{14}$ of *bg*, or the Height of the Point *b* above the Water at H.

48. When the outer Tube is in its loweſt Situation, the Mercury in the inner Shell will be nearly at the Top X Y of the inner one, and in the outer Shell it will have but a ſmall Height as *a* O, or Q*d*. And when the Tube M S is drawn up again, the inner Tube X W will force the Water in the Pump to act upon the upper Part of the Valve I, the under Part of the Valve E, and the Surface of the Mercury at *b* in the inner Shell. The Valve I it ſhuts cloſe, the Valve E it endeavours to puſh up but cannot, till it has firſt reduc'd the Mercury in each Shell to a Level, when they, balancing each other, produce an *Equilibrium* with the external Air acting equally on the Surface of the Mercury in the outer Shell at *a*, and on the Valve E. After this, as the Tube M S continues its Motion upwards, the Mercury will riſe in the outer Shell, the Preſſure now on the inmoſt being greateſt. And for every 14 Feet the Water is forced above the Surface of the inner Shell *b*, the Mercury will riſe in the outer one, one Foot.

49. Becauſe the Structure of this Pump makes the Theory of its Operation not quite ſo eaſy and obvious to thoſe who have not conſider'd it very thoroughly, (as I have found by frequent Experience in my Lectures) I have contrived to repreſent it more plainly, by a Kind of Syphon or Pump of a very ſimple Form, and yet acting on the very ſame Principle as this complex Pump under Conſideration. The Sy-

is ſo far increaſed, as to re-act with great Force on the Surface Y Z of the ſubjacent Water; which

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phon is H G F Z N S M, where H G F is the Water-Pipe as in the Pump; Z N repreſents the inner cylindric Tube, and M S the outer one; O F Z Q is the Barrel in which the Piſton V X moves (which is a Forcer, or without a Valve,) and C R a Conduct-Pipe to carry away the Water forced thro' the Valve E. In all this it is eaſy to ſee the Analogy between this Syphon and the mercurial Pump. Its Operation likewiſe is the ſame.

Fig. 3, and 4. together. 50. For let Mercury be pour'd into the Legs M S and Z N, it will come to a Level at *a*, *b*. Now ſuppoſe the other Part of the Syphon full of Water, and the Piſton rais'd, 'tis plain the Water at H cannot open the Valve I, till a Column of Mercury be rais'd in the Leg Z N above the Surface of that in the outer Leg M S, ſufficient to balance the Weight of the Water in the Pipe H F, then will the Preſſure of the Air be equal upon the Water H, and the Mercury in the Leg M S; and as the Motion of the Piſton is continued, the Water and Mercury will continue to riſe with equal *Momenta*, and therefore the Mercury will riſe 1 Inch for every 14 Inches nearly, till the Piſton ſtops.

51. When this happens, the Surface of the Mercury in the Leg Z N will be at *c*, and that in the Leg M S at *d*; and now, if the Piſton be puſh'd down again, it will cauſe the Water to ſhut the Valve I, and to act on the Mercury at *c*, and on the Valve at E, but the Preſſure of the Air at E will not ſuffer the Valve to riſe till the Preſſure there be greater within than without, which it cannot be till the Surface of Mercury in the outer Leg M S be higher than that in the inner one; wherefore, before the Valve E can open, the Mercury in each Leg muſt come again to the Level *a*, *b*; after which, the Mercury will riſe in the outer Leg M, ſo as to be always *in Equilibrio* with the Water in the Conduct-Tube R C, and the Part C *e* of the inner Leg; ſuppoſing the Mercury now ſtands at *e* in that, and at *f* in the other: And then the ſaid Height of the Water will be nearly 14 times the Height *df*, or *ce*. I think all this is very eaſy to underſtand from the common Principles of Hydroſtatics, and if this be underſtood, the Nature of the Pump muſt, as being the very ſame Machine with a different Diſpoſition of its Parts.

52. From this Theory of the mercurial Pump, 'tis eaſy to infer, that in conſtructing one, the Part Z F O Q ought to be

aſcending through the ſmall Tube *ef* to the *Stop-Cock eg*, is there, upon turning the Cock, ſuf-

placed in the Middle of the Height from the Water H to be raiſed, to the Ciſtern A B which receives and delivers it; or more nicely, the level Surface *abcd* of the Mercury when the Pump is full, and juſt going to work, ought to be in the middle Point of the Line P*g*; the reaſon is, becauſe in the Deſcent and Aſcent of the Tube M S, the Differences between the Altitude of the outer and inner Shells *ad* and *cb* are equal, and in each Caſe a 14th Part of the Height of the Water below or above it.

53. Suppoſe the whole Height from the Water to the Piſton be $Pg = 60$ Feet; then $aP = ag = 30$ Feet, or 360 Inches; then alſo 14)360(26 nearly, that is, the Difference of the Altitudes in the mercurial Shells will be about 26 Inches. The Place therefore where they ought to be on a Level is at leaſt 13 Inches below M, or the Quickſilver muſt never be pour'd in to a Height N*a* greater than about 15 or 16 Inches, or 17 at moſt. If the Height P*g* be greater than 60 Feet, the Tubes M S, *&c.* muſt be proportionably enlarged.

54. To compute the Quantity of Quickſilver in the Shells; let A C B D be the Area of the Top of the inner Tube X Y, and E H F G the Area of the Surface of the inmoſt Mercury Shell ſurrounding it; then becauſe $AB = 6$ Inches, we have $AB^2 \times 0{,}7854 = 36 \times 0{,}7854 = 28{,}274 =$ the Area of the Circle A D B C. Again, ſince the Diameter E F is 6,1, the Area of the Circle E H F G will be $EF^2 \times 0{,}7854 = 29{,}224$; but $29{,}224 - 28{,}274 = 0{,}95$ of an Inch, the Area of the annular Surface of Mercury; wherefore $26 \times {,}95 = 24{,}7$ the cubic Inches of Mercury contain'd in 26 Inches Altitude of the inner Shell; which as it is leſs than the Area of the Plug 28,274; ſhews that before the outer Tube M S can deſcend one Inch, the Water will begin to riſe to H, near 30 Feet below. Pl. XXI. Fig. 5.

55. Alſo becauſe the Circumferences of Circles are as their Diameters, and the Diameters of theſe Shells are as 6,1 to 6,36 (*Art.* 45.) therefore $6{,}1 : 6{,}36 :: 0{,}95 : 0{,}99$ or 1 Inch; that is, every Inch of Altitude in the exterior Shell is 1 cubic Inch, and therefore when the Tube M S (in the forcing Stroke) has deſcended near 1 Inch, the Water will force up the Valve E, the Mercury then being about 26 Inches high in the outer Shell.

fered to paſs through a Pipe *b*, fixed to a *Ball and Socket*, from the Orifice of which it iſſues

56. If the Water be but 30 Feet below the Ciſtern, the Heights of the mercurial Shells will then be but half as great as before, *viz.* 13 Inches one above the other; and then 12,35 cubic Inches of the inner Shell will equal 13 Altitude, which therefore will riſe with a leſs Deſcent than $\frac{1}{2}$ an Inch of the Tube M S, in the inner Shell, and of Aſcent in the outer Shell. Therefore if the Stroke or Deſcent of the Tube M S be 15 Inches, there will be leſs than a 30th Part of the Stroke loſt, or a 30th Part of the Labour beſtow'd in Pumping. And if the Mercury Shells were but half as thick, as here ſuppos'd, the Part of the Labour loſt would be but $\frac{1}{60}$ of the Whole, which is far leſs than in any other Pump of this Kind.

57. Since the Area of the Piſton X W is 28,274 Inches, therefore every Inch of its Motion raiſes the $\frac{28,274}{282}$ Parts of a Gallon, or the $\frac{28,274}{17766}$ Parts of an Hogſhead; that is, 28,274) 17766 (= 628,4 Inches of Motion for an Hogſhead. Hence $52\frac{3}{10}$ Strokes, each 12 Inches, will raiſe one Hogſhead. And this, if the Weight be not greater than 10 Feet to which the Water is raiſed, may be done by an able Man in *one Minute*, working ſo as to be able to hold it all Day; as has been found by Experience. And therefore this is to be look'd upon as the greateſt Effect, which Perſons can expect from any Hydraulic Engine, to raiſe Water of any Kind whatſoever, unleſs he can invent ſomething more compleat and perfect in this Kind, than the Pump we have now been deſcribing.

58. Another excellent Pump, of the lifting Sort, is the Invention of Meſſrs. *Goſſet* and *de la Denille*, and ſet up in the King of *France*'s Garden at *Paris*. This Pump has this Peculiarity, that its Piſton works without Friction. The Form and Structure thereof is as follows; A B C D E F G H I K L is a Sort of Box incloſing the Piſton. This Box conſiſts of two Parts, the upper A B C I K L, and the under one D E F G H, which ſhut upon each other. The Piſton within is *a b*, and is a circular Piece of Wood, about the Circumference of which is nail'd a Piece of well ſeaſon'd Leather, of a circular Form, and ſo wide, that when the Piſton is placed at the Bottom of the Box, the Leather may lie over the Sides thereof at D H all around; the Piſton and Leather thus placed on the upper

with a great Velocity, to a very great Height or

Part, is forced down upon the Leather, and then both Parts ſcrew'd very faſt together. The Manner of which is very eaſy to apprehend from the Figure, where *i g f d b k* is the Leather going from the Piſton thro' the Jointure of the Box.

59. Upon the upper Part of the Piſton is fix'd a circular (or any figured) Piece of Iron or Wood, denoted by *d e f*, in the Top of which, at *e*, is fix'd the Rod of the Piſton Q C, by which the Piſton is drawn up towards the upper Part of the Cavity *l n*, and from thence forced down again in working the Pump. Now as the Diameter of the Piſton is leſs than that of the Cavity, it is plain that in its Motion up and down, no Friction can happen, as there are no Parts for it to rub againſt, which is occaſion'd by the Contrivance of ſuſpending it on the Leather.

60. In the bottom Part is fix'd a Pipe F O to bring up the Water from the Mine or Well at O, which it delivers into the Box by a Valve at *c*. In the Middle of the Piſton is likewiſe another Valve *m* opening upwards. A L M N is a Tube or cylindric Pipe, in which the Water is raiſed to a Ciſtern to run off. 'Tis eaſy to obſerve, that as the Piſton is drawn up, the Water will run in beneath, thro' the Valve *c*, to prevent a *Vacuum*; and alſo, that when the Piſton is forced down, the Water in the lower Parts muſt be forced up thro' its Valve, and when the Piſton is raiſed again, the Water above it will be forced up the Pipe A M to the Ciſtern.

61. Another Thing peculiar to this Pump is, the Shortneſs of the Stroke of the Piſton, which is compenſated by the Largeneſs of its Area, and the greater Number of Strokes that may be made in the ſame Time. The only Objection to this Pump is, that it is always charged with the Weight of ſo much Water, as is equal to a Column of Water, whoſe Baſe is equal to the Area of the Piſton, and the Height equal to that of the Reſervoir above it, as is evident from what we have ſaid in explaining the *Hydroſtatic Paradox*. So that the Tube A L M N has no Advantage in being ſmall, unleſs any particular Circumſtance or Conveniency require it.

62. ARCHIMEDES's SCREW deſerves a Conſideration, not only for its Antiquity, but its Uſefulneſs in raiſing Water. It conſiſts of a long Cylinder, with an hollow Pipe, Tube or Groove coil'd about it, as repreſented in the Figure; where the Cylinder is A B, and the Tube C D open at each End. It is placed in an oblique Poſition to the Horizon with the lower End in the Water to be pump'd away, the other End

Pl. XXII. Fig. 2.

Diſtance, in a ſmall continued Stream, directed

being ſupported on the lower Part of the Winch I K, by which the Screw and Cylinder is turn'd round.

63. As ſoon as the Screw is immerſed in the Water, it immediately riſes therein by the Orifice C, to the Level of the Surface of the Water E F, and if the Point of the Helix or Spiral, which in the Beginning of the Motion is coincident with the Surface of the Water, happens not to be on the lower Side of the Cylinder, the Water will upon the Motion of the Screw move on in the Helix, till it comes to the Point which is on the under Side, and coincident with the watry Surface; when it is arrived to that Point, which ſuppoſe at O, it cannot afterwards poſſeſs any other Part of the Spiral, than that which is upon the loweſt Part of the Cylinder, for it cannot move from O towards H or G, becauſe they are ſituated higher above the Horizon; and ſince this will ever be the Caſe, after the Surface of the Water in the Helix has attain'd the Point O, it is plain it muſt always be on the under Side of the Cylinder.

64. But ſince the Cylinder is in Motion, every Part of the ſpiral Screw from O to D, will by Degrees ſucceed to the ſaid under Part of the Cylinder; the Water therefore in the Helix muſt ſucceed to every Part thereof, from O to D, as it comes on the lower Side; that is, it muſt aſcend on the lower Part of the Cylinder thro' all the Length of the Pipe, till it comes to the Orifice D, where it will run out, as having nothing farther to ſupport it. Hence it appears, how much thoſe Gentlemen are miſtaken, who (affecting the Wonderful) ſay, *that the Water aſcends by deſcending*; whereas, if they would have made the moſt of the Wonder, they might have ſaid truly, *that the Water aſcends, becauſe it cannot aſcend*, i. e. that it aſcends one way, becauſe it cannot aſcend another; but then the Wonder is loſt.

65. I obſerve farther, with reſpect to Pump-Work, that no ſingle Pump, tho' perfectly tight, will raiſe Water by a Piſton playing at a greater Diſtance than 32 or 33 Feet; and even then the Stroke of the Piſton muſt be more than 8 Feet (for ſince $\frac{\frac{1}{4}aa}{b} = p = 8$ Feet; then $a = 32 = b$. See *Art.* 30, 31, 33.) I ſay, no Pump will raiſe Water to a greater Height than 32 Feet by a ſingle Piſton only; nor can this be remedied by ſeveral Valves in the Pipe below the Piſton, as ſome Gentlemen have pretended.

66. For let A B be the Barrel of a Pump, G F the Piſton,

every way, or to any particular Place, by means

Pl. XXII. Fig. 3. BE a Pipe going from the Barrel to the Water at E, 60 Feet below it; and let B, C, D, E, be 4 Valves placed in the Pipe at 20 Feet diſtance from each other. Now when the Piſton is drawn up from B to F, a *Vacuum* will be made between, but the Valve B will not riſe by the Water below it; for the Water in the Part BC cannot riſe itſelf, much leſs can that in the Part CD below it, as being preſs'd with the Weight of the Column above, and its own Weight downwards, therefore no Part of the watry Column BF can move of itſelf, or by itſelf. Now no Power can be applied any where but at E to move the whole Column; nor can the whole be moved without raiſing all the Valves at once; therefore the Power able to raiſe the Valve at E, and conſequently the Valve at B, muſt be able to overcome the Preſſure of a Column of Water 60 Feet high: But the Air can ſuſtain a Column only of 33 Feet high; therefore the Air cannot raiſe the Valve E, nor any of the reſt; therefore the Water cannot riſe in the Barrel AB of a ſingle Pump.

Fig. 4. 67. However, we may raiſe Water by a compound Pump, ſuch as is repreſented in the Figure, where A is the Rod of the Piſton, which at B is divided into two, one of which goes to the Barrel DE, and draws up the Water from the Ciſtern EF; the other Part BC goes down to the ſaid Ciſtern, where it is divided at C into two other Parts; one going to the Pump FG, ſupplies the Water to the Ciſtern EF; the other Part CH goes to another Pump IK below, and draws up the Water from K to the Ciſtern HI. Now theſe Pumps all working at once, will draw Water from any Depth, provided each Pump does not exceed 32 Feet in Height.

Fig. 5. 68. I ſhall conclude this Subject with an Account of one other Method of raiſing Water, by the *natural Agency of Heat and Cold* only. It is as follows, ABCD is a pretty large Veſſel fill'd with Water to the Height EF, the Space above being poſſeſs'd of Air. On the upper Part of the Veſſel is a tall Tube inſerted GH, and deſcends below the Surface of Water to K. On the nether Part is another Tube or Pipe IW. In each Tube is a Valve opening upwards, as at I and G. The Body of this Inſtrument being nicely cloſed every where, ſo that no Air can eſcape, and placed with the lower End in the Water W, and thus continued in the hot Sun of a Summer's Day, the Air will be rarified by the Heat of the Sun in the upper Part, and will compreſs the ſubjacent Water, and force it up thro' the Valve G into of the Ball and Socket (LXXIX.)

the Tube GH, and by the Cold of the following Night it will be condenſed again, and then the Preſſure of the Atmoſphere will force the Water at W up the Pipe WI, to repleniſh the Veſſel each Day. And in this Manner may Water be rais'd in a conſiderable Quantity in the Summer Time, or in very hot Climes.

69. By ſuch a Contrivance as this, ſeveral curious Effects may be produced. Thus if a cylindric Cover be annexed to the top Part of ſuch a Veſſel (diveſted of its Tubes,) it may be raiſed to collect the Sun-Beams at Noon on the Top of the Veſſel, which will ſo greatly rarify the Air within, as to make it produce a conſiderable Stream of Water thro' a ſmall Adjutage in the Side thereof; which Stream may fall on a Wheel to play an Organ at Dinner, *&c.* Thus alſo, an Image may be made to weep in the Sun-Beams, or at the Approach of Fire; with other Devices of the like Kind.

Pl. XXII. (LXXIX.) The greateſt Artifice of this Engine is its Contrivance to produce a *continual Stream,* which is done by Compreſſion, and Elaſticity of Air in the Barrel, call'd the Air Veſſel *a b c d.* For the Air, being an elaſtic Fluid, will be capable of Compreſſion in any Degrees by the Water forced in thro' the Valves at I, K; and ſince the Force of the Air's Spring will always be inverſely as the Space which it poſſeſſes, (as will be ſhewn) it follows, that when the Air Veſſel is half full with Water, the Air will be compreſſed into half the Space it poſſeſs'd at firſt, and therefore its Spring will be twice as great as at firſt.

2. But this Spring at firſt was equal to the Preſſure of the Atmoſphere; for if it was not, it could not have ſuſtain'd or reſiſted the Preſſure of the Atmoſphere which ſtood over it, and conſequently could not have fill'd the Veſſel before the Water was driven in; which yet we find it did, and maintain'd an Equilibrium with the common Air. The Veſſel then being half fill'd with Water, or the Air compreſſed into half the firſt Space, its Spring will be in this Caſe equal to twice the Preſſure of the Atmoſphere; and therefore when the Stop-Cock at *p* is turn'd, the Air within preſſing on the ſubjacent Water with twice the Force it meets with from the external Air in the Pipe *e f,* will cauſe the Water to ſpout out of the Engine to the Height of 32 or 33 Feet, if the Friction is not too great.

THERE is another Sort of *Engine* for raiſing Water, which works by Fire, in the following Manner. ABC is a Copper-Veſſel, partly fill'd with Water to DE, which being ſet over a Fire, and made to boil, will fill the upper Part DBE with a vaſtly elaſtic Vapour, the ſufficient Strength whereof is known by its forcing open a Valve at *e*: This heated elaſtic Steam is, by turning a Cock at F, let into the Barrel *abcd*, where by its elaſtic Force it raiſes the Piſton G, which drives the Air above it through a proper Clack

Pl. XXIII. Fig. 2.

3. That it muſt ſpout to that Height is evident from hence, that only one half the Force of the internal Air is ſpent in overcoming the Reſiſtance of the external Air; and this being deſtroy'd, the remaining Half of the Force will be exerted in projecting the Water, which now muſt be conſider'd as ſpouting *in Vacuo* (the Reſiſtance of the external Air being taken away). But *in Vacuo* Water will riſe to the Height of 32 or 33 Feet (as will be ſhewn hereafter) by the Preſſure of the common Air; therefore ſince the remaining half Force of the internal Air is equal to the Preſſure of the common Air, it muſt cauſe the Spout to riſe to the Height of 32 or 33 Feet; and becauſe the Water keeps entering the Veſſel as faſt as it ſpouts out, it keeps the Air under the ſame Degree of Compreſſion, which therefore muſt conſtantly act with the ſame Tenor on the Water, and produce a continual Stream to the ſame Height.

4. When the Air-Veſſel is $\frac{2}{3}$ full of Water, the Space which the Air takes up is $\frac{1}{3}$ Part, whence its Spring will be 3 times as great as that of the common Air, therefore it will now project the Water with twice the Force of the Atmoſphere, and therefore to the Height of 64 or 66 Feet. When the Air-Veſſel is $\frac{3}{4}$ full of Water, the Air will be compreſs'd into $\frac{1}{4}$ Part, and ſo will protrude the Water with a Force equal to 3 Atmoſpheres, and therefore to the Height of 96 or 99 Feet. And when the Veſſel is $\frac{4}{5}$ full of Water, the Air will take up Part, and will have its Spring 5 times as ſtrong as at firſt, nd will therefore force the Water 132 Feet high.

5. Hence 'tis eaſy to obſerve the Law by which the Spring

at the Top. After this, that the Piſton may by its Weight deſcend, a little cold Water, from the Fountain *f g h i*, is let in at the Bottom by turning a Cock at *k*, which, in Form of a Jet, condenſes the hot Steam in the Barrel into 13000 times leſs Space than before it took up, which makes a ſufficient *Vacuum* for the Piſton to deſcend in. The Piſton G, and Lever H I, being thus put into Motion, do accordingly raiſe and depreſs the Piſton K in the Barrel of the forcing Pump L M on the other Side; which, by the

of the Air will act on the Surface of the Water below it, *viz.* according to the Numbers in the Table below.

Height of the Water.	*Of the Air compreſs'd.*	*Proportion of the Air's Spring.*	*Height of the Spout.*
$\frac{1}{2}$	$\frac{1}{2}$	2	33 Feet.
$\frac{2}{3}$	$\frac{1}{3}$	3	66
$\frac{3}{4}$	$\frac{1}{4}$	4	99
$\frac{4}{5}$	$\frac{1}{5}$	5	132
$\frac{5}{6}$	$\frac{1}{6}$	6	165
$\frac{6}{7}$	$\frac{1}{7}$	7	198
$\frac{7}{8}$	$\frac{1}{8}$	8	231
$\frac{8}{9}$	$\frac{1}{9}$	9	264
$\frac{9}{10}$	$\frac{1}{10}$	10	297

And thus for any greater Degree of Compreſſion.

6. As the Air-Veſſel is the Cauſe of the continual Stream, we may naturally infer, that if ſuch an Air-Veſſel were adapted to the *Common Houſe Pump*, it would then become a *Fire-Engine*, or very uſeful for extinguiſhing Fires that may happen in or near the Houſe. Now this may be effected in a Manner either the ſame with, or analogous to, that which follows. Pl. XXII. Fig. 6. A B C D is the Barrel of the Pump, P H the Rod and Piſton, C W the Pipe going down to the Water of the Well at W. Towards the lower Part of the Barrel is a ſhort Tube, by which the Air-Veſſel F E is fix'd to, and communicates with, the Barrel of the Pump. A M N L is a Collar of Leathers ſo fixed on the Top of the Barrel, and adapted to the Rod, that it

Pipe N, draws the Water from the Depth W, and forces it to riſe and ſpout through the Tube O, continued to any Height at Pleaſure. This Engine is uſed to draw the Water from Coal-Pits and other Mines. Beſides theſe, there are various other Water-Engines of a more complex Structure; but as they all act on the ſame Principles with theſe two deſcribed, 'tis needleſs to ſay any thing of them here (LXXX, LXXXI, LXXXII.)

may move freely in the Leathers, without permitting the Air to paſs in or out between. The Nozzle or Spout D has a Stop-Cock S to ſhut off the Water, or let it paſs out as required. Q is a Piece ſcrew'd on to direct the Stream by a ſmall pliant Leathern Pipe at the End.

7. Now 'tis plain, when the Piſton is raiſed from the Bottom of the Pump Barrel, the Water above it will be forced into the Air-Veſſel, and there compreſs the Air (the Stop-Cock being ſhut); it will alſo compreſs the Air on the Top of the Barrel, for the Water will not be higher than the Spout D at firſt, when the Stop-Cock is ſhut: But afterwards, as the Air is confin'd, it will be compreſs'd at Top, and the Water will riſe to I.

8. This compreſs'd Air, in each Place, will act upon the Water by its Spring, and, upon turning the Stop-Cock, will force it out in a continual Stream, through a Pipe at Q, and that with a greater or leſſer Degree of Force, as Occaſion requires, that being abſolutely in the Power of the Perſon working the Pump. This Invention we owe to Sir *James Creed*, one of the very few Votaries which the Philoſophic Muſes find among Perſons of Quality and Fortune.

(LXXX.) The FIRE-ENGINE is the moſt admirable, curious, and compounded Machine that we find among all thoſe whoſe Inventions have been owing to the Diſcoveries of Modern Philoſophy, and which affords the greateſt Advantages to Mankind. The Fire-Engine is of two Sorts, which I ſhall here give the Theory of, and Figures of each according to the lateſt Perfection in which they are made. The Account given above being only in relation to its firſt State and original Simplicity: But it is now of a very different Form, though acting upon the ſame Principle, as will appear from the following hiſtorical Theory thereof.

2. The Marquis of Worcester, in his *Century of Inventions*, (printed *A. D.* 1663.) is the firſt that I know of who propoſed raiſing great Quantities of Water by the Force of Fire, turning Water into Steam; and he mentions ſome Engine of this Kind, which he ſaw play'd a continual Stream in the Manner of a Fountain, forty Feet high. He alſo ſays, that a Perſon attending it turn'd two Cocks; that one Veſſel of Water being conſumed, another begins to force and re-fill with cold Water; and this alternately and ſucceſſively, the Fire being tended and kept conſtant by the ſame Perſon in the Interim between turning the Cocks.

3. Captain Savery, having read this Account, immediately attempted to raiſe Water by Fire, and was the firſt who erected an Engine for this Purpoſe, of the Form we have ſince had them. To ſecure the Invention to himſelf, he bought up all the Marquis's Books that he could find, and burnt them; and then gave out that he diſcover'd the Method by Accident. He made many Experiments to bring the Machine to Perfection, and erected ſeveral for Gentlemens Seats, but could not ſucceed for Mines, the Depth from which Water was to be raiſed from thence being ſo great, that it required the Steam too dangerouſly ſtrong to be attempted in his Way.

4. Thus the Progreſs of the Engine was ſtopp'd, till long afterwards Mr. *Newcomen* an Ironmonger, and Mr. *John Cawley* a Glazier, (both Diſſenters of the *Baptiſt* Denomination at *Dartmouth*) contrived another Way to raiſe Water by Fire, where the Steam to raiſe the Water from the greateſt Depths of Mines is not required greater than the Preſſure of the Atmoſphere; and this is the preſent Structure of the Engine, and which is now of about 35 or 40 Years ſtanding.

Pl.XXIV. 5. We have already ſhewn (*Annot.* LXXVIII. 67.) that Water may be drawn from any Depth by a Compound Pump. Suppoſe the Depth = 150 Feet, and the Diameter of the Pump-Barrel at *p* be $7\frac{1}{4}$ Inches, then the Weight of a cylindric Column of Water 150 Feet high, and $7\frac{1}{4}$ Inches Diameter, will be about 3000 *lb.* If now the Rod of this Pump were hung by a Chain to the End of a Lever *b b*, as at H, and at the other End another Power were applied, as at P, with a ſuperior Force, the Pump might be work'd, and the Water raiſed by that Power.

6. Now this Power cannot be human, or the Strength of Men; for it will require 100 Men to pull down the Beam, each pulling with a Force of 30 *lb.*; and 100 more to relieve

theſe when weary; for the Pump in a Mine muſt not ſtand ſtill: But 200 Men would be too expenſive. If we allow Horſes, (and one Horſe equal to five Men) there muſt be 20 Horſes working at a time, and 20 more to relieve theſe; which great Number of Horſes, though leſs expenſive than the Men, will be found too great an Expence for moſt Mines: And neither Men nor Horſes would be able to give more than 15 or 16 Strokes in a Minute.

7. Some other Power therefore muſt be applied; and the Philoſopher readily offers his Service with his Air-Pump, and Preſſure of Air. His Propoſal is to fix a large Barrel or Cylinder CC, truly bored, about 8 or 9 Feet long and 22 Inches wide, on the Top of his Air-Pump; and in this is a Piſton well leather'd, whoſe Rod L is faſten'd to a Chain fix'd to the End of the Lever at P. Now, ſays he, allowing 15 *lb.* of Preſſure upon every ſquare Inch, the whole Preſſure of Air upon ſuch a Piſton will be 5800 *lb.* which is near twice the Force to be overcome. But this muſt be effected by exhauſting ſo large a Barrel by the Air-Pump, and which he cannot do more than twice in an Hour; ſo that he can produce only 2 Strokes in the Time that 960 are neceſſary to keep the Water from overflowing the Mine.

8. But though the firſt Expedient fails, he makes a ſecond Propoſition, and that ſucceeds; which is as follows. B is a large Boiler, whoſe Water by the Fire under it is converted into elaſtic Steam. The great Cylinder CC is fix'd upon it, and communicates with it by the Pipe D*d*; on the lower Orifice of which (within the Boiler) moves a broad Plate, by means of the Steam-Cock or Regulator E 10, ſtopping or opening the Paſſage, to prevent or permit the Steam to paſs into the Cylinder, as Occaſion requires. The Diameter of the Pipe D is about 4 Inches.

9. The Steam in the Boiler ought always to be a little ſtronger than the Air, that, when let into the Barrel, it may be a little more than a Balance to the Preſſure of the external Air, which keeps down the Piſton at the Bottom *dn*. The Piſton being by this means at Liberty, the Pump-Rod will, by its great Weight, deſcend at the oppoſite End to fetch a Stroke: For the Weight of 70 or 80 Yards of Iron-Rod amounts to about 9 or 10 Hundred; whereas the Piſton, and other Weights at the other End, is not above 4 or 4½ Hundred. The End of the Lever at the Pump therefore will always preponderate the other, and deſcend when the Piſton is at Liberty.

10. The Handle of the Steam-Cock E 10 being turn'd towards *n*, opens the Pipe D to let in the Steam; and being turn'd towards O, it ſhuts it off that no more can enter. The

Piſton is now raiſed towards the Top of the Barrel at C, and the Barrel is full of Steam. The Lever O 1 muſt then be lifted up, to turn by its Teeth the Injecting-Cock at N, which permits the Water brought from the Ciſtern *g*, by the Pipe *g* MN, to enter the Bottom of the Barrel at *n*, where it flies up in the Form of a Fountain, and ſtriking againſt the Bottom of the Piſton, the Drops being driven all over the Cylinder will by their Coldneſs condenſe the Steam into Water again, and precipitate it to the Bottom of the Barrel.

11. Mr. *Hen. Beighton* made an Experiment to determine the Rarity of the Steam, and found the Content of the Barrel of Steam was 113 Gallons; and ſince there were 16 Strokes in a Minute, therefore $113 \times 16 = 1808$ Gallons of Steam *per* Minute. He alſo obſerved, that the Boiler required to be ſupplied with Water at the Rate of 5 Pints *per* Minute; and ſince 282 Cubic Inches make a Gallon, $35\frac{1}{4}$ make a Pint, and $5 \times 35\frac{1}{4} = 176\frac{1}{4}$ in 5 Pints; alſo the Cubic Inches of Steam are $1808 \times 282 = 509856$; if then we ſay, As $176\frac{1}{4} : 509856 :: 1 : 2893$; or one Cubic Inch of Water is expanded into 2893 Inches of Steam: Conſequently the Steam in the Barrel is reduced to $\frac{1}{2893}$ Part, when turn'd to Water by the Jet of cold Water; and therefore a ſufficient Vacuum is made in the Barrel, for the Piſton to deſcend unbalanced, by the Preſſure of the Air.

12. The Piſton being forced down raiſes the other End of the Lever, and conſequently the Piſton of the Pump which brings up and diſcharges the Water at *p*. Now this whole Operation of opening and ſhutting the Steam-Regulator and Injection-Cock, will take up but little more than 3 Seconds, and will therefore produce very well 16 Strokes in a Minute.

13. That the Ciſtern *g* may always be ſupplied with Water, there is an Arch *x* fix'd near the Arch H at the Pump-End, from whence another Pump-Rod *k* with its Piſton draws Water from a ſmall Ciſtern near the Mouth of the Pit, (ſupplied from the Water raiſed at *p*) and forces it up the Pipe *m m m* into the Ciſtern *g*, which therefore can never want Water.

14. That the Leathers of the Piſton C may be always ſupple, and ſwell'd out ſo as to be always Air-tight, a ſmall Stream of Water is ſupplied from the Injecting-Pipe M by the Arm *z*. On the Top of the Cylinder is a larger Part or Cup L, to hold the Water that lies on the Piſton, leſt it ſhould overflow when the Piſton is got to its greateſt Height, as at W; at which Time if the Cup be too full, the Water will run down the Pipe V to the waſte Well at Y.

15. The Water in the Boiler, which waſtes in generating Steam, is ſupplied by a Pipe F*f* about 3 Feet long, going into the Boiler a Foot below the Surface of the Water. On the Top of this Pipe is a Cup or Funnel F, ſupplied by the Pipe W with Water from the Top of the Ciſtern, which has the Advantage of being always warm, and ſo not apt to check the Boiling of the Water in the Copper.

16. That the Boiler may not have the Surface of the Water too low, (which would endanger burſting) or too high, (which would not leave Room enough for Steam) there are two Gage-Pipes at G; one going a little below the Surface of the Water when at a proper Height, and the other ſtanding a little above it. When every thing is right, the Stop-Cock of the ſhorter Pipe being open'd gives only Steam, and that of the long one Water; but if otherwiſe, both Cocks will give Steam when the Surface is too low, and both give Water when it is too high; and hence the Cock which feeds the Boiler at F may be open'd to ſuch a Degree, as always to keep the Surface of Water to its due Height.

17. The cold Water, conſtantly injected into the Cylinder to condenſe the Steam, is carried off by the Eduction-Pipe *d*TY, going from the Bottom of the Cylinder to the Waſte-Well Y, where going a little under Water it has its End turn'd up, with a Valve Y to keep out the Air from preſſing up into the Pipe, but permitting the injected Water coming the other Way to be diſcharged; whereby the Cylinder is kept empty.

18. Leſt the Steam ſhould grow too ſtrong for the Boiler, and burſt it, there is a Valve fix'd at *b*, with a perpendicular Wire ſtanding up from the Middle of it, to put Weights of Lead upon, by which to examine the Strength of the Steam puſhing againſt it from within. Thus the Steam is known to be as ſtrong as the Air, if it will raiſe up ſo much Weight on the Valve as is at the Rate of 15 *lb.* to an Inch ſquare; becauſe that is the Weight of Air (nearly) on every ſquare Inch, as will be ſhewn in its Place. When the Steam becomes ſtronger than what is required, it will lift up the Valve, and go out. This Valve is call'd the *Puppet-Clack*.

19. The Steam has always a variable Strength, yet never $\frac{1}{10}$ ſtronger or weaker than common Air: For it has been found that the Engine will work well, when there is the Weight of 1 *lb.* on every ſquare Inch of the Valve *b*. This ſhews that the Steam is then $\frac{1}{15}$ Part ſtronger than the common Air. Now as the Height of the Feeding-Pipe from the Funnel F to the Surface of the Water S*s* is not above 3 Feet, and 3½ Feet of Water is $\frac{1}{10}$ of the Preſſure of the Air; if the Steam were $\frac{1}{10}$ Part ſtronger than Air, it would puſh the

Water out at E; which ſince it does not do, it cannot be ſtronger than Air, even in this Caſe, where, the Regulator being ſhut, it is moſt of all confined.

20. When the Regulator is open'd, the Steam gives the Piſton a Puſh, which raiſes it up a little way; then falling a greater Space, it comes to be of the ſame Strength, and ſo a Balance to the Atmoſphere; thus the Piſton, being at Liberty, riſes to the Top W. The Steam, now expanded into the whole Capacity of the Cylinder, is weaker than the Air, and would not ſupport the Piſton were it not for the greater Weight at the other End of the Lever, which keeps it up. The Steam each Stroke drives the injected Water of the preceding Stroke out of the Eduction-Pipe *d*TY; and would itſelf follow, and blow out at the Valve Y, (which is not loaded) if it were ſtronger than the Air, which it never does. If it were exactly equal to the Strength of the Air, it would juſt drive all the Water out at Y, but could not follow itſelf, the Preſſure being on each Side the Valve equal by Suppoſition. If it be weaker than the Air, it will not force all the Water out of the Pipe *d*TY, but the Surface will ſtand ſuppoſe at T, where the Column of Water TY added to the Strength of the Steam is equal to the Preſſure of the Air. When the Steam is $\frac{1}{10}$ weaker than the Air, the Height TY $= 3\frac{1}{2}$ Feet. Now ſince the whole perpendicular Diſtance from *d* to Y is but 4 Feet, and the Steam always ſufficient to expel the Water; it is plain it can never be more than a $\frac{1}{10}$ Part weaker than the Air, when weakeſt.

21. There is Air in all the Water injected; and tho' that Air cannot be taken out or condenſed with the Steam, yet will it precipitate and fall thro' the Steam to the Bottom of the Cylinder, as being much heavier. For it has been ſhewn, that Steam is to Water, as 1 to 2893 in its Denſity, but the Denſity of Air is to that of Water, as 1 to 864 (as will be ſhewn) therefore the Rarity of Steam is to that of Air, as 2893 to 864; the Air will therefore fall thro' the Steam to the Bottom, and from thence be driven out thro' a ſmall Pipe opening into the Cup at 4, on which is a Valve. Now when the Steam firſt ruſhes into the Cylinder, and is a little ſtronger than the outward Air, it will force the precipitated Air to open the Valve at 4, and make its Eſcape; but the Steam cannot follow, becauſe it is weaker than the outward Air, as the Piſton gives it room, by aſcending, to expand. This Valve (from the Noiſe it makes) is call'd the *Sniſting-Clack*.

22. Among the greateſt Improvements of this Engine, we may reckon that Contrivance, by which the Engine itſelf is

made to open and ſhut the Regulator and Injection-Cock, and that more nicely than any Perſon attending could poſſibly do it. For this Purpoſe there is fix'd to an Arch Z, at a proper Diſtance from the Arch P, a Chain, from which hangs a perpendicular Piece or working Beam QQ, which comes down quite to the Floor, and goes thro' it in a Hole, which it very exactly fits. This Piece has a long Slit in it, and ſeveral Pin-Holes, and Pins, for the Movement of ſeveral ſmall Levers deſtin'd to the ſaid Office of opening and ſhutting the Cocks, after the following Manner.

23. Between two perpendicular Pieces of Wood on each Side P, there is a ſquare Iron Axis AB (in the following *Fig.* where this Part is repreſented by itſelf for Diſtinctneſs) which has upon it ſeveral Iron Pieces of the Lever Kind. The firſt is the Piece CED, call'd the Y, from its repreſenting that Letter inverted by its two Shanks E and D; on the upper Part is a Weight F to be raiſed higher or lower, and fix'd, as occaſion requires. This Y is fix'd very faſt upon the ſaid Iron Axle AB.

24. From the Axle hangs a Sort of an Iron Stirrup IKLI, by its two Hooks II, and having on the lower Part two Holes K, L, thro' which paſſes a long Iron Pin LK, and key'd in the ſame. When this Pin is put in, it is alſo paſs'd thro' the two Holes in the Ends E, N, of the horizontal Fork or Spanner EQN, join'd at its End Q to the Handle of the Regulator V 10. From Q to O are ſeveral Holes, by which the ſaid Handle may be fix'd to that Part of the End which is moſt convenient.

25. Upon the Axis AB is fix'd at Right Angles to the Y, an Handle or Lever G 4, which goes on the Outſide of the Piece QQ, and lies between the Pins. Another Handle alſo is faſten'd upon the ſame Axle, *viz.* H 5, and placed at half a Right Angle to the former G 4; this paſſes thro' the Slit of the Piece QQ lying on one of its Pins. Hence we ſee, that when the working Beam goes up, its Pin in the Slit lifts up the Spanner H 5, which turns about the Axle ſo faſt, as to throw the Y with its Weight F from C to 6, in which Direction it would continue to move, after it paſs'd the Perpendicular, were it not prevented by a Strap of Leather fix'd to it at OE, and made faſt at the Ends *m* and *n*, in ſuch Manner, as to allow the Y to vibrate backwards and forwards about a Quarter of a Circle, at equal Diſtances on this Side and that of the Perpendicular.

26. As Things are repreſented in the Figure, the Regulator is open, its Plate TY being ſhown on one Side the Pipe S which joins the Cylinder and Boiler. The Piſton is

now up, and alſo the working Beam near its greateſt Height; the Pin in the Slit has ſo far raiſed the Spanner H 5, that the Weight F on the Head of the Y is brought ſo far from *n*, as to be paſt the Perpendicular, and ready to fall over towards *m*, which when it does, it will by its Shank E, with a ſmart Blow, ſtrike the Iron Pin K L, and drawing the Fork ON horizontally towards the Beam Q; will draw the End 10 of the Regulator towards 6, and thereby ſhut it, by ſlipping the Plate Y under the Holes of the Throat-Pipe S.

27. Immediately after the Regulator is ſhut, the Beam riſing a little higher, with its Pin S on the Outſide upon the lower Part lifts up the End *k* 1 of the Handle of the Injecting-Cock, and opens it by the turning of the two Parts with Teeth. The Jet immediately making a *Vacuum*; the Beam again deſcends, and the Pin *r* depreſſing the Handle *k* 1 ſhuts the Injection-Cock; and the Beam continuing to deſcend, the Pin *p* bears down the Handle G 4, and throwing back the Y, its Shank D throws forward the Fork N Q, and again opens the Regulator to admit freſh Steam. After this every thing returns as before; and thus is the Engine moſt wonderfully contrived to work itſelf.

28. After the Engine had been made, as above deſcribed, for many Years, it receiv'd another Improvement of very great Advantage; and that was, inſtead of feeding the Boiler with warm Water from the Top of the Cylinder (as related *Art.* 15.) by the Pipe W above, and F *f* below; they contriv'd to ſupply it with the ſcalding-hot Water which came out of the Eduction-Pipe *d* T Y, which now, inſtead of going to the waſte Well at Y, was turn'd into the Boiler on the top Part; and as the Eduction-Pipe before went out at the Side of the Cylinder, it was now inſerted in the Bottom of the ſame; and tho' the Preſſure of the Steam in the Boiler be ſomewhat greater in the Cylinder, yet the Weight of Water in the Eduction-Pipe being added to the Force of Steam in the Cylinder, will carry the Water down continually, by overcoming the Reſiſtance in the Boiler.

29. To conclude this Theory of the Fire-Engine, I ſhall here add a moſt curious and uſeful Table of the Calculation of the Power of Fire-Engines for the various Diameters of the Cylinder, and Bore of the Pump, that are capable of raiſing Water from 48 to 440 Hogſheads *per* Hour, at any Depth, from 15 to 100 Yards. It was compoſed by Mr. *Henry Beighton*; and is founded on this Principle, that the Ale Gallon of 282 cubic Inches of Water weighs 10*lb.* 3*oz.* *Averdupois*, and a ſuperficial ſquare Inch is preſs'd with the Weight of 14*lb.* 13*oz.* of Air when of a mean Gravity. But al-

lowing for ſeveral Frictions, and to give a conſiderable Velocity to the Engine, it is found by Experience, that no more then 8*lb.* of Preſſure muſt be allow'd to an Inch ſquare on the Piſton in the Cylinder, that it may make about 16 Strokes in a Minute, about 6 Feet each.

36. Theſe Calculations are but for common Practice; for with large Boilers the Piſton will make 20 or 25 Strokes *per* Minute, and each of them 7 or 8 Feet, and then a Pump of 9 Inches Bore will diſcharge more than 320 Hogſheads *per* Hour, and for other Sizes proportionably. The Uſe of the Table is eaſy; for ſuppoſe it requir'd to draw 150 Hogſheads *per* Hour, at the Depth of 90 Yards; then in the 7th Column, I find the neareſt Number 149, and againſt it, in the firſt Column, I find a 7 Inch Bore for the Pump; then under the Depth 90 on the Right-hand, in the ſame Line, I find 27 Inches for the Diameter of the Cylinder fit for the Purpoſe, and ſo for any other.

Diameter of Bore.	Will hold in a Yard.	Will draw by 6 Feet Stroke.	Weight in one Yard.	At ſixteen Strokes a Minute.	63 Gallons to a Hogſhead.		In one Hour.			The Depths in Yards.												
Inch.	Gallons.	Gallons	lb. Aver.	Gallons.	Hog.	Gal	Hog.	Gal.		15	20	25	30	35	40	45	50	60	70	80	90	100
12	14,4	28,8	146	462	7	21	140		Diameter of the Cylinder in Inches.	18½	21¾	24	28½	30½	32½	34¼	37¼	40				
11	12,13	24	123,5	338	6	20	369	3		17	19¾	22	24¾	26¾	28	29¾	31¼	34¼	37	39½		
12	10,02	20,04	102	320	5	5	304	5		15½	18	20	22	23¾	25¼	27	28¾	31¼	33¾	36	38	40
9	8,12	16,2	82,7	259,8	4	7	247	7		14	16¼	18	20	21½	23	24½	25	28	30½	33	35	36½
8½	7,26	14,5	73,9	232,3	3	43	211	2		13½	15¼	17½	19	20¼	21¾	23	24	26¼	28½	31	32½	35¼
8	6,41	12,8	65,3	205,2	3	16	195	2		12½	14½	16½	18½	19	20½	21½	23	25	27	29	30½	32½
7½	5,66	11,3	57,6	181,1	2	55	172	3		11	13¾	15	16½	18	19	20	21¼	23¼	25	27	28½	30¾
7	4,91	9,8	50	157,1	2	31	149	4		10¾	13	14	15½	16¾	18¾	19	20½	22	24	25½	27	28½
6½	4,23	8,4	43	135,3	2	9	128	5		10	12	13	14	15½	16¼	18	19	20	22	23	24½	26¼
6	3,61	7,2	36,7	115,5	1	52	110	1		9½	11	12	13	14	15½	16	17	19	20½	22	23	24½
5½	3,13	6,2	31,8	99,2	1	36	94	3			10	11	12	13	14	15	15¾	17	19	20	21	22¼
5	2,51	5,0	25,5	80,3	1	7	66	6				10	11	11¾	13	13¾	14	15½	16¾	18½	19¼	20¼
4½	2,02	2,4	20,5	64,6	1	1	60	6					10	11	11¾	12	13½	14	15	16	17	18¼
4	1,6	3,2	16,2	51,2	0	51	48	51						9	10	11	11½	12	13½	14	15	16

(LXXXI) The Method of conſtructing a Fire-Engine, according to the original Inſtitution of the Marquis of *Worceſter* and Capt. *Savery*, wherein the Water was to be raiſed ſolely by the Preſſure of elaſtic Vapour or Steam, is very uſeful, and cheap in reſpect of the other Sort; and when the Height, to which the Water is to be raiſed, does not exceed 100 or 150 Feet, then this Engine is applicable with great Advantage, it requiring but a ſmall Fire, (not bigger than what is generally uſed in a Parlour Chimney) is of a very ſimple and eaſy Structure, and admirably adapted for playing of Fountains to very great Heights.

2. Dr. *Deſaguliers* has given us the Structure of this Engine on a Copper Plate with his laſt Improvements, and becauſe I think this better than that which Profeſſor *Muſſchenbroek* has exhibited in his *Eſſai de Phyſique*, eſpecially for raiſing Water, I ſhall here give the ſaid Figure and its Explication. Pl.XXVI. Fig. 1.

3. The Boiler BB is a large Copper Body of a globular Form, which will beſt of all withſtand the very great Force of Steam, that is in this Caſe neceſſary. Round the Body of this Boiler the Fire and Flame are conducted, as ſhewn at TTT. It has a Copper Cover ſcrew'd on, which contains the Steam-Pipe CD, and two Gage-Pipes N, O, which, by turning their Cocks, ſhew the Height of the Surface of the Water within, as in the other Engine.

4. On the ſame Cover at P is a Valve, over which lies a Steel-yard with its Weight Q to keep it down, the Strength of the Vapour being this Way moſt exactly eſtimated: For being in the Nature of a Lever, of the third Sort, 'tis plain, if the Beam of the Lever be divided into 10 equal Parts, and the firſt of them be upon the Middle of the Valve, and the Weight Q hangs at the 2d, 3d, 4th, &c. Diviſions, that then the Force of the Steam which can raiſe up the Valve will be 2, 3, 4, &c. times as great as the Weight. If the Area of the Valve be a ſquare Inch, and Q = 15*lb.* hanging at the 2d Diviſion is raiſed by a Steam puſhing up the Valve, it will ſhew the Steam will then preſs with the Force of 2 Atmoſpheres, and ſo on to 10 Atmoſpheres; but great Care muſt then be taken about it, leſt the Steam, ſo very ſtrong, burſt the Boiler to Pieces.

5. The Steam is carried from the Boiler to a Copper Veſſel A by means of the Pipe CD, and is let into the ſame by turning the Handle K of the Steam-Cock DI. The Key of this Cock is kept down by the Screw L, held up by the Gibbet DL. The Handle turn'd from K to *k* admits a Paſſage to the Steam into the Copper Receiver A.

6. This Receiver A communicates at Bottom with the Sucking-Pipe ZH going down to the Water H in the Well X; and above with the Forcing-Pipe EE, which goes up a little above the Water of the Reſervoir R; between theſe two Pipes are two Valves F and G, both opening upwards. The Steam being let in upon the Water of the Receiver A, forces it up thro' the Valve F, and the Pipe EE to the Reſervoir, and then the ſaid Receiver is full of hot Steam.

7. This Steam in the Receiver is condenſed by a Jet of cold Water coming from the Forcing-Pipe by the ſmall Pipe MI, being let in, and ſhut off by the Cock at M. The Steam being condenſed by this Jet, will be reduced within a very ſmall Space, and ſo make a *Vacuum*, upon which the Water in the Well will ruſh up the Forcing-Pipe to reſtore the Equilibrium, and thus again fill the Receiver A; the little Air being compreſs'd within a ſmall Compaſs at the Top above *a b*.

8. That there may always be Water in the Force-Pipe for the Jet, there is a little Pipe which brings the Water to it from the Reſervoir, with the ſmall Stop-Cock Y to ſhut it off upon Occaſion. The Valves at F and G are examin'd at any time, by unſcrewing the Pin 1 to looſen the Strap 2, and let down the Flanch 3, all which Parts are ſhewn larger in the Figures below.

9. By the peculiar Contrivance of the Cock at DI, and its Key, the Water is made to paſs from the Force-Pipe to the Boiler, to ſupply the Waſte in Steam. This is plainly ſhewn Pl.XXVI. Fig. 2. in the Sections of the Cock and Key, where 5 is the Top of the Key; 6 is a Hole on one Side, which goes down to the Bottom, to convey the Steam or Jet of Water alternately to the Receiver; 7 is a Notch on the other Side to take in the Water from the Force-Pipe, and conduct it to the Boiler B. How this is done, is eaſy to conceive from a View of the two Sections of the Cock and Key in two Poſitions within it.

10. The Boiler may hold about 5 or 6 Hogſheads, and the Receiver 1 Hogſhead: It will work 4 or 5 Hours, without recruiting; about 4 Strokes *per* Minute will produce upwards of 200 Hogſheads *per* Hour. This Steam makes a *Vacuum* ſo effectually, as to raiſe Water from the Well to the Height of 29 or 30 Feet; and ſuppoſe the Steam able to lift up the Steel-yard, with its Weight hanging at the 6th Diviſion, it will then be able to raiſe a Column of Water about 50 Yards high, as being then 6 Times ſtronger than the Preſſure of the Air, as is eaſy to underſtand from what has been ſaid on this Subject of the Common Water-Engine, (See

Annotat. LXXX. 5.) the Water being raiſed in a ſimilar Manner in both Machines; there by the Preſſure of condenſed elaſtic Air; and here by the Preſſure of rarified elaſtic Steam.

11. The Elaſticity of the Steam increaſing with its Heat, and the Space into which it endeavours to expand itſelf being as its Elaſticity or Strength, and ſince we know (by what precedes) that Water expanded into Vapour, of the ſame Strength of the Air, occupies near 3000 times the Space as before, it follows, that when the Steam has twice that Strength, it will be expanded into 6000; if it be 5 times as ſtrong as the Air's Spring, it will become expanded into 15000 times that Space, and ſo on, beyond the Reach of Experiments. From all which it appears how near a perfect Vacuum we may approach by this Engine, or any other, by means of rarified Vapour.

(LXXXII.) In the *Philoſophical Tranſactions*, N°. 461, there is an Account of an Improvement made in the Fire-Engine by Mr. *Payne*, as follows. He has contrived two Iron Pots or Veſſels of a conical Form inverted, as repreſented by A B E F, on the upper Part of which is fix'd a round globular Copper Head (of about 5½ Feet diameter) as L M N. There is then placed on the Inſide a ſmall Machine H, call'd the *Diſperſer*, with Spouts *a*, *b*, *c*, *d*, *e*, &c. round the Sides fix'd to it, and the Bottom thereof reſts on a Centre-Pin O. In this Machine is fix'd an upright Tube G, with Holes at the Bottom, and a Funnel P on the Top to receive a Spout of Water from a Conduit-Pipe Q, by the Stop-Cock R.

Pl. XXV. Fig. 8.

2. This Veſſel being compleated, two or more of them are placed in a reverberatory Arch for conveying the intenſe Heat of a ſtrong Fire, the Flame whereof encompaſſes the Iron Veſſels, and keep them in a red Heat, during the Time of their Uſe; at which Time the Cog-Wheel I, being turn'd by proper Machinery, whirls the Diſperſer about with great Velocity, and cauſes the Water in it (by a centrifugal Force) to fly thro' the Spouts againſt the Sides of the red-hot Pots, which immediately converts the greateſt Part thereof into Vapour or elaſtic Steam, which is convey'd by a common Pipe and Cock to the Barrel of the Engine to put the Piſton in Motion; and the waſte Water is convey'd away at bottom, by means of a Pipe C D, with a Valve at D to keep out the Air.

3. By many Experiments made on this Engine, Mr. *Payne* has proved the following Particulars, *viz.* That ſuch an ignified Pot or Veſſel will rarify or expand 50 Gallons of Water, Wine-Meaſure, *per* Hour.

4. That a cubic Inch of Water will make in Practice 4000 Inches of Steam; and therefore 50 Gallons will produce 46000000 *per* Hour, or 770000 *per* Minute; hence two ſuch Pots will produce 1540000 cubic Inches of elaſtic Steam *per* Minute.

5. That by an Experiment made at a Fire-Engine, 40 Gallons of Water *per* Hour, made into elaſtic Steam, will effectually give Motion to a 24 Inch Cylinder-Engine.

6. That 112 *lb.* of Coals is ſufficient, in this Method, to rarify 90 Gallons of Water *per* Hour into Steam, whereas in the Common Fire-Engine, with Boilers, 'tis uſual to expend three times as great a Quantity.

7. That 95 Gallons of Water expanded into Steam will work a 36 Inch Cylinder-Engine; from all which he concludes, that his new Invention will ſave at leaſt 60 *per Cent.* in Pit-Coals to work a Fire-Engine.

8. The Method by which he found how much Steam could be produced from a given Quantity of Water was this. He procured a Copper Globe A 12 Inches in Diameter, with two Braſs Cocks BB oppoſite to each other; by one of theſe the Globe was faſten'd to a Pipe E, which was fix'd to the upper Part of one of the Steam Veſſels (as at N) and was fill'd with Steam by the Cock F; upon the other Cock was fix'd a ſmall Valve D, fitted to a Bore of one Inch Diameter; and C is a Handle or Bale to ſuſpend it by.

9. The whole thus fitted weigh'd 15 *lb.* 3 *oz.* *Troy*, or 12 *lb.* $9\frac{1}{2}$ *oz.* *Averdupois*; the Capacity or Content was about 925 cubic Inches. The Globe being fix'd to the Steam-Veſſel, all the Cocks were turn'd, till the Steam by paſſing thro' had ſufficiently heated the Globe; then being ſtopp'd, the Steam condenſed, and the Air let in, when weigh'd in a very nice Balance, the Weight was 15 *lb.* 3 *oz.* 2 *dwt.* ſo that the whole Quantity of Steam was but 2 Penny-Weights, or $\frac{1}{10}$ of an Ounce *Troy*. Whence one Ounce of Water makes 9250 cubic Inches of Steam.

10. The Globe fill'd with Water weigh'd 45 *lb.* 7 *oz.* After this, it being fill'd with Steam, and one of the Cocks put under Water, the Water ruſh'd into the Globe, and the Cock being again turn'd, and the Globe of Water now weigh'd, the Weight was 44 *lb.* 9 *oz.* which deducted from the former Weight, gave 14 *oz*; which ſhew'd that all the Air was nearly excluded, or that a *Vacuum* was approach'd to in the Ratio of 713 to 727; or the Part poſſeſs'd by Air was but the $\frac{14}{727}$ Part of the Whole.

11. The Air being again thus excluded out of the Globe by Steam, and both Cocks turn'd, the Globe was thus weigh'd, and

THE abovemention'd *prodigious Rarefaction* of a ſmall Quantity of Water will be made very ſenſible by the *Æolipile*, whoſe Nature and Uſe in the Production of *a continual and moſt powerful Blaſt of Steam*, or *vapourous Wind*, is a *Phænomenon* equally curious and ſurpriſing, as will appear from the Experiment (LXXXIII.)

and the Weight was 12 *lb.* 10$\frac{1}{2}$ *oz.* then one Cock was open'd and the Air let in, upon which the Scale deſcended, and $\frac{1}{2}$ an Ounce *Averdupois*, being added to the other Scale, reſtor'd the Equilibrium, which was therefore the Weight of the Air, or 925 cubic Inches of Air weigh'd about 220 Grains.

12. But it was found above, that the Globe full of Steam weigh'd only 2 Penny-Weight *Troy*, or 40 Grains; whence the ſpecific Gravity of elaſtic Steam is to that of Air as 40 to 220, or as 1 to 5 nearly; and ſince that of Air is to Water as 1 to 860, therefore the ſpecific Gravity of Steam is to that of the Water, as 1 to $860 \times 5 = 4300$.

13. Again, ſince $\frac{1}{10}$ of an Ounce of Water expelled near 925 cubic Inches of Air; then one cubic Inch of Water (which weighs $\frac{527}{1000}$ of an Ounce *Troy*) will expel about 4630 cubic Inches, but Mr. *Payne* takes 4000 for a Standard in Calculation.

(LXXXIII.) 1. The ÆOLIPILE is an Inſtrument contriv'd for Uſe, as well as Jucundity. It has its Name from the Fable of ÆOLUS, the God of Winds and Tempeſts, who was deſcribed, as having his Palace in the Caves, and locking up the Winds in the Hollows of the Mountains. The Word literally interpreted is, *the Gates of Æolus*, ('Αἰόλου πύλαι) becauſe of the Similitude of its Effect. For it conſiſts of a Copper globular Body, in which is inſerted a ſmall Neck or Pipe, with a very ſmall Orifice, from whence (when the Veſſel has Water in it, and heated) a Vapour iſſues with prodigious Violence, Impetuoſity and Force, and blows in the Manner of a terrible tempeſtuous Wind.

2. The Body of this Inſtrument conſiſts of two Parts, A and B, ſolder'd together with hard Solder, to endure a great Heat of Fire. On the upper Part is alſo ſolder'd the Pipe C D, whoſe Orifice or Hole at D is about the 40th or 50th Part of an Inch in Diameter. The Way to fill it with Water is, to ſet it firſt, while empty, on the Fire, which will Pl. XXII. Fig. 7.

V. The laſt Cauſe of the Motion of Fluids which I mention'd was that of Attraction. We have already ſeen how by this means any Fluid will aſcend above the common Surface in *Capillary Tubes*, &c. But the moſt notable and obvious Motion of Fluids ariſing from Attraction, is that of the Tides; the Waters of the immenſe Ocean, forgetful, as it were, of their natural *Quietus*, move, and roll in ſwelling Tides obſequious to the ſovereign Power of the Moon, and weaker Influence of the Sun.

Pl.XXIII. Fig. 3. To illuſtrate this, let NESQ repreſent the Earth, covered over with Water ABDF; NS the Axis of the Earth, EQ the Equator, TR the Tropic of *Cancer*, *tr* the Tropic of *Capricorn*, M the Moon in her Orbit, S the Sun in his. Now ſince all Bodies are endued with an attracting Virtue, the Moon will attract all the

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heat it, and drive out all the Air; then with a Pair of Tongs you take it off the Fire, and hold the End of the Pipe D under Water, and the Water will run in to poſſeſs the Space deſerted by the Air, by the Preſſure of the Atmoſphere.

3. For an Experiment, it need not be more then half full of Water, which will be a conſiderable Time in being changed into Vapour. The ſtronger the Fire, the better will be the Steam, and the more forcibly will it fly out, and it will continue this ſurprizing Blaſt, till it is wholly evacuated; then the Noiſe ceaſes, and a ſtill fine *Aura* follows the Steam. Great Care muſt be taken not to have the Hole at D ſtopp'd in putting it in the Fire, for if that ſhould happen, the Æolipile will burſt with a great Exploſion (equal to that of a Cannon) and may do very great Miſchief, as I have twice experienced, and once it was like to have been fatal; for then, when it burſt, the Power was ſo great, as to rend aſunder the Body of the Inſtrument, not in the Jointure, but in the ſolid Metal, which was at leaſt 8 Hundredths of an Inch thick. It happily ſtood

Water in the neareſt Hemiſphere FAB, with Degrees of Force which are inverſely as the Squares of the Diſtances from all Parts; and therefore with the ſtrongeſt Force where the Diſtance is leaſt, *viz.* in the Point A directly under her; and this Attraction being in this Hemiſphere contrary to that of the Earth, the Water in all Parts from B and F towards A will have their Gravity gradually decreaſing, and be lighteſt of all at the Part A; and conſequently muſt there ſtand higher than at the Point F, where they are more attracted by the Earth, and ſo heavier, as is evident from the Laws of *Hydroſtatics.*

AGAIN: In the other Hemiſphere FDB, the Attraction of the Moon conſpires with that of the Earth; but decreaſing as the Squares of the Diſtances increaſe, the joint Force of Attraction will every where decreaſe from F and B towards D,

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ſo as to fly againſt the inner Part of the Mantle-Piece, which beat it both Sides together by the Violence of the Blow; which, had it flown into the Room, muſt have been fatal to any one or more who ſhould have been in its Way.

4. To give an Idea of the great Force neceſſary to burſt the Æolipile in the Manner above-mention'd, let us proceed to Calculation. It has been found by Experiment, that a Copper Wire $\frac{1}{10}$ of a *Rhine-Land* Inch in Diameter was broke aſunder by the Weight of $299\frac{1}{4}$ *lb.* hung on to it. Now the *Rhine-Land* Foot is to the *London* Foot, as 139 to 135; the Diameter therefore of the Wire in *Engliſh* Meaſure was $\frac{136}{1350}$ of an Inch, whence the Area of its tranſverſe Section will be $\frac{212531}{25515000}$, or very nearly $\frac{21}{2551}$ of an Inch ſquare..

5. The Diameter of the Æolipile is 4 Inches, its Circumference therefore is 12,566; its Thickneſs about $\frac{8}{100}$, or 0,08 of an Inch, whence 12,566 x 0,08 = 1,00528 = the Area of the rended Circumference of the Æolipile. Then ſay, by the

the Point oppoſite to the Moon; where, again, the Waters will be lighteſt, and therefore ſtand higheſt to preſerve the *Equilibrium*. Whence it appears, that by this Sum and Difference of the Moon's and Earth's Attraction, there will neceſſarily enſue a Protuberance or Swelling of the Waters, which we call *Tides of Flood*, in the two Points A and D directly under the Moon. Alſo in the two Points F and B, as the Waters are there moſt attracted, ſo they will be heavieſt, and conſequently riſe to the leaſt Height from the Earth's Surface, whence they are called *Tides of Ebb*, or the *Ebbing of the Water*.

IF to the Power of the Moon we add that of the Sun, we ſhall have the Tides conſiderably augmented at the *Conjunction* in S, or *Oppoſition* in H, that is, *at the New and Full Moons*, which are call'd *the Spring Tides*; as thoſe which happen

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Rule of Three, If $\frac{21}{2551}$ or $\frac{2}{255}$ require 299$\frac{1}{4}$ or 300 *lb.* to break it, What will 1 ſquare Inch require? The Anſwer is, 38250 *lb.* for $\frac{2}{255}$: 300 *lb.* :: 1 : 38250 *lb.* So the Force of the Steam which burſt the Æolipile was *Thirty-eight Thouſand and two Hundred and fifty Pounds*, or one Thouſand nine Hundred and twelve Tons and an half.

6. This may ſerve as a notable Inſtance of the incredible Force of elaſtic Steam, it being three times as great as the Force of Gun-Powder in the largeſt Cannon, as calculated in *Annotat.* XXV. Hence it appears of what prodigious Uſe this expanded Vapour would be, could we rarify Water into Vapour as ſoon, and with the ſame Eaſe, as we can ſet Fire to Powder. This Property of Air has already ſupplied us with *Air-Guns*; and with much greater Effect might we have *Steam Guns*, and other Inſtruments, if that could be done.

7. To try this, one need only take a Vial with one Drop of Water in it, and ſet it over the Fire, till the Drop begins

when the Sun is at O or P are call'd *Neap Tides*, the Waters at A and D being then loweſt, becauſe the Attraction of the Moon is then counteracted by that of the Sun.

IT is farther to be obſerved, That of *the two Tides of Flood* at A and D, that at A is greateſt to any Place T in *Northern Latitude*, when the Moon is in *Northern Signs*, and above the Horizon; for the Point A is then nearer the *Zenith* of the Place G, than the oppoſite Point D is to the ſame Place at R twelve Hours afterwards; and conſequently, the Height of the Tide T G is greater than that of the oppoſite Tide R*g*. The contrary happens when the Moon is in *Southern Signs*.

THAT there are *two Tides of Flood*, and *two of Ebb*, ſucceeding each other *alternately at the Interval of ſix Hours*, is obvious from the *Figure*.

to riſe in Vapour, and drive out the Air, then putting the Cork faſt into the Mouth of the Vial, ſet it on the Fire, and incline it to the farther Side of the Room, after ſome little Time the Steam will diſcharge the Cork with a conſiderable Exploſion, and throw it as far as a Grain of Gun-Powder would have done.

8. Or thus; let a few Drops of Water be put into a Piſtol or Gun Barrel, cloſed on the Breech-End, and a leaden Bullet thruſt hard into the Barrel; let the End of the Barrel then be put over the Fire to heat, and expand the Water into Steam, which when it begins to come out of the Touch-hole, will ſhew the Air to be all excluded; then let the Hole be ſtopp'd with a ſmall Plug, and in ſome time the Vapour will diſcharge the Bullet, with as much Force as a common Charge of Powder, and with the ſame Report and Execution.

9. And though it may be difficult to contrive any Method of making Vapour of a ſufficient elaſtic Force in large Quan-

That they happen *later each Day near an Hour*, is owing to their exact Correſpondence to the *Motion of the Moon*, which daily *culminates* ſo much later. That they happen *not when the Moon is in the Meridian, but about three Hours after*, is owing to the Force of the Moon being then greater than when in the Meridian of any Place; as the Heat of the Day is greater at Three o'Clock than at Twelve; and the Heat of the Summer is greater in *Auguſt*, than at the 10th of *June*. Laſtly, *that the greateſt Spring Tides happen not at the* 10th *of* March; *and* 12th *of* September, *but in* February *and* October, is becauſe the Sun being neareſt the Earth in *December*, his Influence is then ſtrongeſt, and ſo muſt quicken the Time of the greateſt *Vernal Tides*; and being weakeſt in *June*, the Time of the *Autumnal Tides* will neceſſarily be retarded.

SUCH are the *Phænomena* of the *Tides*, according to a general Theory, and are found to be verified in thoſe Parts of the Earth's Sur-

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tities, ſo as by Conduct-Pipes to charge large Guns therewith, at a Diſtance from the Steam-Veſſel; yet ſomething of this Kind may be effected, by expanding a ſmall Quantity of Water at the Bottom of the Cannon into Vapour, by firſt putting a red-hot Iron Bullet down the Bore to the Water, which it ſoon diſſipates into Vapour, and when confin'd at the Touch-hole, and another with which the Cannon is charged, it will ſoon become ſtrong enough to drive it with Force enough for any propoſed Effect. So that the *Steam-Gun*, or even a *Steam-Cannon*, is perhaps a Matter not merely *in poſſe*, but may be hoped for in Reality, if all proper Expedients be tried, and Experiments made for that Purpoſe.

ſace which are *wholly cover'd with Water*, as the Theory requires; I mean, the vaſt *Atlantic and Weſtern Oceans* (LXXXIV.)

(LXXXIV.) 1. The Theory of the Tides is a Point of great Curioſity and Importance, and has therefore excited ſeveral learned Gentlemen to endeavour an Explanation thereof, in ſuch Ways, as they think the various Phænomena will be beſt and moſt eaſily underſtood. I have conſider'd them all, and can find none which I think ſo natural and obvious as that which I have here propoſed. Nature herſelf here points out all the Phænomena, and that too plainly to want any Aſſiſtance from the various Contrivances of Art; which I find by frequent Experience is more apt to puzzle than afford any *Ecclairciſſement* in this particular Caſe.

2. The Mathematical Principles of Theory are as follow. Let T be the Centre of the Earth, O the Moon, and ACBD the Circumference of the Earth, which ſuppoſe all around environ'd with Water, whoſe outmoſt Surface is repreſented by the dotted Circle, and its Diſtance from the Earth's Surface is indefinitely ſmall in reſpect of the Earth's Semidiameter. Now 'tis plain were the Moon away, or her Attraction at the Earth nothing, the Waters would every where have an equal Share of the Earth's attractive Power (ſuppoſing the Earth quieſcent) and ſo would take their Place at an equal Diſtance from the Centre, and therefore conform themſelves to a ſpherical Figure, agreeable to what was ſhewn under the *Laws of Hydroſtatics*. Plate XXVII. Fig. 1.

3. But ſince the Earth is not quieſcent, but moveable upon its Axis, the Centrifugal Force generated under the Equator (where it is greateſt) will there diminiſh the Power of Gravity upon the Waters, and render them leſs heavy there than on any other Part, and as the Centrifugal Force decreaſes towards each Pole, the Weight of the Water will increaſe, and be greateſt under either Pole. And ſince Fluids, which communicate with each other, maintain an Equilibrium, and if they are of different Weights, they have alſo different Denſities, and conſequently require different Altitudes to ſuſtain each other *in Equilibrio*; therefore it follows, that the Waters under the Equator, which are lighteſt, muſt have a greater Altitude (or Diſtance from the Earth's Surface) to be *in Equilibrio* with the Waters under the Pole, which are heavier, and will therefore have a leſs Altitude. Hence the Figure of an univerſal Ocean will not be ſpherical, but a flatted Sphe-

roid (like that of the fluid Earth, and for the ſame Reaſon, in *Annotat*. XXXIV) whoſe longeſt Diameter is that of the Equator continued.

4. But as this reſults from the diurnal Rotation of the Earth only, which is always uniform, and every where conſtantly the ſame, it can cauſe no Alteration in the Depth of the Water, in any particular Place, and therefore can have no Share in the Cauſe of the *Flux and Reflux* of the Sea, at leaſt ſo little as not to deſerve Notice, and cannot be diſcern'd by the Senſes. However, what that is will be ſhewn in the Sequel of this Note.

5. The alternate Riſing and Falling of the Waters is therefore owing to ſome other Cauſe or Cauſes than what can be found in or upon the Earth; that is, it muſt be owing to ſome of the Heavenly Bodies which we ſee at a Diſtance from us. This the Ancients made no Doubt of; and aſcribed it to the Influence of the Moon, with good Reaſon, as they ſaw ſo plain a Congruity between the Motion of the Waters, and that of the Moon. They could not alſo but obſerve, that at each New and Full Moon the Tides were greater than at other Times, and were therefore aſſur'd, the Sun was likewiſe concern'd in the Cauſe of the Tides. Yea, *Pliny* aſſures us, that they were ſo nice as to obſerve the annual *Phænomena* of the Tide; and knew that the Tides were greater in Winter than in Summer. But they had no Theory to account for the diurnal, menſtrual, and annual Differences of the Tide. This was reſerv'd for the Invention of the celebrated Author of our Philoſophy, whoſe Footſteps, therefore, we ſhall here inſiſt on as near as poſſibly we can, in explicating all the particular *Phænomena* of the Tides.

6. Since (as we have ſhewn) all Bodies mutually attract each other, the Moon at O will attract the Earth at T, and its Waters, as well as be attracted by it. But ſince the Law of Attraction is to decreaſe, as the Squares of the Diſtances increaſe, the Water at A, P, C, B, or at A, D, *p*, B, will be differently affected by the attractive Power of the Moon at O; the Attraction at A will be greateſt as being neareſt, that at B leaſt, and at C or D, it will be at a Mean, and the ſame nearly as at T the Centre of the Earth, becauſe of the very great Diſtance T O in reſpect of T C.

7. Let us conſider the Caſe of one aqueous Particle at P, and compare it with that of A in the neareſt Hemiſphere C A D; and of another at *p* compared with that of B in the fartheſt Hemiſphere C B D. Draw the Lines O P and O *p*; in O P continued out let O K repreſent the mean Attraction,

or that of the Particle C, and let it be made as $OL : OK :: OK^2 : OP^2$; then will $OL : OK :: \frac{1}{OP^2} : \frac{1}{OK^2}$, or $OK : \frac{1}{OK^2} :: OL : \frac{4}{OL^2}$, wherefore O K and O L will repreſent the attractive Forces of the Moon on the Particles at C and P, as being inverſely as the Squares of the Diſtances.

8. In like Manner, if we take $Ol : Ok :: Ok^2 : Op^2$, then will Ol be the Force with which the Particle p is affected by the Moon. Continue out O T, and draw L M parallel to P T, and lm to pT. Then is the Force O L reſolvable into the two Forces L M, and M O, and the Force Ol into lm, and mO. Now ſince L M and lm directly conſpire with P T, and pT, they will expreſs Forces to be ſuperadded to the Attraction of the Earth upon the Particles P and p; and therefore unleſs theſe additional Forces were every where the ſame, the Waters about the Earth's Surface, cannot have an equal Power of Gravity towards the Earth's Centre; and ſo cannot exiſt in a ſpherical Form.

9. In order to ſee this, we have (by ſimilar Triangles LOM and POT) $LM : PT :: LO : PO$; whence $LM = \frac{PT \times LO}{PO}$; but ſince L O is as $\frac{1}{PO^2}$ (by *Art.* 7.) we have $LM = \frac{PT}{PO^3}$, or ſince P T is given $LM = \frac{1}{PO^3}$, and for the ſame Reaſon $lm = \frac{1}{pO^3}$, that is, $LM : lm :: pO^3 : PO^3$; but ſince P O is leſs than pO, L M will be much greater than lm, conſequently the Water at P and p will be affected with very different Forces, and therefore will not tend equally to the Earth's Centre T, or be at an equal Diſtance from it. And this Difference of Forces will be greateſt of all at the Points A and B, for there theſe perturbating Forces will be as AO^3 and BO^3 inverſely, which Cubes have the greateſt Difference of all others.

10. Now this is only one Part of the Attraction of the Moon which diſturbs the Waters; the other Part M O makes this Perturbation ſtill greater; for if T O be the Force with which the Earth's Centre is attracted by the Moon O, 'tis plain the Waters can never accede directly towards the Moon with the ſame Velocity that the Earth does, unleſs when M O and mO are equal to T O, that is, at the Points C and D.

From MO ſubduct TO, the Remainder TM will be the Exceſs of Force, by which the Waters in every Point P, of the Hemiſphere CAD will be accelerated towards the Moon more then the Earth itſelf; and Tm will expreſs the Force by which the Earth tends towards the Moon more than the Water at any Point p in the remote Hemiſphere CBD.

11. The Proportion of the Part of the Force MO, in reſpect to OP, is thus found; as MO : OT :: LM : PT; wherefore $MO = \frac{OT \times LM}{PT}$; but ſince both OT and PT are conſtant, we have MO as LM, *i. e.* MO is as $\frac{1}{PO^3}$. Therefore at A, MO will be as $\frac{1}{AT^3}$; and at B, mO will be as $\frac{1}{OB^3}$, that is, in reſpect to the Points A and B, we have MO : mO :: OB^3 : AO^3; and ſince theſe Cubes are the greateſt of all others, the Waters will be accelerated in A towards the Moon with the greateſt Difference of Attraction, and at B with the leaſt; conſequently the Water will there on this Account alſo recede moſt from a ſpherical Figure.

12. Therefore, ſince both Parts of the Moon's Force are porportional to the Cubes of the Diſtance of the Luminary inverſely, they will both conſpire to make the Waters at A and B recede from the Centre of the Earth T, and put on a ſpheroidical Figure, whoſe longeſt Axis BA goes towards the Moon's Centre O. The Force MO = MT + TO, but ſince the Part TO is always the ſame, the perturbating Part will be only TM, which will be as MO — TO, and ſo when MO = TO, then TM = o; which happens when P is at C or D; and then alſo LM coincides with PT, and both with CT; whence ſince in that Caſe the Line OK becomes OC = OT, the perturbating Forces will in Part vaniſh (*viz.* TM = o) and the other Part LM will make the Waters at C heavier than in any other Place; and ſince every thing happens inverſely, with regard to the Particle p, it follows the Waters about the Points C and D will be at a leſs Diſtance from the Earth's Centre than any where elſe, whence CTD will be the ſhorteſt Diameter of the aqueous Spheroid ACBD.

13. The Sun at S is ſhewn to produce the ſame Sort of Effects upon the Water, about the Earth; only in a leſſer Degree. For tho' the *Momentum*, or abſolute Force of the Sun upon the Waters is much greater than that of the Moon, yet the Parts of the Force which diſturb the Figure of the

aqueous Globe, *viz.* LM and lm, and TM, and Tm will in this Caſe be leſs than before, on account of the much greater Diſtance TS than TO. As the perturbating Forces of the Moon in the Points A and B are as BO^3 to AO^3, thoſe of the Sun will be as SB^3 to SA^3. But the Difference of theſe latter Cubes is much leſs than that of the former, becauſe of their vaſtly larger Roots SB and SA. For Example, Suppoſe $OA = 10$, $OB = 11$; and $SA = 100$, and $SB = 101$; then $OA^3 : OB^3 :: 1000 : 1331 :: 100 : 1033$ nearly. But $SA^3 : SB^3 :: 1000000 : 1030301 :: 100 : 103$ nearly.

14. Hence we ſee, that both the Luminaries produce a Tide in two oppoſite Parts of the Earth's Surface A and B, in a Space of a Day proper to each, *i. e.* in the Space of a *Lunar and Solar Day*, or the Time in which the Moon, or the Sun, after leaving the Meridian, comes to it again. The Lunar Day (becauſe of the Moon's quick Motion conſpiring with that of the Earth) is about 50 Minutes longer than the Solar Day of 24 Hours. And thoſe Tides will always be moving the ſame way, ſometimes together, ſometimes ſeparately, that of the Moon always following after that of the Sun.

15. In order to compute the Forces of the Sun and Moon to raiſe the Waters, we will ſuppoſe (at preſent) that P is the Moon, and T the Earth, then LM will be the Addition of the Moon's Gravity to the Earth ariſing from the Action of the Sun S. When P is at C, then LM will be as CT, which let us call A. The Gravity of the Earth towards the Sun will be expounded by TS, which call F, that is, let $CT : TS : A : F$. Let P be the periodical Time of the Earth about the Sun, and p that of the Moon about the Earth; alſo let f be the Force of Gravity in the Moon towards the Earth; then will $F : f :: \frac{TS}{P^2} : \frac{TC}{p^2}$ (by *Annotat.* XXXIV. 6.) therefore $F : f :: \frac{F}{P^2} : \frac{A}{p^2}$; conſequently, $\frac{FA}{p^2} = \frac{Ff}{P^2}$, that is, $\frac{A}{p^2} = \frac{f}{P^2}$, whence $A : f :: p^2 : P^2$; *or the additional Gravity to the Moon at* C, *is to the Gravity of the Moon towards the Earth, as the Square of the Moon's Period to the Square of the Earth's Period.* Now $p = 27$ Days 7 Hours $43'$; and $P = 365$ Days 6 Hours $9'$, whence $p^2 : P^2 :: 1000 : 178725 :: 1 : 178\frac{725}{1000} :: A : f$.

16. Now the Semidiameter of the Earth being put $= 1$, and $G =$ Power of Gravity on its Surface; ſince the Mean-

Diſtance of the Moon from the Earth is 60 Semidiameters of the Earth, we ſhall have $f : G :: 1^2 : 60^2 :: 1 : 3600$; therefore $3600 f = G$, and $f = \frac{G}{3600}$; which being put for f, in the Proportion above, we have $A : \frac{G}{3600} :: 1 : 178,725$; whence $178,725\, A = \frac{G}{3600}$. Therefore $A : G :: 1 : 178,725 \times 3600 = 643400$.

17. What we have hitherto ſaid is upon Suppoſition that the Moon revolves about the Centre of the Earth at Reſt; but this is not the Caſe, for both the Earth and Moon revolve about one common Centre of Gravity (as we have ſhewn *Annotat.* XXXVI.) and allowing the Maſs of Matter in the Moon to that in the Earth as 1 to 39,788, we find the true Mean-Diſtance of the Moon from the Earth revolving about the common Centre of Gravity, to be 60,5 Semidiameters of the Earth; but the Force by which it is retain'd in its Orbit at 60 Semidiameters, is to the Force retaining it at $60\frac{1}{2}$ (revolving in the ſame Time as before) as 60 to $60\frac{1}{2}$ (as was ſhewn *Annotat.* XXXIV. 8.) Wherefore $60 : 60\frac{1}{2} :: f : \frac{60,5 f}{60} = f$ truly equated. But here $f = \frac{60 f}{60,5}$, and before we had $f = \frac{G}{60^2}$, therefore $\frac{60 f}{60,5} = \frac{G}{60^2}$, and ſo $\frac{60,5 G}{60 \times 60 \times 60} = f$; wherefore $A : G :: 1 : 643400 \times \frac{60}{60,5} = 638059,78 = b$, that is $A = \frac{G}{b}$.

18. After the ſame Manner it is ſhewn, if any Body P revolve about the Earth at its Surface, it would at C have an additional Gravity a, which would be as $\frac{G}{x}$; therefore $A : a :: 60\frac{1}{2} : 1 : \frac{G}{b} : \frac{G}{x}$; whence $x = 60\frac{1}{2} a = 60\frac{1}{2} \times 638059,78 = 38602616,7$. Now becauſe the Waters will be affected juſt in the ſame Manner as ſuch a Body revolving at the Earth's Surface, by the Influence of the Sun, (becauſe both will yield to an impreſs'd Force) *therefore the Power of the Sun to increaſe the Gravity of the Waters at* C *and* D, *is to the Power of Gravity as* 1 *to* 38602616,7.

19. In order to eſtimate this Force at A and B, we muſt find the Value of L M there, by which the Particle at A is

impell'd towards T; and of TM, by which it is impell'd in a contrary Direction towards S. Then will the Difference of theſe two Forces, *viz.* $TM - LM$, be the Force required. In order to this, it muſt be conſider'd, that on account of the prodigious Diſtance of O in Compariſon of P, the Angle POT will be exceeding ſmall, and the Lines QL, OM, may be look'd upon as parallel, and therefore $LM = PT$, and $PL = TM$, very nearly.

20. Then when P coincides with A, and K with T, we have $LM = AT = PK$, and $TM = PL = AT + KL$; wherefore $TM - LM = KL$, which therefore is the whole Force by which the Waters at A are drawn from the Earth's Centre T. In the ſame Manner it is ſhewn that $Tm - lm = kl =$ to the whole perturbating Force at B.

21. Again, ſince SP is nearly equal to SK, and it is $SL : SK :: SK^2 : SP^2$, and therefore $SL - SK : SK :: SK^2 - SP^2 : SP^2$, that is, $KL : SK : PK \times \overline{SK + SP} : SP^2$ (becauſe the Difference of the Squares of any two Numbers is equal to their Sum multiplied by their Difference; thus $\overline{a+b} \times \overline{a-b} = aa - bb$.) Therefore, ſince $SK = SP$, we have $KL : SP :: PK \times 2SP : SP^2 :: 2PK : SP$; wherefore $KL = 2PK$, and $PL = TM = 3PK$.

22. Hence, when P coincides with A, we have $PK = AT = PT = LM$, and therefore $TM = PK = 3LM$; whence $TM - LM = KL = 2LM = 2AT = 2CT$. But CT expreſſes the additional Gravity at C; therefore $KL = 2TC$, is the Force by which the Waters are drawn from T towards S; and ſince $TC = a = \frac{G}{38602616{,}7}$, we have $KL = \frac{2G}{38602616{,}7} = \frac{G}{19301308{,}3}$; that is, the Diminution of the Gravity at A is double the Increaſe of it at C, and is the $\frac{1}{19301308{,}3}$ Part of the whole Gravity.

23. Hence the Sum of both the Forces $KL + LM = \frac{2G}{38602616{,}7} + \frac{G}{38602616{,}7} = \frac{3G}{38602616{,}7} = \frac{G}{12867538{,}9}$; or the whole Force of the Sun to agitate the Waters of the Ocean will be the $\frac{1}{12867538{,}9}$ Part of the Gravity or Force of the Earth's Attraction. This is the Force in any given Place to which the Sun is vertical, and in his Mean-Diſtance from the Earth. In other Poſitions of the Sun, the Force will vary in the Proportion of the *verſed Sine of double the*

Sun's Altitude above the Horizon directly, and the Cube of its Diſtance from the Earth inverſely.

24. It has been ſhewn that the centrifugal Force of the Parts of the Earth under the Equator is $\frac{1}{289}$ Part of Gravity, (*Annotat.* XXXIV. 20.) wherefore the centrifugal Force is to the Sun's Force to agitate the Sea, as 12867538,9 to 289, or as 44524,4 to 1. Alſo ſince the Diameter of the Equator is to that of the Poles as 230 to 229 (as was there ſhewn), if we ſay as 229 : 230 :: 4000 : 4017,5, the Difference 17,5 Miles, or 924000 Feet, is the Effect or Elevation at the Equator, occaſion'd by the centrifugal Force, Say then, As 44524,4 : 924000 :: 1 : 2,07 = 2 Feet $\frac{4}{5}$ of an Inch = 24,84 Inches, which therefore is the Height which the Sun can elevate the Waters to at A and B above that at the Points C and D.

25. In this Computation, *Profeſſor* EULER has thought fit to find great Fault with Sir ISAAC NEWTON, and his Principles, calling this Altitude of 25 Inches an *enormous and erroneous Effect*, becauſe his own Calculation produces but ½ a Foot, or a little more, for the Height to which the Sun can elevate the Sea. But ſince Meſſrs. *Bernoulli* and *Mac Laurin* have each of them ſeparately inſtituted the Calculus, and reſolv'd the Problem, with ſuch an exact Agreement with each other, and with Sir *Iſaac*, there is no room to doubt of the Truth of it; and it appears to depend on as certain Principles, and more evident than thoſe which Mr. *Euler* has advanced in Contradiction to Sir *Iſaac Newton*.

26. The Power of the Sun to move the Waters has been computed by means of the periodical Times of the Earth and Moon; and after the ſame Manner might be computed the Force of the Moon to diſturb the Waters, were there any Satellite revolving about the Moon to compare its periodical Time with the Moon's periodical Time; but as there is none, we muſt have recourſe to other Methods, the eaſieſt of which is, to collect the Proportion of the Force of the Sun and Moon from the Proportion of their Effects on the Waters of the Sea; that is, by obſerving the Heights to which they riſe when they act together, and when they act contrarily; thus *Sturmius* obſerved at the Mouth of the *Avon*, at the third Stone below *Briſtol*, that the Water in the Syzygies was about 45 Feet, and in the Quadratures about 25 Feet. Alſo at *Plymouth*, Captain *Colpreſs* obſerved thoſe Elevations to be as 20½ to 11½, or 41 to 23. At *Havre de Grace* the Altitudes have been obſerved as 17 to 11. The Mean of all theſe Proportions is 10 to 6. Let S = the Sun's Force, and

$L =$ the Moon's Force; then will $L + S : L - S :: 10 : 6 :: 5 : 3$; whence $L : S :: 4 : 1$, or the Force of the Moon will be about 4 times greater than that of the Sun.

27. And this would be a Determination ſufficiently exact, were the Sun and Moon in the Equator, in their Mean-Diſtances from the Earth; alſo were the Tides to be greateſt and leaſt in the Syzygies and Quadratures, and the Diſtance of the Moon the ſame in each; but ſince all thoſe Things are found otherwiſe in Fact, we ſhall have the abovementiọn'd Proportion varied by them in ſuch manner, that it will become at laſt $1{,}017522\,L + 0{,}7986355\,S : 0{,}9830427 \times 0{,}8570327\,L - 0{,}7986355\,S :: 5 : 3$; or, putting Letters for the Numbers, $a = 1{,}0175$, $b = 0{,}7986$, $c = 0{,}983$, $d = 0{,}857$, we have $aL + bS : cdL - bS :: 5 : 3$.

28. Hence $aL + bS = 5$, and $cdL - bS = 3$; whence $\frac{5 - aL}{b} = S$, which ſubſtituted in the other Equation gives $cdL - 5 + aL = 3$, or $cdL + aL = 8$, whence $L = \frac{8}{cd + a} = 4{,}301$; then $\frac{5 - aL}{b} = 0{,}7808 = S$. Wherefore $S : L :: 0{,}7808 : 4{,}301 :: 1 : 5{,}568$; but if we take the Mean-Proportion of Tides as 9 to 5, as Sir *Iſaac Newton* has done, then $S : L :: 1 : 4{,}4815$. Since the Sun elevates the Water 25 Inches, the Moon will elevate it $25 \times 5{,}508$ Inches $= 137{,}7 = 11\frac{1}{2}$ Feet. The Sum of both Forces ſhould elevate the Water $13\frac{1}{2}$ Feet at the New and Full Moon; and the Difference of the Forces ſhould raiſe the Water $9\frac{1}{2}$ Feet, as in the Quarters; and theſe Heights anſwer pretty well to Obſervations: For in the *Atlantic* and *Æthiopic* Oceans the Tides are found from 6 to 15 Feet Altitude, and generally higher in the great *Pacific* Ocean. In ſhort, that the Phænomena of the Tides, in this reſpect, may anſwer to the Theory, the whole Globe ſhould be cover'd with very deep Water; at leaſt the Ocean ſhould not be leſs than 90 Degrees wide, or one Quarter of the Globe.

Plate XXVII. Fig. 2.

29. The Sum of all that has been ſaid is this: If NOPQ be the Surface of the Earth, T its Centre; IFKGLHCE a Circle repreſenting the ſpherical Surface of the Waters covering the Earth, and affected only with the Gravity of the Earth; upon placing an attracting Body at S at the Diſtance ST, the Waters will no longer continue their ſpherical Figure, but be immediately drawn into a ſpheroidical Figure ACBD, in ſuch manner, as to be depreſs'd at C and D to M and K, and elevated from L and I to A and B; and the Elevation AL or BI is double the Depreſſion CM of

cember 28, the Water aſcended 19 Feet 2 Inches, and on *June* 19 (of the preceding Year) it aſcended but to the Height of 18 Feet 4 Inches: The Diſtance of the Moon, *&c.* being the ſame in each Caſe, which will be found nearly to anſwer to the Ratio of the Cubes of the Diſtances inverſely, or of the apparent Diameter of the Sun directly.

Plate XXVII. Fig. 6.

37. For ſince the apparent Diameter A B of any Body decreaſes with the Angle A C B under which it is ſeen, ſo that at C and D it will be as the Angles A C B and A D B; and theſe Angles decreaſe as their Radii (or the Diſtances) C F, D F, increaſe; it follows that the apparent Diameter A B is every where inverſely as the Diſtance, that is, at C or D we have A B as $\frac{1}{CF}$ or $\frac{1}{DF}$. Whence if the apparent Diameter of the Sun be $= D$, and T S the Diſtance of the Sun, then ſince $D = \frac{1}{TS}$, and the Sun's Force (F) to move the Sea as $\frac{1}{TS^3}$ (as ſhewn in *Art.* 9.) therefore $F = \frac{1}{TS^3} = D^3$; that is, *the Force of the Luminary to move the Sea is directly as the Cube of the apparent Diameter thereof.*

38. As the Force of the Moon to move the Waters is at a Medium about 5 times greater than that of the Sun, which is the $\frac{1}{12867539}$ Part of Gravity; that of the Moon will be the $\frac{1}{2573508}$ Part of Gravity; and therefore by far too ſmall to be ſenſible in the Motion of Pendulums, or any Kind of Static or Hydroſtatic Experiments. In the Motion of the Waters of the Sea only can it produce any ſenſible Effect.

Fig. 7.

39. To eſtimate the Elevation of Waters which the Earth will produce in the Moon; let us ſuppoſe the Quantity of Matter (Q) in the Earth to be to that in the Moon (q) as 40 to 1, that is, $Q : q :: 40 : 1$, and let us firſt ſuppoſe the Earth and Moon of equal Bulk, and repreſented by A I K and B D E, and the Force (F) of the Earth, at the Surface of the Moon B, will be to the Force (f) of the Moon at the Surface of the Earth A, directly as the Maſſes of Matter in each (becauſe of the equal Diſtances T B and L A), that is, $F : f :: Q : q :: 40 : 1$. Again, let L B be to L C as the Diameter of the Earth to that of the Moon, which is as 365 to 100, then will the Force at B be to the Force at C as L B to L C, which let be as F to f, then $F : f :: 365 : 100$, whence $F = \frac{365 f}{100}$; and above we have $F = 40 f$, therefore $4000 f = 365 f$; and ſo $f : f :: 365 : 4000 :: 1 : 11$ nearly; that is, *the Power of the Earth to raiſe the Water at the Moon* C, *is*

to that of the Moon at the Earth A as 11 to 1, very nearly.

40. If then the Moon can raiſe the Water here 10 Feet, the Earth can raiſe the Water there to the Height of 110 Feet; but becauſe the Moon always turns about her Axis in the ſame Time as ſhe revolves about the Earth, and by this Means ſhews always the ſame Face to the Earth, the Waters (if any there be) will be raiſed on this and the oppoſite Side, and always continue over the ſame Part of the Moon's Surface, ſo that there can be no different Heights of Water there, and conſequently no Tides, except what ſmall ones may happen on Account of the Sun's Attraction, and her various Inequalities of Motion, and Diſtance from the Earth.

41. The Air, which is 860 times lighter than Water, muſt yield in Proportion more eaſily to the attracting Force of the Sun and Moon; and therefore the Tides will in this profound and vaſt *Aërial Ocean* be very conſiderable and importing. But of this the Reader may expect a further Account when we come to treat directly of the *Atmoſphere* in the next Lecture.

The End of the FIRST VOLUME.

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